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Abstract

Credit risk-free long-term interest rates can typically be decomposed into two components: expectations of the future path of the short-term policy rate and the term premium. Changes in term premium are considered to have been an important driver behind developments in long-term bond yields in recent years. As policy rates of major central banks approached their effective lower bound in the aftermath of the global financial crisis, their ability to provide the necessary degree of monetary stimulus using conventional policy measures became very limited. In this particular context, central banks had to move beyond conventional policy instruments and instead deploy a set of unconventional tools (such as large-scale asset purchase programs and forward guidance) that were tailored to target the longer-end of the yield curve. There is a growing body of empirical evidence suggesting that these unconventional measures turned out to be effective in compressing the term premium component of interest rates. This paper, after providing a definition of the term premium and a succinct overview of different ways to measure it, presents the empirical results obtained from calibrating a Gaussian affine term structure (GATSM) based term premia model to the euro area. In addition to discussing the GATSM model's assumptions and specifications, it also describes the calibration algorithm employed, which is based on genetic algorithms. Thereafter, it provides some insight into the time profile of the euro area term premium in the post global financial crisis (GFC) era and in particular how it has evolved after key ECB policy decisions since 2008.

JEL Classification: E43, E44, E52, E58.

Keywords: monetary policy, term premia, term structure models, genetic algorithms.

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CONTENTS

1. INTRODUCTION

The term structure of interest rates can be a valuable source of information for central bankers as it provides insights into market expectations about the economy and their evolution in response to changes in economic conditions. The yield curve can also serve as a tool to assess the impacts of central bank policy decisions and communications. However, a proper interpretation of yield curve information by central banks requires separating expectations of future short-term policy rates from the so-called "term premium" component embedded in interest rates. The term premium reflects the excess return that an investor demands as compensation for holding a bond with a long maturity (for example a bond maturing in 10 years) relative to rolling over a short-term bond until this long-dated maturity[1]. Locking into a long-dated fixed income investment for a period of time is not equivalent to rolling over a short-term investment for the same period, because holding the long-dated bond exposes the investor to the risk that interest rates may increase unexpectedly during the holding period. An unexpected increase in interest rates causes a market loss on the investment position in fixed-rate securities. As will be explained in further details later on, the term premium compensates investors for taking on such interest rate or duration risk[2].

In recent years, understanding the term premium has attracted considerable attention. Indeed, explicit reference to the term premium – which was previously considered a rather obscure part of academic jargon – has become commonplace in policy discussions and central bank communications. This may be linked to the fact that the decade following the Global Financial Crisis (GFC) of 2008 has been marked by historically low interest rates, even at the long end of the yield curve. The observed gradual decline in benchmark 10-year government bond yields across major economies has generated much interest in trying to understand both the source of this decline as well as its economic implications. To shed more light on the observed decline in long-term yields, a growing number of academics have resorted to use dynamic term structure models which divide yields into these two components – a policy rate expectations component and a term premium component. The expectations component reflects the average of current and future expected short-term policy rates over the maturity of the bond. If the pure expectations hypothesis of the term structure were to hold, this would be all that mattered in terms of explaining movements in long-term rates. But broad empirical[3] evidence suggests that the pure expectations hypothesis fails to hold true in practice and that, in addition, there exists a time-varying premium that investors require in order to hold a long-term bond instead of simply rolling over a series of short-term bonds. This has implication for the conduct of monetary policy as its transmission (in particular of unconventional policy measures) depends not only on the expectations component, but also the term premium component.

Accordingly, a central bank can seek to influence the level and shape of the yield curve by acting on two components of the long-term interest rates: the expectations component and the term premium. The expectations component reflects market expectations of the future path of the short-term policy rate. All else equal, a path that is lower and shallower tends to produce a lower level of long-term yields and a flatter yield curve. Since the onset of the GFC, the ECB has adopted a series of unconventional monetary policy measures to bring inflation rates back to levels below, but close to, 2% over the medium term. These measures have included targeted longer-term refinancing operations (TLTROs), lowering the deposit facility rate into negative territory, and an expanded asset purchase program (APP) targeting a variety of investment-grade private and public sector securities. This set of measures has been underpinned by forward guidance on the key ECB interest rates which was first

[1] This definition of the term premium is commonly employed in the literature as well as in central bank speeches (see for example the ECB speech of Peter Praet "Maintaining price stability with unconventional monetary policy measures", MMF Monetary and Financial Policy Conference, London, October 2017).

[2] Duration risk refers to the risk associated with the sensitivity of a bond's price to a one percent change in interest rates. The higher a bond's duration, the greater its sensitivity to interest rates changes.

[3] See for example Sarno, Thornton and Valente (2003) who provide empirical evidence that the expectations hypothesis fails to hold in practice.

introduced in July 2013 when the ECB's Governing Council stated that it expected interest rates to remain low for an extended period of time. Since then the ECB's forward guidance has been adapted on a number of occasions such that nowadays it clarifies the Governing Council's intentions not only with respect to the expected future path of short term policy rates, but also with regard to the horizon of its asset purchase program.

This paper presents the empirical results obtained from calibrating a Gaussian affine term structure (GATSM) based term premia model to the euro area and it is organised as follows: Section 2 provides a non-technical overview of what the term premium is, what the different ways to measure it, and what are the broad classes of models to extract term premia. A particular focus is given to the class of Gaussian affine term structure models (GATSM) and how these can be used to extract the term premium component. Section 3 discusses how to calibrate the GATSM-based term premia model to yield curve data as well as the technical challenges and hurdles which need to be considered. Section 4 then goes on to present the results obtained from calibrating the term structure based term premia model to the euro area (EA) as well as to the US. In Section 5 we discuss the time profile of the EA 10-year term premium in the post GFC era and in particular how it evolved after key ECB unconventional monetary policy decisions were being implemented. Finally, Section 6 concludes the paper and Section 7 (technical appendix) provides a succinct technical overview of the GATSM-based term premia model presented in this paper.

The paper is designed in such a way as to appeal to two groups of readers: (1) those with prior knowledge of term-structure modelling and term premia models and (2) readers without such technical background knowledge. The latter group might primarily be interested in the practical application of the term premia model to the fields of macroeconomics, monetary policy and financial markets (bond pricing, interest rate risk management …). This group of readers is invited to start by reading Section 2 which provides a non-technical overview of the term premia model presented in this paper, and then subsequently jump to Sections 4 and 5 which in turn discuss the empirical results obtained from the term premia model (both for EA and the US) as well as how well the 10-year term premium in the EA responded to key monetary policy events. The first group of readers, with a background in term-structure modelling and knowledge of the term premia literature, will be essentially interested in the mathematical underpinnings of the term premia model presented in this paper as well as the calibration method proposed (based on genetic algorithms). This group of readers is thus invited to skip the non-technical review and directly start with Section 3, which discusses the calibration algorithm in more detail in addition to the yield curve dataset used for the calibration, followed by Section 4, which discusses the calibration results obtained. Section 7 in addition offers a succinct technical overview of the GATSM-based term premia model presented in this paper. As the paper is designed to address both readers with and without knowledge of term-structure modelling, reiterations of certain key aspects throughout the paper are inevitable.

2. A NON-TECHNICAL OVERVIEW OF THE TERM PREMIA

2.1 WHAT IS THE TERM PREMIUM?

The oldest and most debated theory underlying the term structure of interest rates is the expectations hypothesis. According to this theory, the expected return from holding a long-term bond until maturity (for example expiring in 10 years) should be the same as the expected return from rolling over a series of short-term bonds (for example each having a one year maturity) such that the total rollover maturity is equal to that of the long-term bond. Another equivalent way to re-express this hypothesis is to say that the long bond yield is equal to the average of the expected short-term rates, or also that the forward rate (the interest rate which investors fix now to borrow or lend in the future) should be equal to the interest rate expected to prevail in the future. Empirically, the expectations hypothesis has largely failed to fully explain the behavior of interest rates. Several academic papers including Fama (1984), Fama and Bliss (1987), Campbell and Shiller (1991), Stambaugh (1988), Cochrane and

Piazzesi (2005), Sarno Thornton and Valente (2003), amongst others, have pointed out evidence in support of non-zero and time-varying risk premia in bond markets, thus violating the expectations hypothesis.

Although the expectations hypothesis provides a simple and intuitively appealing interpretation of the yield curve, it ignores one very fundamental reality which investors must face in financial markets: interest rate risk. Except if held until maturity, the nominal return on a long maturity bond is uncertain (i.e. the market price of a long-term bond can vary a lot even from one day to the next), and investors require compensation for bearing this risk. The "term premium" refers to such compensation and any other sources of deviation from the expectations hypothesis. In addition to pure market interest rate risk, there could exist other factors influencing the term premium, such as for example the lack of liquidity of some traded bonds, credit risk, or even the particular investor behavior in some bond markets (referred to as the preferred investor habitats[4]). An even more common example is the "flight to quality" effect in some major (credit risk-free) government securities markets at times of economic turmoil. News on important geopolitical events, for instance, might induce a particularly strong demand for relatively safe assets, temporarily pushing down bond yields and thus compressing the term premium component.

Before turning to a more formal definition of the term premium, the starting point is that interest rates can be decomposed into two components: the expectation of the future path of the short-term policy rate[5] and the term premium. In the absence of the other factors mentioned above, the term premium is the compensation that investors require for bearing the risk that short-term rates do not evolve as they expected (i.e. the pure interest rate risk). This elementary decomposition of interest rates into its two components serves to define what is commonly referred to as the "yield" term premium:

Yield-to-maturity of the long-term bond = [Average of the expected future short-term policy rates from the present to the maturity of the long-term bond] + [Term premium].

Or

Term premium = [Yield-to-maturity of the long-term bond] – [Average of the expected future short-term policy rates from the present to the maturity of the long-term bond].

It is now worth considering a more formal definition of the "yield" term premium which not only makes a link to forward rates but also to the so-called "forward" term premium. Suppose that we observe at current time t a tradeable bond with a time to maturity of T years. Let us further define the following variables and a short rate horizon period defined to be 1-day for example (corresponding to the ECB's deposit facility rate, DFR, which is an overnight facility):

$TP_{t,T}$ = yield term premium for a yield with maturity T years from today.

r_t = 1-day rate or "short rate" (i.e. corresponding to the ECB's DFR for example).

$f_t^{t+i-1,t+i}$= today's forward short rate[6] for the 1-day period going from $t+i-1$ to $t+i$ (where i denotes some arbitrary future point in time prior to the maturity date of the bond).

[4] The preferred investor habitat refers to a theory on the investing behavior of bond buyers, stating that individual investors have a preferred range of bond maturity lengths, and will only go outside of this range if a comparatively higher yield is promised.

[5] Throughout the paper, the terms "short rate" and "short-term rates" refer to short-term central bank policy rates (i.e. corresponding to the ECB's deposit facility rate for example).

[6] A forward rate agreement (FRA) is a bilateral over-the-counter contract directly negotiated between two parties fixing the rate of interest on a notional loan or deposit for a period of time in the future. Interest rate futures are the exchange-traded equivalents of FRAs. An example is the EURIBOR 3M future being traded on the ICE electronic platform. However,

N = number of days until the bond's maturity which is in T years from today.

Today's "forward" term premium for the period (one day in our example) going from $t+i-1$ to $t+i$ can be defined as:

$$TP_t^{t+i-1,t+i} = f_t^{t+i-1,t+i} - E_t\left(r_t^{t+i-1,t+i}\right)$$

In other words, for each forward period (i.e. day in our example) into the future until the maturity of the bond, there exists a forward term premium specific to that forward period. As soon as a given forward term premium takes on a value which is different from zero, it implies a departure from the expectations hypothesis. What if we calculate the average of the forward term premiums over all of the days until the maturity T of the bond? This average forward term premium is nothing else but the so-called "yield" term premium:

$$TP_{t,T} = \left(\frac{1}{N}\right) * \sum_{i=1}^{N-1} TP_t^{i,i+1} = y_{t,T} - \left(\frac{1}{N}\right) * \sum_{i=0}^{N-1} E_t(r_{t+i})$$

where $y_{t,T}$ is the market's quoted yield-to-maturity of the bond.

By rearranging the above equation, we obtain the definition for a bond's yield-to-maturity as broken-down into its two components:

$$y_{t,T} = \left(\frac{1}{N}\right) * \sum_{i=0}^{N-1} E_t(r_{t+i}) + TP_{t,T}$$

According to the above equation, the yield-to-maturity of a bond can thus be broken down as the sum of the average expected short-rates (until maturity of the bond) plus a residual component. As previously mentioned, this residual component captures interest rate risk (hence is commonly referred to as the term premium); however, depending on the specific type of instrument being traded, it can in addition embed other factors such as credit risk (i.e. the case of Greek government bonds), liquidity constraints and/or flight-to-quality effects. Moessner (2018) calculates term premia for individual euro area countries and shows that term premia includes liquidity and credit risk premia for some countries[7]. For instance, term premia in Greece, Portugal and Ireland rose strongly during the euro area sovereign crisis of 2010-2011, as concerns about sovereign risk in peripheral countries increased. One can obtain a "cleaner" estimation of the term premia (in the sense of covering only risks related to changes in short-term rates) by running the model calculations on the German government curve for instance, as German government bond yields are typically considered a good proxy for euro area risk-free interest rates and markets for German government bonds are generally very liquid[8]. One would then expect that the term premia extracted from the German government curve should reflect solely (or almost) interest rate risk. Recall that this is the risk that short rates will not evolve as initially expected, which is nothing else than to say that future short rates are random variables (statistically speaking) which can be characterised by a probability distribution.

To illustrate this in a graphical manner, consider Graph 1 below with a purely hypothetical example of how the short rate might be projected to evolve in the future. For each date in the future, the short rate

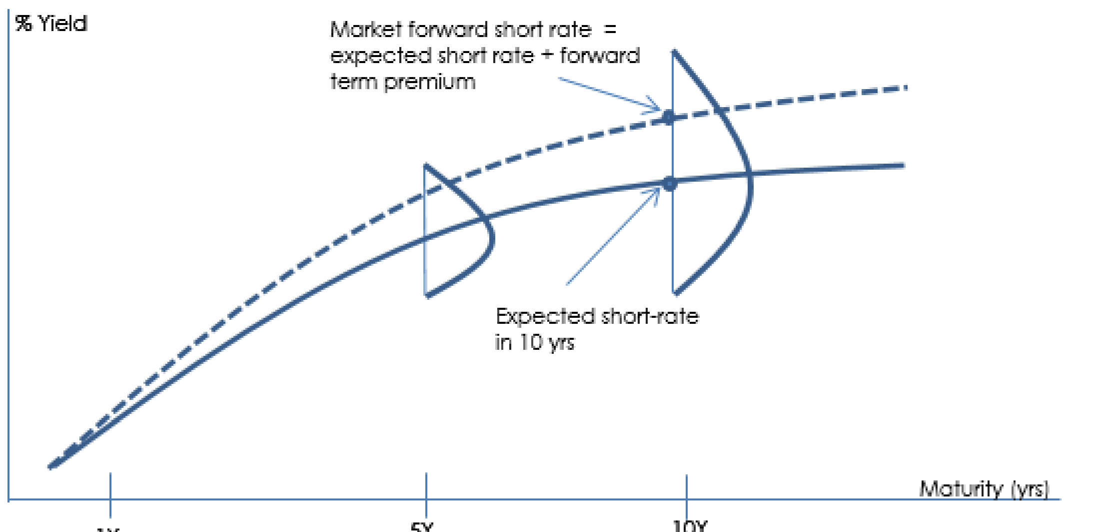
tradeable products and markets in forward rates typically exist only for a few interest rate index tenors (usually 3M, 6M, 12M) and when there exist no such tradeable contract, the forward rate is implied or calculated from the zero-coupon yield curve using GATSM (which is the case in this paper).

[7] Calibrating GATSM to other government bond markets so as to capture these other elements (i.e. credit risk premia) is outside the scope and aim of this paper which focuses exclusively on interest rate risk.

[8] Credit/default risk is rather minimal, even almost inexistent, for German government bonds while liquidity is typically considered very high.

(unknown today) is “expected” to evolve according to the solid blue-colored curve. Therefore, for each date in the future, the expected short rate can be graphically depicted as the mean of a probability distribution. Moreover, as the graph illustrates, the longer the horizon (10 years for example), the more widespread is the distribution around the expected short rate and thus the higher is the interest rate uncertainty or risk. This makes intuitive sense, as it is more difficult to make a forecast today of where the short rate will be in 10 years from now versus where it will stand in 1 year from now. Investors will thus require compensation for this uncertainty, which should be proportionally greater at the 10-year horizon as compared to the 1-year or even 5-year horizon. One would thus expect the market’s forward short rate in 10-years to embed a bigger forward term premium than the corresponding forward term premium for a forward short rate in 1-year from now. The resulting forward short rates (each embedding a forward term premium) are depicted by the dotted blue-colored curve, which lies above the solid blue-colored curve due to the (gradually rising) forward term premiums. If one were to calculate the average of all the forward term premiums up to the 10-year maturity, this would correspond to the 10-year “yield” term premium.

Graph 1: **Expected short rates and forward term premia**



Now that the basic notions underlying the yield term premium have been covered, the next section will very succinctly delve into the complicated and much debated task of how to extract the yield term premium for the market's interest rate curve.

2.2 WHAT ARE THE DIFFERENT WAYS TO EXTRACT THE TERM PREMIUM?

At first glance, it would appear trivial to measure the term premium using the definition provided in the previous section, but in reality the task of estimating financial markets’ expectations about the future course of short-term interest rates over a fairly long horizon is technically challenging. Indeed, the key concept underlying the term premium is the uncertainty surrounding investor expectations about the future course of short-term interest rates over the lifetime of the long-term bond. The fact that such uncertainty is not observable foreshadows some of the difficulties in measuring term premia that many researchers have had to deal with over the past years. Three broad classes of methods have emerged to try to gauge the term premia: (1) term structure based models, (2) regression based models and (3) estimates based on survey data. The three approaches are briefly introduced in this section (as it is not the aim of this paper to provide an in-depth technical comparison of the various existing models of term premia).

The first class of models, dynamic term structure models, or the so-called “no-arbitrage models”, has been increasingly used to extract short rate expectations and term premiums from the observed market yield curve. The no-arbitrage concept implies, among other things, that securities with the same risk characteristics (same payoff in all states of the world) should have the same price. This condition constrains the way bond yields of various maturities can move relative to one another, simplifying the formulation of the dynamics of the entire yield curve. A workhorse amongst the no-arbitrage models is the so-called Gaussian affine term structure model (GATSM). An affine term structure model is a financial model that relates zero-coupon bond prices (i.e. the yield curve) to a model for the short rate. It is particularly useful for deriving the zero-coupon yield curve from quoted bond prices[9]. “Affine” means that the bond yields depend linearly on the risk factors. Although the assumption of linearity may appear simplistic at first glance, when the risk factors[10] are defined as unobserved (statistical) variables, such a specification can accommodate a rich array of possible term structure models (such as the Nelson Siegel family of yield curve models). “Gaussian” refers to the distributional assumption for the risk factors, which also helps to simplify the yield dynamics considerably. Krippner (2012) calibrates an arbitrage-free Nelson Siegel term structure model to US yield curve data and subsequently provides an analytical framework to extract the term premia component.

Because term structure models have a tendency to capture the high persistence of yields, reflecting their propensity to be highly correlated over time[11], some researchers have embedded survey data and even macroeconomic factors in their term structure models. Kim and Wright (2005) for example include survey data on future three-month interest rates into their term structure model to estimate US yield curve dynamics. Other models incorporate macroeconomic variables into the term structure model and typically the choice of these macroeconomic variables is based on what investors typically look at when investing in bonds. One example is the Hordahl and Tristani (2014) model, which incorporates data on nominal and real (index-linked) interest rates, the output gap (as a measure of economic slack) as well as survey data on both future short-term interest rates and future inflation rates. Another term structure based model of the term premia embedding survey data is the one used by the Bank de France (BoF) based on Monfort et al (2017). The authors build a new class of affine term structure models which is able to accommodate a short-term rate that stays at the zero lower bound (ZLB) for extended periods of time while longer-term rates feature high volatilities[12].

The second class of models used to produce measures of the term premia is the family of regression based models. Under the joint assumption of the expectations hypothesis and rational expectations (i.e. expectations that are unbiased and incorporate all available information), the difference between the forward short rate and the ex post realised short rate should not be forecastable with ex-ante variables (i.e. variables available when the expectations were formed). If, in fact, ex-ante variables help to predict this difference, it would imply the presence of a term premium or in other words a failure of the expectations hypothesis. In such a case, one may then use the predictable component of the rate difference resulting from the regression as a measure of the term premium. The regression of the forward rate minus the ex-post realised short-term rate on explanatory variables nests several well-known models, and the most popular within this category is the ACM term premium model of Adrian, Crump and Moench (2013). Basically, their approach takes the risk factors to correspond to the first few principal components of the observed yield curve data, and then models the factor dynamics as a classical vector auto-regression model. ACM show that the parameters of the term structure model are

9 Typically, quoted bond prices are available for certain maturity dates (most often 1Y, 2Y …10Y, 20Y, 30Y), and hence the GATSM enables one to obtain calculated yields for other (intermediate) maturity dates.

10 The underlying risk factors are also commonly referred to as latent factors or state variables and hence this terminology will be used interchangeably throughout the paper.

11 Interest rates usually exhibit a strong persistence and it can take a long time before they revert to a long-term equilibrium level. This shows up in the poor statistical evidence of mean reversion; see for example Willem van den End (2011) who provides statistical evidence on the persistence of interest rates.

12 They introduce of a new univariate non-negative affine process called Autoregressive Gamma-zero (ARG) and its multivariate affine extension (VARG) which assumes that the factors follow a Gamma distribution instead of a Gaussian distribution [see Monfort et al 2017 for further details].

then obtained in three steps using standard OLS regressions. ACM find evidence in favor of a five-factor model, and subsequently use this as their baseline specification.

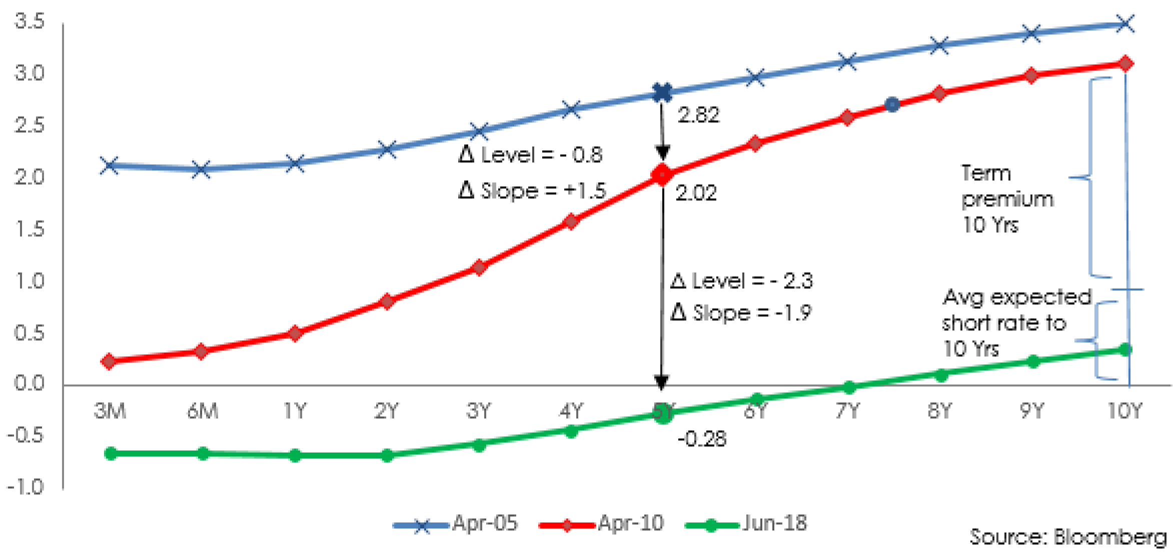
A third alternative approach to the term structure and the regression based estimates of the term premia is to use survey forecasts of financial market participants as a model-free proxy for "expected" future short rates (i.e. the solid blue-colored line in Graph 1 which corresponds to the mean of the forward rate distribution and thus excludes the forward term premium component). The forward premium in this case is then simply calculated as the market quoted forward short rate minus the expectation of the short rate implied by the survey. However, there are several caveats to using this approach which include namely the risk that participants in the survey base their estimations on observable/quoted market forward rate data which already embeds a forward term premium component. Furthermore, there is also the issue of data quality or rather the lack of reliable survey data on future short policy rates for the euro area. Overall, the literature using forecast surveys to estimate term premia is scarce and more commonly survey data are usually incorporated into the model-based approaches to enhance the estimation procedures.

Since this paper presents the results of calibrating a term structure based term premia model, Section 2.3 below will provide the reader with further insight on the basic notions underlying Gaussian affine term structure models (GATSM) and how they can be used to extract the term premium component of interest rates.

2.3 A GATSM-BASED TERM PREMIA MODEL

The first step towards acquiring an understanding of the term premia model presented in this paper is to grasp the basic idea underlying term structure models of the yield curve (commonly referred to as Gaussian Affine Term Structure Models or GATSM). The term structure of interest rates refers to the relationship between the yields-to-maturity of a set of bonds and their times-to-maturity. It is a simple descriptive measure of the cross-section of bond prices observed at a single point in time. An affine term structure model hypothesises that the term structure of interest rates at any point in time is a mathematical function of a small set of common state variables or factors. Furthermore, these term structure models compress a large amount of cross-sectional and time series yield information into the behavior of a reduced number (typically two or three) of unobserved factors. The dynamics of these factors determine the shape of the yield curve at each point in time (i.e. the cross-sectional dimension) as well as through time (the times series dimension). Once assumptions regarding the dynamics of the underlying factors are specified, the dynamics of the entire term structure are also determined.

Graph 2: **German Government Yield Curve Evolution**



Source: Bloomberg

To illustrate the concepts, consider the example in Graph 2, which shows the evolution of the German government yield curve from April-05 to June-18. As one can observe directly from the graph, the evolution of the curve from April-05 to June-18 consisted basically of a downwards parallel shift, while the (intermediate) evolution of the curve from April-05 to April-10 involved a steepening of the curve in addition to an overall reduction in rates across the maturity spectrum. So the question to be answered with the help of the example depicted in Graph 2 is how can the GATSM explain the evolution of the curve across time (i.e. the time dimension) and how can the model also pinpoint the shape of the curve at a given point in time (i.e. the cross-sectional dimension)?

Firstly, regarding the evolution in time of the yield curve, a few words on what these factors represent will help to shed more light on the link between the evolution of the extracted factors in time and the associated evolution of the yield curve. The GATSM model employed in this paper extracts two underlying factors which are generally deemed to be associated with the "level" and "slope" components of the yield curve. In essence, the GATSM used in this paper captures the time dynamics of the entire term structure of interest rates by extracting these "level" and "slope" components from historical yield data and thereby generates a time series of "level" and "slope" coefficients[13]. Referring back to Graph 2, once a GATSM is calibrated to historical German government bond yields and the two underlying factors have been extracted, it is sufficient to know the values of these two factors in April-05, April-10 and June-18 to explain the observed movements in the yield curve (i.e. parallel shifts and/or steepening/flattening). To understand why this is so, consider two proxies for the two factors (level and slope factors) extracted using our model, which are computable directly from the yield curve:

Proxy for the factor "level" = $y_t(60)$ in basis points

Proxy for the factor "slope" = $y_t(120) - y_t(3)$ in basis points

where $y_t(m)$ refers to the zero-coupon yield (expressed in basis points) of maturity m (in months). The choice of the proxy for the "slope" as being equal to the difference between the 10-year and 3-month yields is rather common while the choice of a proxy for the "level" is more variable across empirical studies[14]. To simplify the current example the proxy for the "level" component was chosen to be the 5-year yield (i.e. mid part of the curve). This particular proxy for the "level" component is chosen for didactical purposes (i.e. to simplify the illustrative example) but it nevertheless remains a valid proxy, as evidenced by the correlation coefficient between this proxy variable (the 5-year yield) and the level factor which is 0.91. The correlation coefficient between the proxy for the "slope" and the slope factor, which is 0.95, validates the choice of this second proxy as well[15].

Reverting back to Graph 2 and using the market proxies for the factors, one can now associate the changes in the values of the two factors to the observed changes in the yield curves. Visually from the graph, one can perceive that the change in the yield curve from April-05 to April-10 was comprised of both a downwards parallel shift combined with a steepening of the yield curve. This is evidenced by change in the values of the two factors: the "level" factor has decreased by 80 basis points while the "slope" factor has increased by 150 basis points. The evolution of the yield curve between April-10 and June-2018, characterised by both a downwards parallel shift combined with a flattening this time, is also corroborated by the change in the two factors. The factor for the "level" has decreased by 230

[13] In other words, the estimated coefficients are time varying; at each point in time (i.e. each end-of-month) where we observe a market yield curve, the GATSM generates "level" and "slope" components specific to that point in time.

[14] See for example Alfonso and Martins (2010) who define market proxies for the "level" and "slope" as being, respectively, equal to the simple average of 3, 24 and 120 month yields (level component) and to the difference between 120 month and 3 months yields (slope component).

[15] Alfonso and Martins (2010) obtain correlation coefficients of a similar magnitude: 0.86 for the correlation between their extracted factor for the "level" and their market proxy for the level (defined as the simple average of 3, 24 and 120 month yields) and 0.93 for the correlation between their extracted factor for the "slope" and their market proxy for the slope (defined as the the difference between 120 month and 3 months yields).

basis points and the factor corresponding to the "slope" has decreased by 190 basis points (representing the flattening of the curve). Overall, in the period between April-05 and June-18, the evolution of the curve was essentially a downwards parallel shift which is also substantiated by the change in the factors between those two dates: the "level" factor has decreased by 310 basis points largely outweighing the change in the "slope" component which has fallen by only 40 basis points (confirming what one can visually observe in the Graph 2). As this simple example demonstrates, the GATSM extracted factors represent the underlying source of uncertainty in the model of the term structure, as the changes in the factors account for the changes and evolution of the yield curve in time.

Secondly, regarding the shape of the yield curve at any moment in time (cross-sectional dimension), consider again the red-colored curve in Graph 2 where the red squares represent the observed yields in April-10 for maturities 3M, 6M, 1Y, 2Y… 10Y. The question is how can the GATSM help to obtain the red-colored smooth line which connects these red-squares (i.e. the observed market yields)? GATSMs allow us to calculate any point on the yield curve depicted by the smooth red-colored line and even extrapolate below the 3M rate for example to obtain the 1week rate or the overnight rate or beyond the 10Y rate to calculate the 20Y or even 30Y yields. The starting point is to realise that at any given point in time the two factors extracted by the GATSM provide us with a general indication of the overall "level" and "slope" of the yield curve on that date. However, this is not sufficient for the GATSM model to pinpoint the exact shape of the yield curve (which is not linear and typically includes some concavity and/or convexity). In order for the GATSM to be able to generate a yield curve characterised by a more complex shape and which fits the observed market yields at any given point in time, some assumptions need to be made regarding the behavior of the short policy rate that drives the entire yield curve. Loosely speaking, the evolution of the short rate in time is assumed to be composed of a long-term trend term and a variance diffusion term which accounts for random market shocks. Moreover, the drift term not only includes a long-term mean parameter but also a mean-reversion parameter. In other words, when the level of the short rate deviates too much from its long-term mean, it will revert back to this mean at a speed governed by the mean-reversion parameter. This mean-reversion process is hampered in its ability to get back to its long-term mean level due to the diffusion or shock term. Once the GATSM model has been calibrated to pick up on the trend and mean-reversion behavior (i.e. parameters) of the short rate, this information is combined with the overall indication of the "level" and "slope" (as provided by the value of the extracted factors) in order to obtain each of the three curves depicted in Graph 2.

Now that the basic mechanics of the GATSM have been explained with the help of an illustrative and non-technical example, let us now revisit the definition of the term premium presented in Section 2.1 in the context of a typical term structure model. Recall the basic definition of the term premium which is given by:

$$\mathrm{TP}_{\mathrm{t,T}} = y_{t,T} - \left(\frac{1}{\mathrm{N}}\right) * \sum_{\mathrm{i=0}}^{\mathrm{N-1}} \mathrm{E}_{\mathrm{t}}^{\mathbb{P}}(\mathrm{r}_{\mathrm{t+i}})$$

The only difference with the formulaic definition presented in Section 2.1 is the appearance of the $\mathbb{P}$ in the expectations operator. To calculate the term premium with a GATSM model, one needs to extract what is known as the "physical" process for the short rate. The "physical process", denoted by $\mathbb{P}$, means that we extract from historical yield data, using GATSM, financial markets' expectations about the future course of short-term interest rates. This implies that the term premium for a given maturity date is equal to the difference between the observed market yield for that maturity and the average of expected short rates up to that maturity, according to the actual expectations that economic agents have about the future values of the short rate (i.e. the physical process for the short rate)[16].

[16] See Section 7.1 for a more detailed and technical discussion on what is meant by extracting the "physical" process for the short rate, which is a key concept underlying the calculation of the term premium component of interest rates.

To conclude this section, reverting back to Graph 2 once again, the term premium for a maturity of 10 years, coming from the April-10 yield curve (red-colored curve) for example, can be obtained by first extracting from historical yield data the average expected short rate (following a "physical" process) up to the 10 year maturity and then calculating the term premium as the difference between the German observed market 10-year government bond yield and the average expected short rate. However, in practice, the technical challenge in using GATSM to calculate the term premia is to properly extract this "physical" process for the short rate. The next section will dive deeper into the dataset used to calibrate the ECFIN term premium model as well as the calibration algorithm employed.

3. CALIBRATING THE TERM PREMIUM MODEL

3.1 THE CALIBRATION DATASET

In this section, a brief description of the dataset used to calibrate the ECFIN term structure based term premia model is presented. Although the focus of this paper is on the calibration of the term premia to the euro area, the ECFIN term premia model was also calibrated to US historical yield data (to have a basis for comparison and also as an additional robustness check). For the euro area, the term premia model was calibrated to two different historical datasets: the German government bond benchmark curve and the EONIA swap curve. It is generally considered that these two curves in particular are informative and provide insight on the transmission of monetary policy, as they have become widely accepted proxies for credit risk-free interest rates in the euro area. As such, they serve as the bedrock for pricing virtually all credit products and related derivatives. These curves are vital for the transmission of monetary policy, as they have a significant influence on broad asset valuations and the pricing of bank loans and, ultimately, they affect the investment and saving decisions of households and firms.

Historical data on German government bond yields was obtained from Bloomberg for the following yield curve maturities: 3M, 6M, 1Y, 2Y … 10Y, 15Y, 20Y and 30Y. The dataset, of a monthly frequency, goes back to December 1994. The US dataset is comprised of historical US treasury yields, with a monthly frequency, going back to January 1990 and for the following yield curve maturity points: 3M, 6M, 1Y, 2Y … 10Y, 15Y, 20Y and 30Y. Calibrating the term premia model in addition to historical EONIA swap curve served as an additional robustness check, given that EONIA swap yields are typically considered a rather good proxy for euro area risk-free interest rates (outside of the German bond market that is). As euro area OIS markets were generally not well developed in the early years of the monetary union, reliable data at least for the longer maturities is not available pre-2006. We therefore only use OIS data from 2006 onwards and apply the percentage evolution of Euribor swap rates before January 2006 to back-fill the OIS swaps dataset to June 2002. The historical dataset from the OIS yield curve, with a monthly frequency, is constructed for the following yield curve maturity points: 1M, 3M, 6M, 1Y, 2Y, 3Y, 4Y, 5Y, 7Y, 10Y, 15Y, 20Y and 30Y.

3.2 THE CALIBRATION ALGORITHM

The particular GATSM chosen and implemented in this paper is a 2-factor arbitrage-free Nelson Siegel term structure model. One of the advantages of choosing a 2-factor model is that, being more parsimonious[17] as compared to a 3- or 4- factor model, there are fewer model parameters to estimate which in turns renders model calibration somewhat easier.

The model's underlying state variables/factors are estimated using a Kalman filter, which is useful in situations such as this one, where the underlying state variables are not directly observable. In the

[17] A parsimonious model is a model that accomplishes a desired level of predictive power with as few predictor variables as possible

particular case of the 2-factor model, as already explained in Section 2.3, the first state variable can be interpreted as the "level" component of the yield curve while the second state variable represents the yield curve's "slope" component. The key to estimating these "hidden" factors lies in the relationship between the bond prices and the underlying state variables. Indeed, the calibration algorithm begins with an observed system of equations called the measurement system; this system represents exactly this affine relationship between market zero-coupon rates (which is a simple logarithmic transformation of the bond price function) and the unobserved state variables. A second, unobserved system of equations called the transition system describes the dynamics of the state variables as they were originally formulated in the model (i.e. the stochastic process for the short rate). Together, the measurement and transition equations represent the state-space form of the model.

Once the initial conditions (for the state variables and state variance matrix) have been specified (the so-called "priors") and given a set of starting values for the model parameters (which define the stochastic process for the short rate) to be optimised, the Kalman filter then uses this state-space formulation to recursively make inferences about the unobserved values of the state variables (transition system) by conditioning on the observed market zero-coupon rates (measurement system). These recursive inferences are then used to construct and maximise a log-likelihood function to find an optimal parameter set for the system of equations[18]. Stated otherwise, in order to compute the "optimal" model parameters, the Kalman filter algorithm is embedded within an optimisation routine which seeks to obtain those parameters which provide the best possible fit to the observed yield curve data. A very commonly used algorithm to maximise the model's fit to the observed yield curve, is the Nelder-Mead simplex search method of Lagarias et al (1998). One known weakness of this algorithm is that it often provides a "local" solution (for the optimal model parameters) meaning that the algorithm may result in being trapped in a local optimum of the mathematical function instead of converging to the function's global optimum point.

Kim and Orphanides (2005) point out that in order to obtain accurate information about the "physical" dynamics of the short rate underlying the GATSM, one has to calibrate the term structure model to a sufficiently long dataset. In many empirical studies, however, the practical implementation of GATSMs runs into the problem that data samples are too short to provide a reliable characterisation of the physical dynamics of the interest rate process. This is linked to the highly persistent nature of interest rates[19] which implies that in a historical data sample spanning 5 to 10 years, one may not observe a sufficient number of "mean-reversions" and hence it becomes very difficult to estimate properly the drift parameter of the underlying short rate. They discuss alternative ways to overcome this problem. One of them is to embed survey data on interest rates into the calibration algorithm; the basic idea is that this additional information on the expected path of the short rate can help to pin down the model parameters related to the "physical" drift of the short rate. Another approach is to impose restrictions on some of the model parameters (i.e. those that define the dynamics of the short rate) during the calibration process[20]. Furthermore, they explain that if the historical dataset is too short, this could result in the situation whereby likelihood function (which is to be optimised) may have multiple local maxima resulting in different estimates for the economic quantities of interest. This problem of having multiple local maxima during the calibration is even more relevant when constraints are imposed to parameters.

To reduce the risk of being exposed to the potentially detrimental effects of the "short sample" bias problem, the optimisation of the likelihood function and the calibration of the associated model parameters was implemented in this paper using a constrained genetic optimisation algorithm. The box

[18] A more detailed and technical explanation of the Kalman filter implementation is provided in Section 7.2

[19] See for example Willem van den End (2011) who provides statistical evidence on the persistence of interest rates.

[20] Kim and Orphanides (2005) suggest, for instance, to restrict those parameters with large standard errors to zero and re-estimate the model. They also discuss the possibility of constraining the parameters defining the mean-reversion process of the short rate directly.

below provides a succinct description of what a genetic algorithm does and why it is particularly useful in this context (i.e. to avoid being trapped in local maxima during the optimisation process).

Box: **WHY A GENETIC ALGORITHM?**

A genetic algorithm (GA) is a method for solving both constrained and unconstrained optimisation problems based on a natural selection process that mimics biological evolution. Genetic algorithms embed a probabilistic search which is founded on and mimics the idea of an evolutionary process. The GA procedure is based on the Darwinian principle of survival of the fittest. An initial population is created containing a predefined number of individuals (or solutions), each represented by a genetic string (incorporating the variable's information). Each individual has an associated fitness measure, typically representing an objective value. The concept that fitter (or better) individuals in a population will produce fitter offspring is then implemented in order to reproduce the next population. Selected individuals are chosen for reproduction (or crossover) at each generation, with an appropriate mutation factor to randomly modify the genes of an individual, in order to develop the new population. The result is another set of individuals based on the original subjects leading to subsequent populations with better individual fitness. Therefore, the algorithm identifies the individuals with the best optimising fitness values, and those with lower fitness will naturally get discarded from the population. Ultimately this search procedure finds a set of variables that optimises the fitness of an individual and/or of the whole population.

What makes genetic algorithms (GAs) a superior optimisation tool?

(1) Most other traditional algorithms are serial and can only explore the solution space to a problem in one direction at a time whereas GAs can explore the solution space in multiple directions at once. If one path turns out to be unfruitful, they can easily eliminate it and continue work on more promising avenues, giving them a greater chance at each step of finding the global optimal.

(2) A notable strength of genetic algorithms is that they perform well in problems for which the solution space is complex - ones where the fitness function is discontinuous, noisy, changes over time, or has many local optima. GA has proven to be effective at escaping local optima and discovering the global optimum in even a very rugged and complex fitness landscape.

(3) Another aspect in which genetic algorithms prove particularly efficient is in their ability to manipulate many parameters simultaneously (when the dimension of the solution space is high).

The GATSM-based term premia model implemented in this paper has altogether 11 parameters which need to be calibrated, with constraints imposed on some of the parameters defining the "physical" dynamics of the short rate. The solution space, being high-dimensional and complex, is thus particularly well suited for a constrained GA algorithm.

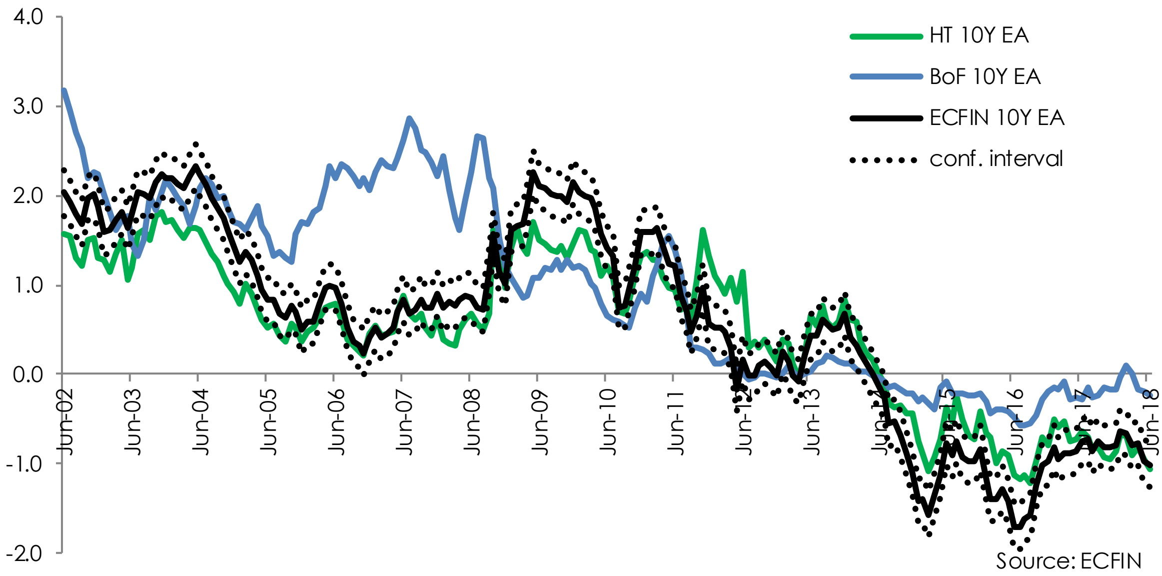
In a nutshell and to conclude, the GA optimisation algorithm presents advantages over traditional optimisation techniques which tend to be sensitive to the initial starting point (i.e. the starting values assigned to each of the 11 parameters to be estimated by the model) and which may result in ending up being trapped in local maxima of the Kalman filter log-likelihood function.

Now that the datasets and calibration algorithm employed in this paper have been discussed, the next section will present the empirical results obtained for the term premia.

4. EMPIRICAL RESULTS FOR THE TERM PREMIA

In the current state of literature, the estimation of term premia on euro area data is relatively scarce in comparison to US-based term premia estimates for which a number of models already exist (both hybrid term structure based models embedding survey/macroeconomic variables or regression based models). The results of calibrating our GATSM-based term premia model is shown in Graph 3 below which plots the 10-year term premium estimates coming from our term premia model alongside the HT, Hordahl and Tristani (2014) model estimates as well as estimates from a model used by the Bank of France, based on Monfort et al (2017)[21].

Graph 3: **EA 10Y Term Premia**



It is worth noting that the models depicted in Graph 3 make use of different euro area benchmark rates: the HT model relies on French government bond yields, the BoF model on the OIS swap curve and the ECFIN model is based on German government bond yields. Despite the use of different datasets and despite different modeling assumptions underlying the three models, it is reassuring to observe that the various term premia estimates for the euro area concur on the overall trend and dynamics. Moreover, to provide a first indication of model robustness, we also plot in Graph 3 the confidence intervals (defined as +/- 2 standard deviations) stemming from the ECFIN model estimates and which correspond to the dotted black-colored lines lying above and below the solid black-colored line. All three estimates (HT, BoF and ECFIN) point to a general downwards trend of the 10-year term premium since the onset of the GFC, in particular during the effective lower bound (ELB) period when the ECB had to move beyond conventional policy instruments and instead deploy a set of unconventional tools (such as large-scale asset purchase programs and forward guidance) that were tailored to target the longer-end of the yield curve.

The ECFIN model was also calibrated to the EONIA swap curve and the time profile of the 10-year term premia turns out to be close to the one obtained by calibrating the model to the German government bond curve[22]. The reason for calibrating in addition to the EONIA swap curve is twofold. Firstly, it serves as an additional model robustness check, as one would expect the time profile of the 10-year term premia estimated from the EONIA swap curve to match (even if not perfectly as these are different markets) the one obtained from the German government bond curve. In the academic literature dealing with term structure based term premia models, one most often finds term premia estimates calibrated to government bond yield data since, as mentioned in Section 3.2, the historical dataset needs to be sufficiently long in order to capture accurately the "physical" process of the factors and the short rate. As explained in Section 3.1, since reliable EONIA swap data (at least for the longer maturities) were not available pre-2006, the calibration dataset was therefore constructed with EONIA

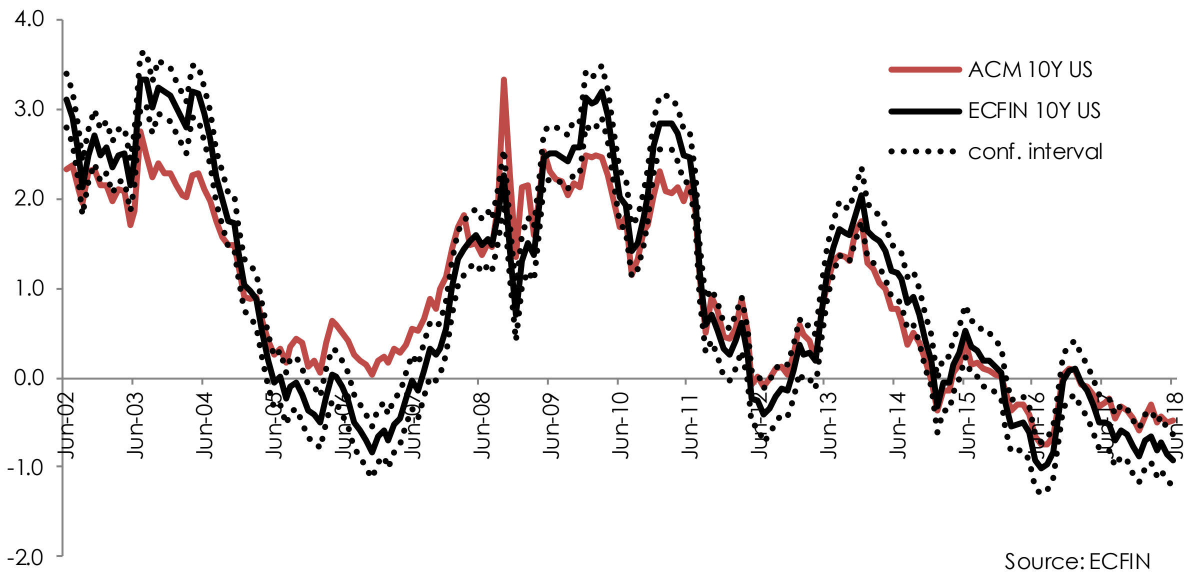
[21] The HT and BoF models are introduced and succinctly described in Section 2.2.

[22] Graph 6 in Section 5 shows the 10-year term premium obtained after calibrating the ECFIN model to the OIS swap cuve.

swap data post-2006 and pre-2006 the percentage evolution of Euribor swap rates was applied to back-fill the OIS swaps dataset to June 2002. Despite this workaround, the time profile of the estimated 10-year term premium presents similar dynamics as compared to the one obtained from calibrating to the German government bond curve.

Secondly, the OIS swap curve is often used in analysing the transmission of a central bank's monetary policy[23]. Although the German government curve is still considered today as a good proxy for risk-free rates (due to the quasi inexistence of credit/default risk), it can still capture factors other than pure interest rate risk such as, to cite but one example, flight-to-safety effects observed during times of heightened financial market stress. Although the EONIA market is not as prone to these particular effects, one has to bear in mind that it is nevertheless a different market with different dynamics as compared to the bond market. For example, the OIS and EURIBOR swap markets have become a popular hedging vehicle for banks and corporations active in euro financial markets. Despite the fact that the German government bond market and the EONIA swap market have different driving forces, both are today commonly used as proxies for risk-free rates and thus it is reassuring to observe that the 10-year term premium estimated from both of these markets yields a comparable time profile (in terms of trend and dynamics).

Graph 4: **US 10Y Term Premia**



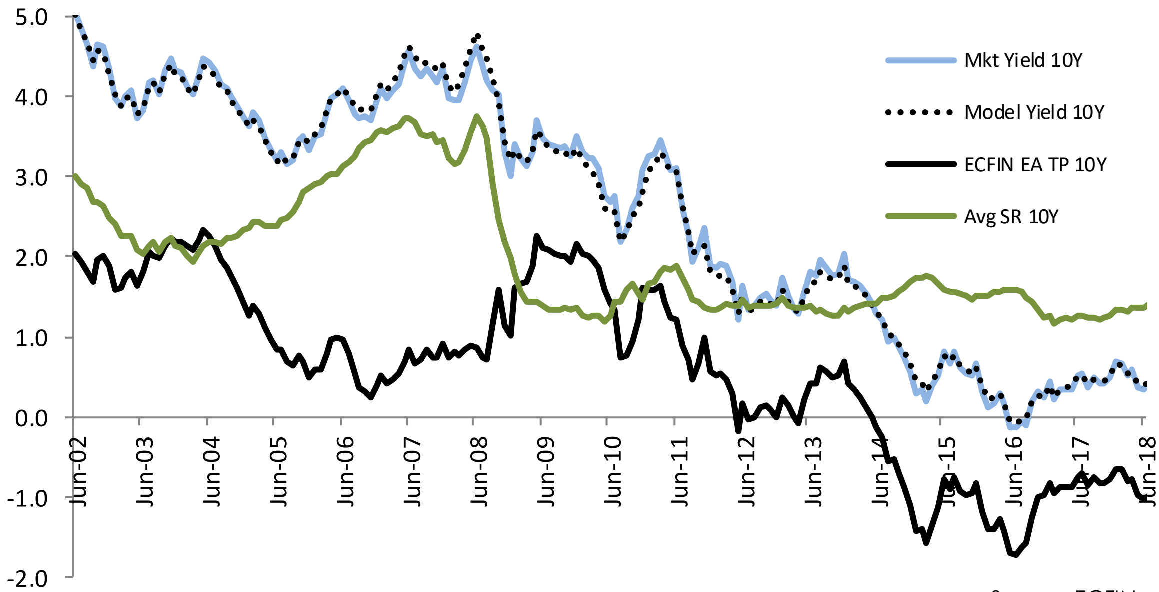
The earliest models of the term premia which were developed were almost exclusively all calibrated to the US treasury curve. Still today, the "reference" term premia model for the US is the one developed by Adrian, Crump and Moench or ACM (2013), which is currently used by the Federal Reserve Bank of New York, and which is calibrated to historical US government bond yields. Graph 4 above illustrates the 10-year term premium estimates obtained from the ACM yield-only regression based model alongside the ECFIN term structure based model estimates. Both models were calibrated to historical US government bond data. It is also reassuring to observe that the ECFIN 10-year term premium estimates (i.e. obtained with the same ECFIN model that was used to produce the euro area 10-year term premium depicted in Graph 3 calibrated to German government bond data) coincide rather well with the "reference" US term premia model of Adrian, Crump and Moench (2013). Both the ACM and the ECFIN model estimates rely solely on yield curve data (i.e. do not incorporate survey data or macroeconomic variables), but both have very different modeling assumptions (the ACM model is a regression-based while the ECFIN model is term-structure based) and yet they produce term premium estimates that match rather closely, thus serving as another robustness check

[23] Section 5 will discuss in more details how the EA 10-year term premium evolved after key ECB unconventional policy decisions since 2008.

for the ECFIN term premia model. Moreover, Graph 4 also displays the confidence intervals (defined as +/- 2 standard deviations) stemming from the ECFIN model estimates and which correspond to the dotted black-colored lines lying above and below the solid black-colored line. As for the case of the euro area 10-year term premium estimates in Graph 3, the confidence intervals for the US are also quite narrow and on a number of occasions encompass the estimates produced by the ACM model.

As just discussed, to ascertain the robustness of the ECFIN term premia estimates we calibrated the model to both EA and US data and compared to other existing benchmark models which exhibit the same dynamics across time; moreover, we went one step further in our robustness checks and also calibrated the model to the EA OIS swap curve data, obtaining the results which are comparable to the ones from calibrating to the German government bond curve. Finally, Graph 5 below illustrates for the euro area the decomposition of the 10-year German government bond yield into its two components: the average of the short rate up to the maturity of 10-years and the 10-year term premium. What is also quite perceptible from this graph is that the model-estimated 10Y yield (dotted black-colored line) is consistently (i.e. across time) close to the market-observed 10Y interest rate (solid blue-colored line), thus demonstrating that the calibration algorithm employed is successful in fitting observed market yields.

Graph 5: **EA 10Y Yield Decomposition**



Source: ECFIN

It is also interesting to note from Graph 5 that as policy rates of major central banks approached the effective lower bound (ELB) in the aftermath of the global financial crisis, the general downwards trend of the 10-year yield coincided with the compression of the 10-year term premium within this same period. Since the onset of the effective lower bound period as from 2012, the "market expected[24]" average short rate up to the 10-year maturity remained relatively stable, with the majority of the overall decrease of the 10-year yield accounted for by the compression of the term premium component. The next section will further discuss this observed decline in the term premia, namely in relation to central bank unconventional measures which have been implemented during the effective lower bound period.

[24] Recall that the term premium component involves calculating the "physical" $\mathbb{P}$-measure process for the short rate which captures actual expectations that economic agents have about the future values of the short rate.

5. THE TERM PREMIA AND UNCONVENTIONAL MONETARY POLICY

By changing its key policy rates, a central bank can directly impact the short end of the curve – the anchoring of the expectations component. In normal times, medium to long-term rates would only adjust to the extent that market participants would see a change in policy rates as the beginning of an incremental series of changes. But with the ECB's deposit facility rate (DFR) cut to zero for the first time in July 2012 and staying there until June 2014 when it entered into negative territory, this channel had become less effective in the euro area. The leeway to cut policy-controlled short-term interest rates turned out to be insufficient to provide the degree of accommodation that was considered necessary to support the economy and achieve the ECB's price-stability objective. By mid-2014, the euro area economy was facing disinflationary pressures that risked spiraling into outright deflation. With the deposit facility rate cut to zero already in July 2012, the ECB's ability to provide the necessary degree of further monetary stimulus using conventional policy measures was very limited. Under these conditions, the focus of providing monetary accommodation shifted from an approach based on adjusting the short term policy rates (which steers the very short end of the yield curve) to one that attempts to affect the whole spectrum of interest rates across the yield curve. In general, a central bank can only influence the yield curve indirectly through unconventional measures[25]. The ECB Governing Council thus had to deploy another set of unconventional tools that were tailored to the specific challenges of that time.

Specifically, the instruments the ECB has deployed since June 2014 included a negative deposit facility rate (DFR), which provides the floor of the interest rate corridor[26], a reinforced form of forward guidance on the evolution of the key policy rates in the future, targeted longer-term refinancing operations (TLTROs), and the asset purchase program (APP). To the extent that forward guidance reduced uncertainty about the future path of policy rates, it has not only affected the expectations component but also the term premium. Yet, the main channel through which the ECB – and other major central banks – have exerted measurable downward pressure on the term premium is through asset purchases. Indeed, there is now a growing body of evidence suggesting that central banks have had the effect of lowering long-term rates by removing duration risk from the market.[27]

The APP applies further pressure on longer-term interest rates along the yield curve, mainly by compressing the term premium component of interest rates. In essence, the APP extracts duration risk from the market. If, at times of disinflation and weak growth, long-term borrowing is to be made more affordable so as to promote investment and consumption, then the central bank can try to absorb part of the duration risk that otherwise would have to be held by private investors. This can be done by purchasing long-dated bonds, as the ECB did under its APP. With less long-dated bonds to hold in the aggregate, private investors have an incentive to rebalance their portfolios towards other, riskier market segments. This is because central bank purchases free up risk-taking capacity in the private sector and drive down risk-adjusted returns on the assets targeted by the purchase programs, hence inducing investors to consider alternative investments. Via this portfolio-rebalancing channel, the ECB's non-standard measures have compressed risk premia across a wide range of asset classes.

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[25] The existence of an effective lower bound (ELB) on nominal interest rates makes it more difficult for central banks to achieve their inflation objectives with conventional policy tools. In other words, to the extent that the effective lower bound on nominal interest rates is binding, such that policy rates cannot be lowered further, the central bank needs to resort to other "unconventional" tools to implement its monetary policy objectives.

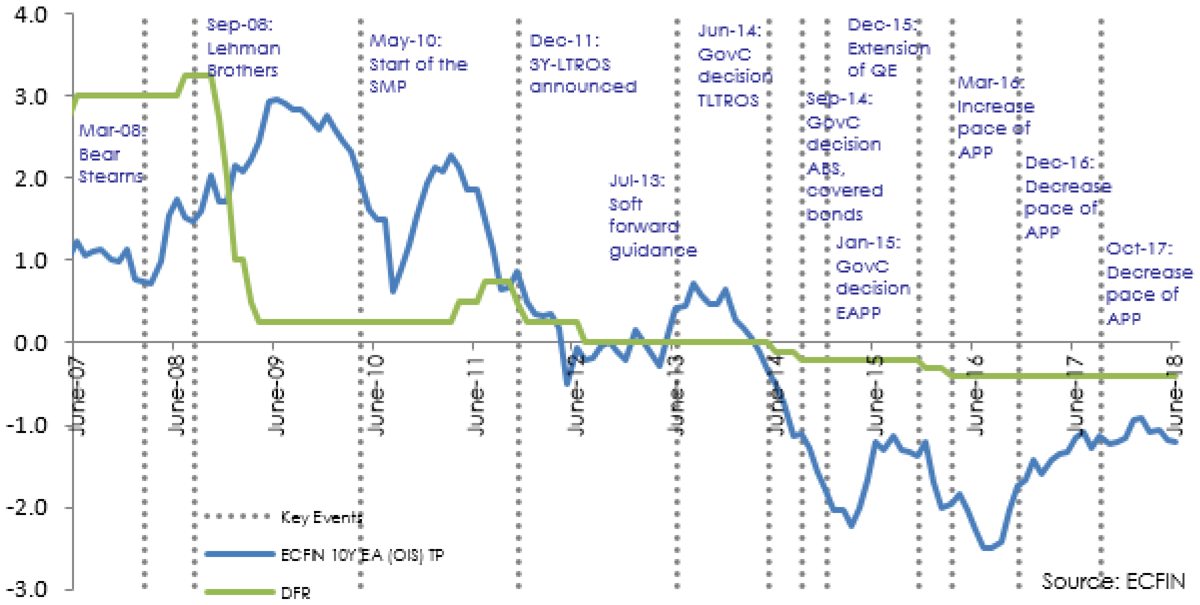
[26] The ECB's interest rate corridor corresponds to the difference between the Maginal lending facility (MLFR) rate, at which banks can borrow funds overnight from the Eurosystem, and the Deposit facility rate (DFR), at which banks can deposit funds onvernight with the Eurosystem.

[27] See for instance Li and Wei (2012) who provide evidence showing that the various US Federal Reserve asset purchase programs have compressed the term premium component of US treasury yields.

The forward guidance on policy rates, which took the form of the ECB's stated expectation that its key interest rates will remain "at their present levels for an extended period of time", was calibrated in a way that anchored the short-to-medium maturities of the yield curve (those portions most sensitive to short-term interest rate expectations and, therefore, to forward guidance) around levels that are sufficiently steady and low. In this respect, a mildly negative DFR has proved to be particularly powerful in controlling and anchoring these maturities which are key to pricing bank credit in the euro area. The notion that zero was not the effective lower bound has exerted additional flattening pressure on the short-to-intermediate maturities of the yield curve, those to which banks tend to index loans with adjustable interest rates.

With this context in mind, it is interesting to observe the evolution of the euro area 10-year term premium in the post GFC era and in particular how it evolved alongside key ECB unconventional monetary policy decisions. Graph 6 below depicts the 10-year term premium estimated according to the ECFIN term premia model, but this time it was calibrated to the EONIA swap curve (instead of German government bonds). Generally, as the overnight swap contracts only involve the exchange of net payments and no principal, OIS rates can be seen as risk-free and thus can better reflect monetary policy than for example government bond yields, which might mirror other developments, e.g. with regard to flight-quality or scarcity effects.

Graph 6: **Evolution of EA 10Y Term Premia post GFC**



In the run-up to the GFC, there were some early signs, such as the failure of the global investment bank Bear Stearns, a major early casualty which increased market perceptions of risk, and which in turn led to a sizeable up-spike in the 10-year term premium. As is also visible from the graph, a second marked increase in the 10-year term premium followed some time after the bankruptcy of Lehman Brothers in September 2008, which is often considered to mark the beginning of the GFC period. Since the onset of the GFC, the 10-year term premium has followed a downwards trajectory until mid-2016 when it started to reverse direction. One can clearly observe from Graph 6 a rather good match between some key ECB unconventional monetary policy decisions since 2010 (see Table 1 below containing a synopsis of these events) and the subsequent move of the 10-year term premium[28]. This seems to corroborate the original aim of non-standard measures (notably the asset purchase program)

[28] A detailed analysis of the impact of each ECB monetary policy decision on the term premium would ideally require the use of event-study econometric techniques and would also need to take into account other factors such as the spillover effects of US monetary policy decisions. This is beyond the scope and aim of this paper.

which was to target essentially the long-end of the yield curve and in particular the term premium component.

Table 1

May-10	Securities Markets Programme (SMP) implemented from May-10 to Sep-12 whereby the ECB intervened in the form of outright secondary market purchases of government debt securities issued by 5 countries (Greece, Ireland, Portugal, Spain, and Italy).
Dec-11	Two 3-year LTROs were conducted in Dec-11 and Feb-12, with allotment of €490 bn and €530 bn respectively.
Jul-13	In the context of its forward guidance, the ECB communicates that it "expects the key ECB interest rates to remain at present or lower levels for an extended period of time".
Jun-14	The ECB introduces a negative deposit facility rate (DFR). The ECB launches a series of targeted LTROs to be held quarterly until Jun-16. Under the first four TLTROs banks took up €384 bn of liquidity.
Sep-14	The DFR is cut by a further 10 bps. The ECB announces programme for purchases of ABS and covered bonds.
Jan-15	The ECB announces purchases of sovereign bonds, with total private and public purchases at a pace of €60 bn per month expected to run until Sep-16.
Dec-15	The expected APP duration is extended by 6 months to Mar-17. This was simultaneously accompanied by a 10 basis points cut in the deposit rate.
Mar-16	Increase of monthly APP buying pace to €80bn and the DFR was cut to the current level of -0.4%.
Dec-16	The expected APP duration is extended to end-2017 with the pace of net asset purchases reduced from €80 bn to €60 bn as from Apr-17.
Oct-17	The expected APP duration is extended until Sep-18 with the monthly pace of net asset purchases reduced to €30bn as from Jan-18.
Jun-18	The APP is extended until Dec-18 with the monthly pace of net asset purchases reduced to €15bn as from Oct-18.

In practice, however, distinguishing between the effects of the APP and of forward guidance on the two components of long-term interest rates is not straightforward. The credibility of the ECB Governing Council's expectation to follow a certain course of action for setting the policy rates in the future (i.e. forward guidance) is almost certainly enhanced by the APP, as these purchases are a concrete demonstration of a desire to provide additional stimulus. In other words, there is a signaling channel inherent in asset purchases, which reinforces the credibility of the forward guidance on policy rates via the expectations channel. Conversely, forward guidance may affect the term premium component of longer-term yields by reducing the uncertainty about the future path of short-term interest rates and thereby reducing perceived duration risk in the market. Consequently, the net stimulus provided by asset purchases depends in part on expectations of how policymakers will adjust short-term interest rates in the future. This being said, one discernable pattern that emerges from Graph 6 is that the implementation of the ECB's unconventional monetary policy measures (in particular the APP) since 2012 has been accompanied by a compression in the term premium component of interest rates.

To conclude this section, it is worth considering the implications for investors of the negative term premia which has turned out to be a persistent phenomenon (both in the EA and the US) over the past couple of years. The existence of such negative term premia implies that investors are willing to hold bonds without being rewarded for bearing interest rate risk (and even willing to incur negative returns on their bond holdings) because they seek to protect their overall portfolio of assets against the risk of a large deflationary shock in the future. Stated otherwise, if demand shocks are creating low inflation economic downturns, nominal bond yields will fall (namely as a result of falling inflation) implying that bond prices will rise amidst falling stock prices (associated with the economic slowdown). Therefore, in this context, bond prices will act as a hedge or insurance against falling stock prices during economic slowdowns characterised by deflation. Investors would then be willing to accept a negative term premium for holding bonds in this particular environment, as would someone accept to pay a premium for holding insurance. In other words, it is the change in the way that investors price interest rate risk (as just described) which contributes to explaining the persistence of negative term premia. But this argument has to be considered whithin the wider context of unconventional monetary policy measures, and in particular the implementation of the APP, which had the effect of compressing the term premium component of interest rates (as previously discussed in this section).

6. CONCLUSION

Credit risk-free interest rates can typically be decomposed into two components: expectations of the future path of the short-term policy rate and the term premium. The expectations component of interest rates reflects the average of current and future expected short-term rates over the maturity of a bond. If the pure expectations hypothesis of the term structure were to hold, this would be all that mattered in terms of explaining movements in long-term rates. But broad empirical evidence suggests that the pure expectations hypothesis fails to hold true in practice, and that there is indeed a time-varying premium that investors require in order to hold a long-term bond instead of simply rolling over a series of short-term bonds. Changes in term premium are estimated to have been an important driver behind developments in long-term bond yields in recent years. As policy rates of major central banks approached their effective lower bound in the aftermath of the global financial crisis, their ability to provide the necessary degree of monetary stimulus using conventional policy measures became very limited. In this particular context, central banks had to move beyond conventional policy instruments and instead deploy a set of unconventional tools (such as large-scale asset purchase programs and forward guidance) that were tailored to target the longer-end of the yield curve. There is a growing body of empirical evidence to suggest that these unconventional measures targeting the longer-end of the interest rate curve have had the effect of compressing the term premium component of interest rates.

This paper presents the results of extracting the term premium component using a two-factor arbitrage-free Nelson Siegel term structure model. More specifically, the model parameters were calibrated to market data using a Kalman filter and the optimisation of the Kalman filter likelihood function was based on a genetic algorithm. The model was calibrated to both euro area and US government bond yields and in the process we extracted from historical yield data financial markets' expectations about the future course of short-term policy rates. In addition, for the euro area, the model was also calibrated to the OIS swap curve which is frequently used to analyse the transmission of monetary policy. The empirical results obtained, both for the euro area and the US, when compared with other benchmark models of the term premia concur on the overall trend and dynamics of the 10-year term premium, despite the use of different datasets and different modeling assumptions underlying the other models. While employing genetic algorithms contributes to improving the overall calibration quality, it does not eradicate the usual caveats associated with any estimation exercise. More generally, term premia estimates are model dependent and thus any such model should be seen as a useful simplifying tool although it may not necessarily capture all various real-life influences.

As policy rates of major central banks approached their effective lower bound in the aftermath of the global financial crisis, the general downwards trend of the 10-year benchmark yield coincided with the compression of the 10-year term premium within this same period. As the empirical results presented in this paper reveal, since the onset of the period (since mid-2012) characterised by zero and thereafter negative ECB policy rates, the "market expected" average short rate up to the 10-year maturity remained relatively stable, with the majority of the overall decrease in the 10-year yield accounted for by the compression of the term premium component. When plotting the estimated 10-year term premium (calibrated to the EONIA swap curve) alongside key ECB unconventional monetary policy decisions, one can observe a rather good match between some of these monetary policy events and the relative parallel compression of the term premium compared to the pre-2012 period (i.e. when ECB policy rates were positive). The empirical results obtained thus seem to corroborate the growing body of evidence suggesting that unconventional monetary policy measures (in particular the asset purchase program) targeting the longer-end of the interest rate curve were effective in compressing the term premium component of interest rates.

7. TECHNICAL ANNEX

7.1 A TERM STRUCTURE-BASED TERM PREMIA

An affine term structure model is a financial model that relates zero-coupon bond prices (i.e. the discount curve) to a model for short rate. It is particularly useful for deriving the zero-coupon yield curve from quoted bond prices. Thus the starting point for the development of the affine class is the postulation of a stochastic process for the short rate and the related state variables, or factors, which drive the dynamics of the term structure. These factors are the underlying source of uncertainty in the model of the term structure. Under a Gaussian affine term structure model or GATSM, the short rate $r(t)$ at time t is a linear function of the state variables $x(t)$ at time t:

$$r(t) = a_0 + b_0' x_n(t)$$

where $r(t)$ is a scalar, a_0 is a constant scalar, b_0 is a constant Nx1 vector containing the weights for the N state variables $x_n(t)$. Under the "physical" $\mathbb{P}$-measure, $x(t)$ evolves according to a correlated vector Ornstein-Uhlenbeck process[29]:

$$dx(t) = \kappa[\theta - x(t)]dt + \sigma dW(t)$$

where θ is a constant Nx1 vector representing the long-run level of $x(t)$, κ is a constant NxN matrix that governs the deterministic mean reversion of $x(t)$ to θ, σ is a constant NxN matrix representing the correlated variance of innovations to $x(t)$, and $dW(t)$ is an Nx1vector with independent Wiener components[30].

The future evolution of the short rate under the $\mathbb{P}$-measure is therefore determined by the $\mathbb{P}$-measure process for x(t). Note that the $\mathbb{P}$-measure is also often referred to as the "physical" process, and it refers to the market's expectations about the future values of the short rate (and associated state variables). However, bonds in financial markets are priced under the risk-adjusted $\mathbb{Q}$-measure. The $\mathbb{Q}$-measure is also known as the "risk-neutral" measure and the expected returns for all assets under this measure are equal to the risk-adjusted short rate. Hence, the $\mathbb{P}$-measure process for x(t) and r(t) must

[29] In mathematics, the Ornstein–Uhlenbeck process is a stochastic process that can be considered to be a modification of the random walk in continuous time in which the properties of the process have been changed so that there is a tendency of the walk to drift towards its long-term mean, with a greater mean-reversion when the process has drifted further away from it's long-term mean.

[30] That is, $dW_n(t) \sim N(0,1)\sqrt{dt}$ where $N(0,1)$ represents the unit normal distribution.

be adjusted for the market prices of risk to arrive at the risk-neutral $\mathbb{Q}$-measure, which is subsequently used to represent the observed market term structure (i.e. quoted bond prices).

In GATSMs, the market prices of risk are typically specified as a linear function of the state variables, which allows the market prices of risk to vary over time:

$$\Pi(t) = \frac{1}{\sigma}[\gamma + \Gamma \cdot x(t)]$$

where $\Pi(t)$ is an Nx1 vector containing the market prices of risk for each state variable, γ is a constant Nx1 vector containing the constant component of the market prices of risk, and Γ is a constant NxN matrix that specifies how the market prices of risk vary with the state variables.

Under the risk-adjusted $\mathbb{Q}$-measure, x(t) also evoles as correlated vector Ornstein-Uhlenbeck process:

$$\mathrm{dx(t)} = \tilde{\mathrm{k}}\left[\tilde{\theta} - \mathrm{x(t)}\right]\mathrm{dt} + \sigma \mathrm{d}\widetilde{\mathrm{W}}(\mathrm{t})$$

where $\tilde{\kappa} = \kappa + \Gamma$, $\tilde{\theta} = \tilde{\kappa}^{-1}(\kappa \cdot \theta - \gamma)$, and $\mathrm{d}\widetilde{W}(t) = dW(t) + \Pi(t)dt$. The future evolution of the short rate under the $\mathbb{Q}$-measure is therefore determined by the $\mathbb{Q}$-measure process for *x*(*t*).

In light of the above description of the stochastic process of the short rate, let us revisit the basic definition of the term premium as it was defined in Section 2.3:

$$\mathrm{TP}_{t,\tau} = \mathrm{Y}(t,\tau) - \left(\frac{1}{\mathrm{N}}\right) * \sum_{\mathrm{i}=0}^{\mathrm{N}-1} \mathrm{E}_{\mathrm{t}}^{\mathbb{P}}(\mathrm{r}_{\mathrm{t+i}})$$

where

$\mathrm{Y}(t,\tau)$ = market zero-coupon interest rate or yield for time to maturity τ years from today.

$\mathrm{TP}_{t,\tau}$ = is yield term premium for an interest rate with maturity τ years from today.

$\mathrm{r_t}$ = is the one-day rate or “short rate” (i.e. corresponding to the ECB’s DFR for example).

N = number of days until the bond’s maturity which is in in τ years from today.

In a continuous-time framework[31], the above definition can be re-expressed as follows:

$$\mathrm{TP}_{t,\tau} = \mathrm{Y}(t,\tau) - \frac{1}{\tau}\int_0^\tau \mathbb{E}_t[r(t+\tau)[x(t)]$$

But the expression $\mathbb{E}_t[r(t+\tau)[x(t)]$ is nothing else than the expected value of the short rate at $t+\tau$ under the "physical" $\mathbb{P}$-measure process for r(t), conditional on x(t). It is convenient at this point to express the market yield $\mathrm{Y}(t,\tau)$ as the sum of the two components: (1) the GATSM model estimated yield $R(t,\tau)$ and (2) the model’s fitting error $e_{t,\tau}$ such that $Y(t,\tau) = R(t,\tau) + e_{t,\tau}$. The expression for GATSM (i.e. model-based) zero-coupon interest rates for time to maturity τ is obtained using the standard continuous-time term structure relationship:

[31] In a continuous-time framework, the time interval for the short rate, instead of being a day, tends to an infinitely small number. This enables the use of calculus to derive relevant expressions and it facilitates the analysis.

$$R(t,\tau) = \frac{1}{\tau}\int_0^\tau f(t,\mu)d\mu$$

where µ is a dummy variable for τ used to evaluate the integral. Note that $f(t,\tau)$ denotes the forward short rate in τ years from today.

In GATSMs, forward rates have the following expression:

$$f(t,\tau) = \widetilde{\mathbb{E}}_t[r(t+\tau)[x(t)] - VE(\tau)$$

where $\widetilde{\mathbb{E}}_t[r(t+\tau)[x(t)]$ is the expected value of the short rate at $t+\tau$ under the "risk-neutral" $\mathbb{Q}$-measure, conditional on $x(t)$, and $VE(\tau)$ is the volatility effect[32].

This thus implies the following:

$$R(t,\tau) = -\frac{1}{\tau}\int_0^\tau VE(\tau) + \frac{1}{\tau}\int_0^\tau \widetilde{\mathbb{E}}_t[r(t+\tau)[x(t)]$$

After substituting the definition for $\mathrm{Y}(t,\tau)$ back into the original definition for the term premium, we obtain that the term premium can "in essence" be defined as the difference between the expected evolutions of the short rate under the risk-neutral $\mathbb{Q}$-measure and the physical $\mathbb{P}$-measure:

$$\mathrm{TP}_{t,\tau} = \frac{1}{\tau}\int_0^\tau \widetilde{\mathbb{E}}_t[r(t+\tau)[x(t)] - \frac{1}{\tau}\int_0^\tau \mathbb{E}_t[r(t+\tau)[x(t)] - \frac{1}{\tau}\int_0^\tau VE(\tau) + e_{t,\tau}$$

As both the volatility effect and the model's fitting error are neglibible[33], the term premium basically boils down to the difference between the expected evolutions of the short rate under the "risk neutral" $\mathbb{Q}$-measure and the the "physical" $\mathbb{P}$-measure. This being said, incorporating the fitting error $e_{t,\tau}$ into the term premium nevertheless presents the practical advantage that the sum of both the average expected short rate and term premium components add-up exactly to the observed market yield. This is particularly convenient if both time series (expected short rate and term premium time series) are used in econometric analysis of market observed yields.

The term premia estimates presented in this paper are the ones obtained by implementing the above equation in the specific case of two-factor arbitrage-free Nelson-Siegel term structure model. So let us now zoom in on the calculation of the term premia for this specific case. Closed-form expressions for the short rate and for zero-coupon interest rates exist in the particular case of the two-factor arbitrage-free Nelson Siegel model:

$$r(t) = x_1(t) + x_2(t)$$

$$R(t,\tau) = a(\tau) + [b(\tau)]' \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

[32] The volatility effect VE(τ) captures the influence that volatility in the short rate has on expected returns due to Jensen's inequality. For a more complete discussion of the derivation of the forward rate ahd the volatility effect, see Krippner 2015, "Zero Lower Bound Term Structure Modeling", p51.

[33] The fitting error turns out to be immaterial both for the euro area and US calibrations (see for instance Graph 5 which illustrates that the quality of the model's fit to observed euro area 10Y yields).

where $r(t)$ is the short rate and $R(t, \tau)$ is the GATSM zero-coupon interest rate for time to maturity τ.

The expressions $a(\tau)$ and $b(\tau)$ are themselves functions of τ and of the parameters $\tilde{k}$, $\tilde{\theta}, \sigma$ which define the "risk neutral" $\mathbb{Q}$-measure process for the short rate [see Krippner 2015, "Zero Lower Bound Term Structure Modeling", p 65]:

$$a(\tau) = -\sigma_1^2 \cdot \frac{1}{6}\tau^2 - \sigma_2^2 \cdot \frac{1}{2\phi^2}\left[1 - \frac{1}{\tau}G(\phi,\tau) - \frac{1}{2\tau}\phi[G(\phi,\tau)]^2\right] - \rho\sigma_1\sigma_2 \cdot \frac{1}{\phi^2}\left[1 - \frac{1}{\tau}G(\phi,\tau) + \frac{1}{2}\phi\tau - \phi G(\phi,\tau)\right]$$

$$[b(\tau)]' = \left[1, \frac{1}{\tau}G(\phi,\tau)\right]$$

where $G(\phi, \tau) = \frac{1}{\phi}[1 - exp(-\phi\tau)]$.

Once the GATSM (model-based) zero-coupon interest rates are calculated, the fitting error is easily obtained as the residual with quoted market interest rates: $e_{t,\tau} = Y(t,\tau) - R(t,\tau)$.

All that remains now is to calculate the expected path of the short rate under the "physical" $\mathbb{P}$-measure which is derived using the following expression:

$$\mathbb{E}_t[r(t+\tau)[x(t)] = a_0 + b_0' \cdot \mathbb{E}_t[x(t+\tau)[x(t)]$$

In the particular case of the arbitrage-free Nelson Siegel two factor model, the factor loadings are given by

$$a_0 = 0;\ b_0 = \begin{bmatrix}1\\1\end{bmatrix}$$

The remaining term (i.e. the conditional expectation of the factors or state variables at time τ under the "physical" $\mathbb{P}$-measure) is defined [see Krippner 2015, "Zero Lower Bound Term Structure Modeling", p 49] as follows:

$$\mathbb{E}_t[x(t+\tau)[x(t)] = \theta + \text{expm}(-\kappa\tau) \cdot [x(t) - \theta]$$

where expm is the matrix exponential[34]. Regarding the practical implementation of the above expression, the entire path for a time-to-maturity grid was generated for the factors (and hence the short rate) with suitably small spacing and subsequently the average was calculated.

Now that the theoretical foundations underpinning how to extract the term premium component using GATSMs model have been established, the next section will discuss how to calibrate the model to market data.

[34] In mathematics, the matrix exponential is a matrix function on square matrices analogous to the ordinary exponential function.

7.2 CALIBRATING THE TERM-PREMIA MODEL

A Kalman filter is used to calibrate the two-factor arbitrage-free Nelson-Siegel model and thereby estimate the following 11 parameters, of which the first 10 originate from the definition of the stochastic process for the short rate, that is

$$\text{Parameter set} = \{\phi, k_{11}, k_{12}, k_{21}, k_{22}, \theta_1, \theta_2, \sigma_1, \sigma_2, \rho_{12}, \sigma_\eta\}$$

plus the variable σ_η which represents the measurement equation (Kalman filter) standard deviation.

The state variables and the parameters when expressed in matrix form can be linked directly to the expression defining the stochastic process for the short rate (a correlated vector Ornstein-Uhlenbeck process). In the case of the two-factor arbitrage-free Nelson Siegel model, they can expressed in the following form [see Krippner 2015, "Zero Lower Bound Term Structure Modeling", p 61]:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix};\ a_0 = 0;\ b_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; k = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix};\ \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\sigma = \begin{bmatrix} \sigma_1 & 0 \\ \rho_{12}\sigma_2 & \sigma_2\sqrt{1-\rho_{12}{}^2} \end{bmatrix};\ \tilde{k} = \begin{bmatrix} 0 & 0 \\ 0 & \phi \end{bmatrix};\ \tilde{\theta} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Calibrating the arbitrage-free Nelson Siegel two-factor model involves embedding the Kalman filter into an optimisation algorithm so as to estimate the above parameters for the specified model. The state variables associated with those parameters are also calculated by the Kalman filter (as an output of the optimisation algorithm). The Kalman filter is based on a state equation, which specifies how the state variables evolve over time, and a measurement equation, which specifies how the state variables explain the observed data at each point in time. In our particular case, the state variables are in the vector x (t), the measurement equation is the GATSM yield curve expression as a function of x (t), and the data is the observed yield curve at each point in time. The objective function of the optimisation algorithm is to maximise the log-likelihood function given by the expression

$$logL(parameters, \sigma_\eta, \{ZC_1, \dots, ZC_T\}) = -\frac{1}{2}\sum_{t=1}^{T}[K \cdot log(2\pi) + log(|\mathcal{M}_t|) + \eta_t' \mathcal{M}_t^{-1}\eta_t]$$

where $\{\eta_1 \dots \eta_T\}$ is the time series of Kx1 vectors containing the unexplained component of the yield curve data at time t relative to the arbitrage-free two-factor Nelson Siegel model (obtained using the measurement equation) and $\{\mathcal{M}_1 \dots \mathcal{M}_T\}$ is the time series of KxK matrices obtained at each time step of the Kalman filter algorithm. The constant K refers to the number of yield curve tenors used in the calibration sample, which in the case of the German bond yield curve for example corresponds to 15: 3M, 6M, 1Y, 2Y … 10Y, 15Y, 20Y and 30Y.

The state equation for the GATSM is a first-order vector autoregression of the following form:

$$x_t = \theta + expm(-k\Delta t)(x_{t-1} - \theta) + \varepsilon_t$$

where the subscripts t correspond to an integer index representing the progression of time in steps of Δt between observations (in the case of monthly data Δt = 1/12), $expm(-k\Delta t)$ is the matrix exponential of $-k\Delta t$, and ε_t is the vector of innovations to the state variables. The variance of ε_t is:

$$var(\varepsilon_t) = \int_0^\tau exp(-ku)\sigma\sigma' \exp(-k'u)\, du$$

which is a 2x2 matrix. The measurement equation for the GATSM is given by:

$$\begin{bmatrix} Y_t(\tau_1) \\ \vdots \\ Y_t(\tau_K) \end{bmatrix} = \begin{bmatrix} a(\tau_1) \\ \vdots \\ a(\tau_K) \end{bmatrix} + \begin{bmatrix} [b(\tau_1)]' \\ \vdots \\ [b(\tau_K)]' \end{bmatrix} \cdot x_t + \begin{bmatrix} \eta_t(\tau_1) \\ \vdots \\ \eta_t(\tau_K) \end{bmatrix}$$

where K is the index for the yield curve tenor τ_K, $Y_t(\tau_K)$ is the observed interest rate at time t for time to maturity τ_K, $a(\tau)$ and $b(\tau)$ are functions (defined in Section 7.1) evaluated at τ_K, and $\eta_t(\tau_K)$ is the component of $Y_t(\tau_K)$ that is unexplained by the GATSM model (i.e. fitting error). The variance of η_t is specified to be homoskedastic and diagonal:

$$\Omega_\eta = diag\left[\{\sigma_\eta^2, \ldots, \sigma_\eta^2\}\right]$$

where Ω_η is a KxK diagonal matrix with entries σ_η^2. Furthermore, reflecting standard practice, the vectors η_t and ε_t are assumed to be uncorrelated over time.

The estimated model parameters for both the euro area and the US are presented below:

Table 2

	EA	US		EA	US
$\boldsymbol{\phi}$	0.1598	0.2278	$\boldsymbol{\theta_1}$	0.0236	0.0428
$\boldsymbol{k_{11}}$	0.0273	0.0251	$\boldsymbol{\theta_2}$	0.0206	0.0403
$\boldsymbol{k_{12}}$	0.0098	0.0018	$\boldsymbol{\sigma_1}$	0.0061	0.0047
$\boldsymbol{k_{21}}$	0.1535	0.1325	$\boldsymbol{\sigma_2}$	0.0102	0.0099
$\boldsymbol{k_{22}}$	0.1093	0.0477	$\boldsymbol{\rho_{12}}$	-0.4948	-0.4060
			$\boldsymbol{\sigma_\eta}$	0.0019	0.0021

Kim and Orphanides (2005) explain that in order to obtain accurate information about the “physical” dynamics of the short rate underlying the GATSM, one has to calibrate the term structure model to a sufficiently long dataset. This is linked to the highly persistent nature of interest rates which implies that in a historical data sample spanning 5 to 10 years, one may not observe a sufficient number of “mean-reversions” and hence it becomes very difficult to estimate properly the drift parameter of the underlying short rate. One way to overcome this problem is to embed survey data on interest rates into the calibration algorithm; the basic idea is that this additional information on the expected path of the short rate can help to pin down the model parameters related to the “physical” drift of the short rate. Another approach is to impose restrictions on some of the model parameters (i.e. those that define the dynamics of the short rate) during the calibration process. Kim and Orphanides (2005) also point out another possible consequence of using a short sample, which is that the likelihood function to be optimised may have multiple local maxima (especially if constraints are imposed to model parameters). In light of this "short sample" bias problem, the optimisation of the likelihood function and the calibration of the associated model parameters was implemented in this paper using a constrained genetic optimisation algorithm.

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