Pricing and hedging GDP-linked bonds in incomplete markets

Stavros A. Zenios
University of Cyprus
The Wharton School, USA
Visting scholar BRUEGEL.

(Joint work with Andrea Consiglio, Univ. of Palermo, IT.)

Outline

- Define the instruments
- A policy question: Are they expensive?
- A model in incomplete markets
- Findings

Consiglio and Zenios (2018) *Journal of Economic Dynamics & Control*, Online Jan. 8.

Define the instruments

Coupon-indexed bonds

Link coupon to GDP growth (Borensztein and Mauro, 2004) by

$$c_t = \max\left[c_0 + (g_t - \bar{g}), 0\right]. \tag{1}$$

Called *floaters* in IMF (2017).

Principal-indexed bonds

Index principal (Kamstra and Shiller, 2009) by

$$B_t = B_0 \frac{Y_t}{Y_0}. \tag{2}$$

Called *linkers* in IMF (2017).

Define the instruments

Sovereign contingent convertible debt (S-CoCo)

Instrument with a built-in trigger to allow standstill of payments when an indicator breaches a threshold.

Brooke et al. (2013); Consiglio and Zenios (2015)

Debt vs Equity financing for sovereigns.

Are GDP-linked bonds expensive?

- Reduce probability of default though countercyclical payments or reduced nominal value of debt
- Carry a premium which increases debt level
- Benefits of reduced default probability justify premium?
- What about advanced economies when probability of default is essentially zero (neglected risk)?

Think of these are *insurance premia*. Insure what and why?
(Joint with M. Demertzis, Bruegel.)

Are GDP-linked bonds expensive?

- Others estimated appropriate premia for sovereigns to benefit.
- We use pricing in incomplete markets to determine what the premia will be.
- Thresholds

  $\Rightarrow$ Emergine economics 250–350bp

  $\Rightarrow$ Advanced economies 50–100bp

Pricing issues for GDP-linked bonds

- Two approaches to priceing (determine premium with respect to a benchmark)
  - CAPM–like approach, where market equilibrium is assumed and use CAPM to obtain premia
  - Contingent claim approach
- CAPM-like approach faces the difficulty to estimate the sensitivities with respect to (which?) market index
- Contingent claim approach looks more robust, but there is an intrinsic obstacle in the non–tradeability of GDP

Pricing in incomplete markets

- We consider the GDP-linked bond as a contingent claim
- Black & Scholes fails since underlying not traded
- B&S price is the cost of hedging the option, i.e., selling and buying the underlying to make the process risk-free

The Super-Replication framework

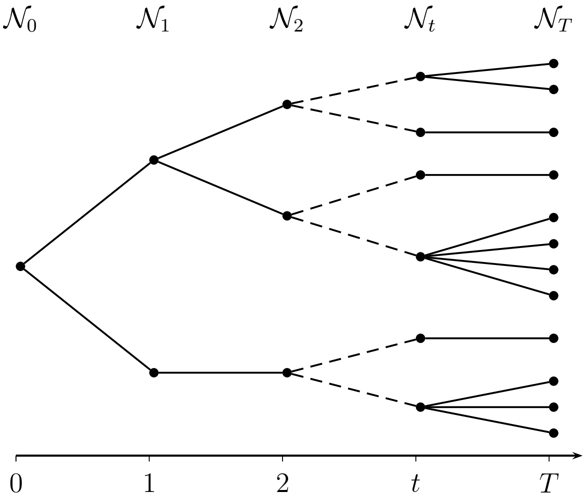
Buyers price is maximum amount investors are willing to pay to purchase cashflow F without risk of having negative terminal wealth. This is a stochastic linear programming problem on a *scenario tree* (King, 2002):

$$\underset{V,\theta}{\text{Maximize}} \quad V$$

s.t.

$$\begin{aligned} S_0 \cdot \theta_0 &= F_0 - V & \\ S_n \cdot (\theta_n - \theta_{a(n)}) &= F_n & (n \in \mathcal{N}_t, t \geq 1) \\ S_n \cdot \theta_n &\geq 0, & (n \in \mathcal{N}_T). \end{aligned}$$

Scenario trees



Moment-matching arbitrage-free scenario trees

- Multi-assets with specified expected value, standard deviation, skeweness and kurtosis
- Assets are not independent and linear correlation is used to describe dependency
- Arbitrage-free

Consiglio et al. (2016) A parsimonious model for generating arbitrage-free scenario trees. *Quantitative Finance* 16:201–212, 2016.

The Super-Replication framework

- Stochastic linear program on scenario tree for sellers price
- Buyer and seller prices coincide in complete markets, else we have a spread
- Dual prices to estimate risk premium
- Risk premium is endogenous so model is internally consistent
- Model chooses the best market portfolio, does not assume it

Calculating the risk premium

Consider a stochastic payoff x with price $\mathcal{P}_0$ obtained from

$$\mathcal{P}_0 = \frac{\mathbb{E}_P(x)}{1 + r_f} + \text{cov}(m, x). \quad (3)$$

r_f is the risk free rate of return and m is the stochastic discount factor, with $1 = \mathbb{E}_P((1 + r_f)m)$.
$\mathcal{P}_0$ computed under the risk neutral probability measure is

$$\mathcal{P}_0 = \frac{\mathbb{E}_Q(x)}{1 + r_f}. \quad (4)$$

Combining (3) and (4) we write the risk premium as

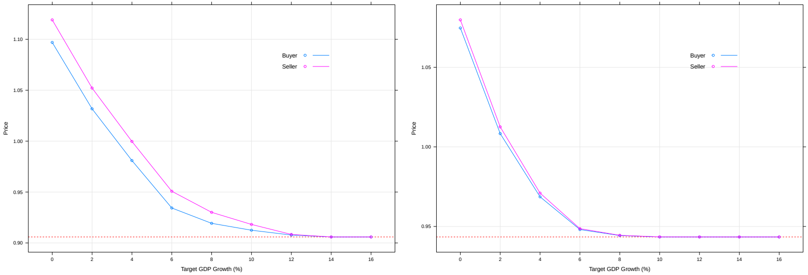
$$\text{cov}(m, x) = \frac{\mathbb{E}_Q(x)}{1 + r_f} - \frac{\mathbb{E}_P(x)}{1 + r_f}. \quad (5)$$

Calibrations

Calibrate arbitrage-free tree using asset returns from the Global Returns Data (Dimson et al., 2002) and GDP data from Schularick and Taylor (2012).

- US
- UK
- Germany
- Italy
- South Africa

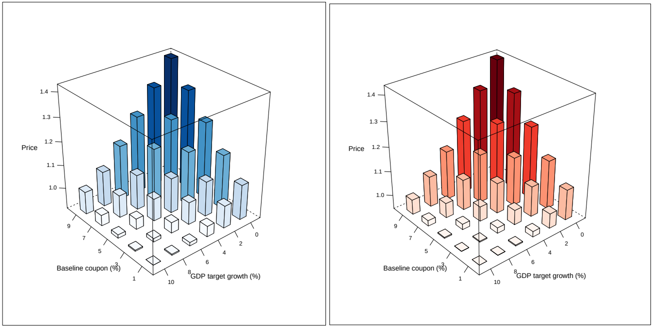
Prices and bid-ask spreads



(a) UK (b) US

Figure: Buyer and seller prices for coupon-indexed bonds.

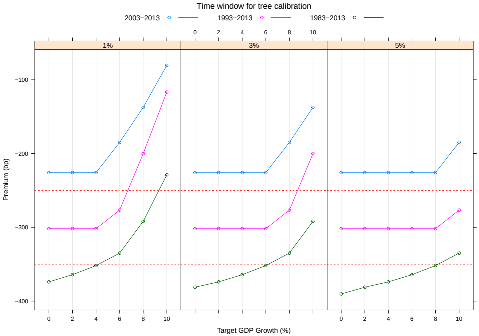
Price sensitivity to design parameters



(a) UK (b) US

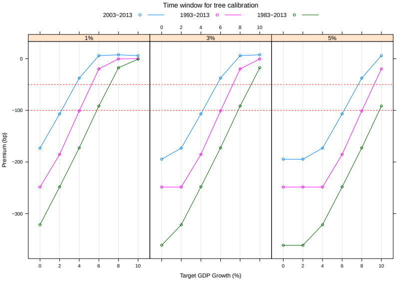
Figure: Buyer price sensitivity to changes in base coupon and target growth for coupon-indexed bonds.

Premia and thresholds for emerging markets

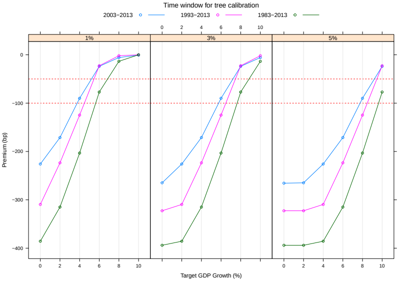


(a) South Africa

Premia and thresholds for advanced economies

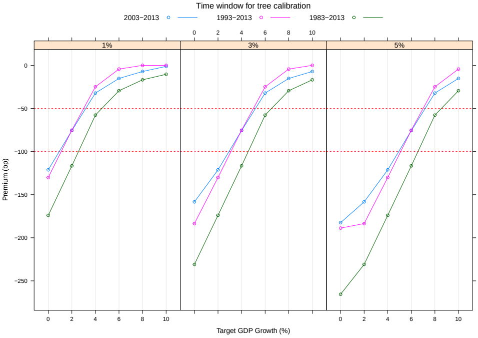


(b) UK

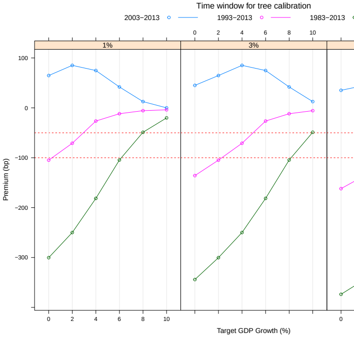


(c) US

Premia and thresholds for advanced economies

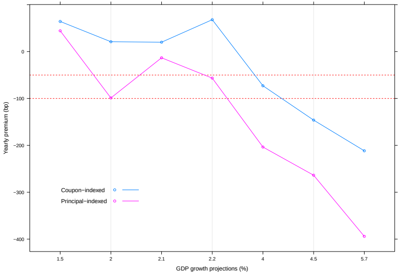


(d) Germany

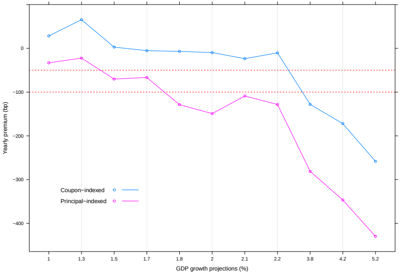


(e) Italy

Coupon-indexed or principal-indexed bonds



(f) UK



(g) US

Conclusions

1. Risk premium sensitive to bond design
2. GDP-linked bond designs exist that can be beneficial for sovereigns
3. Risk premia and prices are sensitive to input data. Our conclusions are robust when tested using different calibration windows for the US and Germany, less so for the UK, and depend crucially on the calibrated correlations for Italy and South Africa.

Conclusions

1. Germany and US: a broad range of bond design parameters with attractive risk premia. For US, a coupon-indexed bond with base coupon 1% and target growth 4% has risk premium 90bp–140bp. For Germany the same design carries a premium less than 50bp for the most recent calibrations and slightly larger in the long run.
2. UK: bond with coupon 1% and target growth 4% has risk premium 50bp to 100bp for the more recent calibrations, and as large as 175bp for the longer time window. For coupon 3%, risk premium increases to 105bp for the more recent calibrations and 250bp for the longer time windows, so it is important to determine with higher accuracy the correlations. Similar conclusions for Italy.

Conclusions

1. South Africa: several designs within the 350bp threshold, and even within the 250bp threshold when calibrated on recent data. When calibrated over the long-term horizon, the premia for reasonable designs do not satisfy the threshold. If we believe that the correlations calibrated using the most recent data reflect the country's future prospects, then GDP-linked bonds are beneficial.

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