Estimating a production function with natural capital

Marta Kornafel

Department of Mathematics
Krakow University of Economics

Natural capital measurement and modelling workshop (DG ECFIN-OGWG)
EC, Brussels 30.11-1.12.2023

Modelling *natural capital*

Marta Kornafel

Department of Mathematics
Krakow University of Economics

Natural capital measurement and modelling workshop (DG ECFIN-OGWG)
EC, Brussels 30.11-1.12.2023

M. Kornafel, I. Telega *Dynamics of natural capital in neoclassical growth model*, International Journal of Sustainable Economy 12(1):1-24, https://doi.org/10.1504/IJSE.2020.107862

- Socio-economic development has a significant impact on the natural environment through the use of resources, generation of waste and pollution.
- Nature is increasingly seen as another form of capital (natural capital), which is a source of benefits for present and future generations (*ecosystem services*).
- Therefore, it is important to question the dynamics of long-term economic growth processes in the context of preserving natural capital and its impact on social welfare.
- The aim of the work is to supplement the neoclassical growth model with a natural capital factor (N) and to analyze possible long-term equilibrium states at positive values of N.

Neoclassical economics vs. ecological economics

1. substitutability of production factors
2. Natural resources (market goods) and ecosystem services. Utility of consumption and quality of life.
3. The direction of the evolution of the natural capital stock is the result of several contradictory factors – the ability of natural capital to regenerate and the pressure exerted by the production and consumption processes.
4. The problem of critical thresholds and exponential economic growth.

Main results

- Neoclassical growth with exhaustible resources: Dasgupta & Heal (1974), Solow (1974), Stiglitz (1974) and Hartwick (1977) – model DHSS (Benchekroun, Withagen, 2011).
- Models aiming to explain the collapse of ancient societes: Brander & Taylor (1998), Dalton & Coats (2000), D'Allesandro (2007), Roman, Bullock & Brede (2017)
- Two-sector economic model with complementary production factors (Leontief function) and aggregate natural capital stock X of the form $\dot{X} = g(X) - M$, $g(X) = \gamma X(1 - X/S)$, $\gamma > 0$: Comolli (2006).
- Combination of neoclassical growth theory with the concept of dematerialisation of the economy. Material requirement as a measure of natural capital consumption: Bringezu, Schutz and Moll (2003), Rodrigues et al. (2005).

The model

- K – manufactured capital, N – natural capital, A – technology. $L = L_0$ is assumed to be constant, $L_0 = 1$.
- C-D production function: $Y = K^{\alpha} A^{1-\alpha}, \qquad \alpha \in (0, 1)$.
- Division of product: $Y = I + V + C$
- Dynamics of capitals:

$$\dot{K} = Y - C - V - \delta K$$

$$\dot{N} = rN\left(\frac{N}{CT} - 1\right)\left(1 - \frac{N}{CC}\right) - P + V^{\omega}, \quad \omega \in (0, 1)$$

- material requirement: $P = \gamma Y = \gamma_0 A^{-a} Y^{n}$
- $(*)$ utility: $U = U(C, N) = \ln C + \phi \ln N$

Steps of analysis

(I) model with constant technology – dynamics

(II) the role of technology – dynamics

(III) investment in natural capital – dynamics

(I) Dynamics of the system: case of $A = 1$, $V = 0$ (1)

Assumptions:

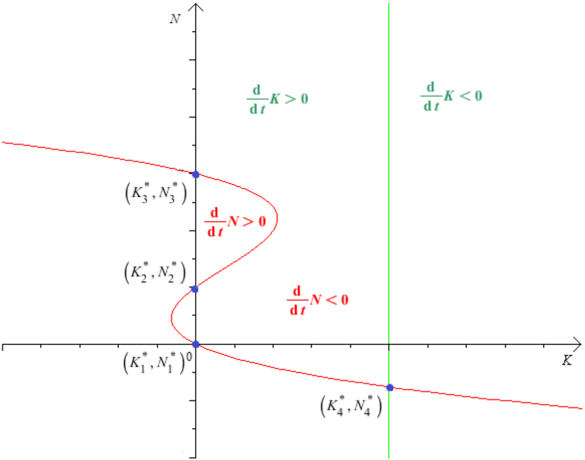
- $A = 1$ – constant technology
- s – saving rate, $\dot{K} = sY - \delta K$
- $P = \gamma_0 K^{\alpha n} \quad \Longrightarrow \quad \dot{N} = rN\left(\frac{N}{CT} - 1\right)\left(1 - \frac{N}{CC}\right) - \gamma_0 K^{\alpha n}$

Stationary points are solutions of the system:

$$\begin{cases} \dot{K} = 0 \\ \dot{N} = 0 \end{cases} \Longrightarrow \begin{cases} sK^{\alpha} - \delta K = 0 \\ rN\left(\frac{N}{CT} - 1\right)\left(1 - \frac{N}{CC}\right) - \gamma_0 K^{\alpha n} = 0 \end{cases}$$

It may have four, five or six stationary points, depending on the environmental capacity, critical treshold and the rate r.

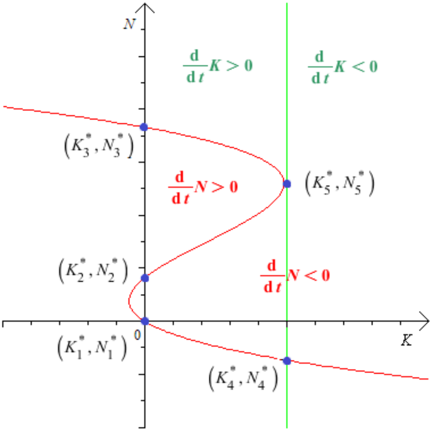
(I) Dynamics of the system: $A = 1$, $V = 0$: stationary points (2)



(a) $\Delta > 0$

Figure: Possible stationary points of the system

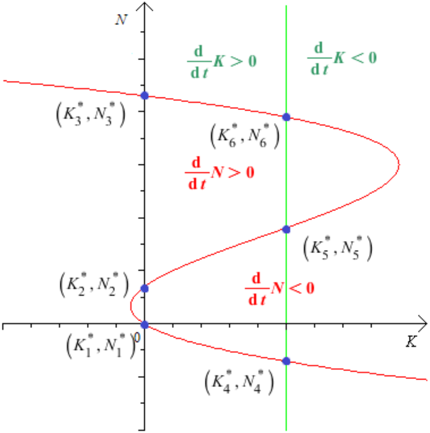
(I) Dynamics of the system: $A = 1$, $V = 0$: stationary points (2)



(b) $\Delta = 0$

Figure: Possible stationary points of the system

(I) Dynamics of the system: $A = 1$, $V = 0$: stationary points (2)



(c) $\Delta < 0$

Figure: Possible stationary points of the system

(I) Dynamics of the system: $A = 1$, $V = 0$: solvability (3)

- Isoclines:

$$\begin{cases} K = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}} \\ \frac{\gamma_0}{r} K^{\alpha n} = -\frac{1}{CT \cdot CC} N^3 + \frac{CC+CT}{CT \cdot CC} N^2 - N \end{cases}$$

$$-\frac{1}{CT \cdot CC} N^3 + \frac{CC + CT}{CT \cdot CC} N^2 - N - \frac{\gamma_0}{r} \left(\frac{s}{\delta}\right)^{\frac{\alpha n}{1-\alpha}} = 0$$

- Decisive quantites:

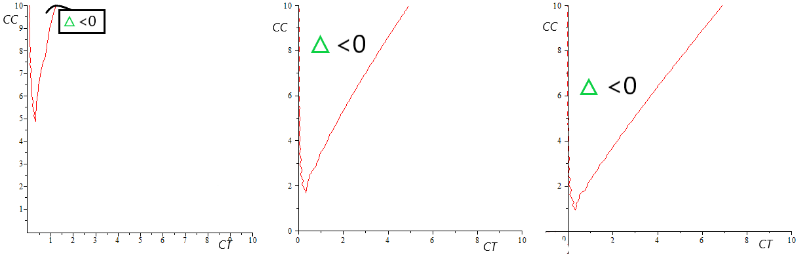
$$\frac{1}{r} \cdot \gamma_0 \left(\frac{s}{\delta}\right)^{\frac{\alpha n}{1-\alpha}} \equiv \frac{E}{r}$$

$$\begin{aligned} \Delta \quad := \quad & \left[-\tfrac{2}{27}(CT + CC)^3 + \tfrac{E}{r} CT \cdot CC + \tfrac{1}{3}(CT + CC) \cdot CT \cdot CC\right]^2 \\ & + \tfrac{4}{27}\left[CT \cdot CC - \tfrac{1}{3}(CT + CC)^2\right]^3 \end{aligned}$$

- Cardano formulas imply:

no internal (positive) solution iff $\Delta > 0$
one internal (positive) solution iff $\Delta = 0$
two internal (positive) solutions iff $\Delta < 0$

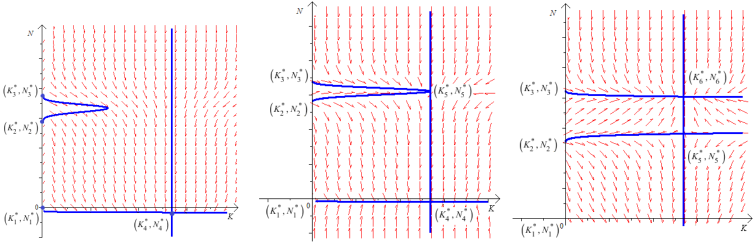
(I) Dynamics of the system: $A = 1$, $V = 0$: solvability (4)



(a) $E >> r$ (b) $E = r$ (c) $E << r$

Figure: Combination of (CT, CC) allowing (at fixed rate r) existence of positive internal solutions.

(I) Dynamics of the system: $A = 1$, $V = 0$ (5)



(a) $\Delta > 0$ (b) $\Delta = 0$ (c) $\Delta < 0$

Figure: Dynamics of the system (exemplary values of parameters: $\alpha = 0.3$, $s = 0.1$, $n = 0.2$, $\delta = 0.05$, $g_0 = 0.2$; for the case (a): $r = 0.15$, $CT = 6$, $CC = 8$; for the case (b): $r = 0.15$, $CT \approx 5$, $CC \approx 7.23$; for the case (c): $r = 0.25$, $CT = 4$, $CC = 9$). Isoclines in blue.

(I) Dynamics of the system: $A = 1$, $V = 0$: bassin of attraction (6)

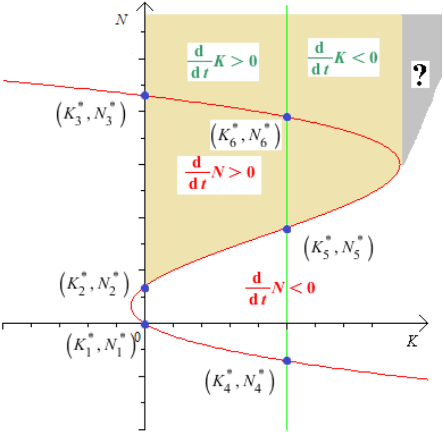


Figure: Bassin of attraction of the point (K_6^*, N_6^*) (yellow)

(I) Analysis of quantity Δ

limit	**interpretation**
$\lim_{CC \to 0^+} \Delta(CC, CT, r) = 0$	when natural capital disapears ($CC = 0$), then the model reduces to the standard Solow model with one stationary point and typical dynamics.
$\lim_{CC \to +\infty} \Delta(CC, CT, r) = -\infty$	if the carrying capacity increases, there are two positive stationary points (man-made capital, catural capital)
$\lim_{CT \to 0^+} \Delta(CC, CT, r) = 0$	if there is no critical treshold for natural capital, the model boils down to the one presented in [Rodrigues et al., 2005].
$\lim_{CT \to CC^-} \Delta(CC, CT, r) = \frac{E}{r}\left(\frac{4}{27}CC + \frac{E}{r}\right) CC^4 > 0$	if the level of the critical treshold is close to the carrying capacity, there is no positive stationary points (natural capital is exploited)
$\lim_{r \to +\infty} \Delta(CC, CT, r) = -\frac{1}{27} CC^2 CT^2 (CC - CT)^2 < 0$	the increasing regeneration rate leads to two positive stationary points
$\lim_{r \to 0^+} \Delta(CC, CT, r) = +\infty$	the decreasing regeneration rate leads to exploitation of natural capital

(II) Dynamics of the system: influence of technology, $V = 0$ (1)

Quantities explicitly dependent on A

$$\frac{\dot{A}}{A} = g_A$$

$$Y = K^\alpha A^{1-\alpha}$$

$$P = \gamma_0 A^{-a} Y^n$$

Introducing:

$$k = \frac{K}{A}, \quad y = \frac{Y}{A},$$

the model in those new terms is described by equations:

$$\begin{aligned} y &= k^\alpha \\ \dot{k} &= s k^\alpha - (\delta + g_A) k \\ \dot{N} &= rN\left(\frac{N}{CT} - 1\right)\left(1 - \frac{N}{CC}\right) - \gamma_0 A^{n-a} k^{\alpha n} \end{aligned}$$

The positive capital solution obviously is:

$$k^* = \left(\frac{s}{\delta + g_A}\right)^{\frac{1}{1-\alpha}}$$

(II) Dynamics of the system: influence of technology, $V = 0$ (1)

Quantities explicitly dependent on A

$$\frac{\dot{A}}{A} = g_A$$

$$Y = K^{\alpha} A^{1-\alpha}$$

$$P = \gamma_0 A^{-a} Y^{n}$$

Introducing:

$$k = \frac{K}{A}, \quad y = \frac{Y}{A},$$

the model in those new terms is described by equations:

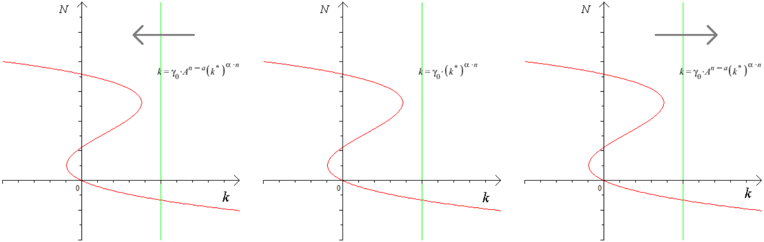
$$\begin{aligned} y &= k^{\alpha} \\ \dot{k} &= sk^{\alpha} - (\delta + g_A)k \\ \dot{N} &= rN\left(\frac{N}{CT} - 1\right)\left(1 - \frac{N}{CC}\right) - \gamma_0 A^{n-a} k^{\alpha n} \end{aligned}$$

The positive capital solution obviously is:

$$k^* = \left(\frac{s}{\delta + g_A}\right)^{\frac{1}{1-\alpha}} < \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}.$$

(II) Dynamics of the system: influence of technology, $V = 0$ (2)

$$\begin{cases} k = k^* \\ \gamma_0 A^{n-a}(k^*)^{\alpha n} = rN\left(\frac{N}{CT} - 1\right)\left(1 - \frac{N}{CC}\right) \end{cases}$$



(a) $n < a$ (b) $n = a$ (c) $n > a$

Figure: Possibility of internal solutions in the model with technology.

(∗) For (b): $E = \gamma_0 \left(\frac{s}{\delta + g_A}\right)^{\frac{\alpha n}{1-\alpha}}$, $\Delta < 0$.

(∗) empirical studies: $n > a$, hence $k^* \searrow 0$ (for $g_A = const$)

(II) Dynamics of the system: influence of technology, $V = 0$, $g_A = g(g_K)$ (3)

after Rodriguezs et al (2005):

$$g_A = \frac{\dot{A}}{A} = g\left(\frac{\dot{K}}{K}\right), \quad g(\cdot) = 0 \text{ for } \frac{\dot{K}}{K} \leq 0,$$

where g is a concave and continuous function for $g_K = \frac{\dot{K}}{K} > 0$, $g'(0) > 1$ and bounded from above.

If natural capital N^* is to be kept constant, i.e. $\dot{N}^* = 0$, we should require $P = \gamma_0 A^{-a} Y^n$ to be constant as well. Therefore:

$$-a g_A + n g_Y = 0$$

and in conclusion:

$$g_K = \frac{1}{\alpha}\left(\frac{a}{n} - 1 + \alpha\right) g(g_K).$$

(II) Dynamics of the system: influence of technology, $V = 0$, $g_A = g(g_K)$ (4)

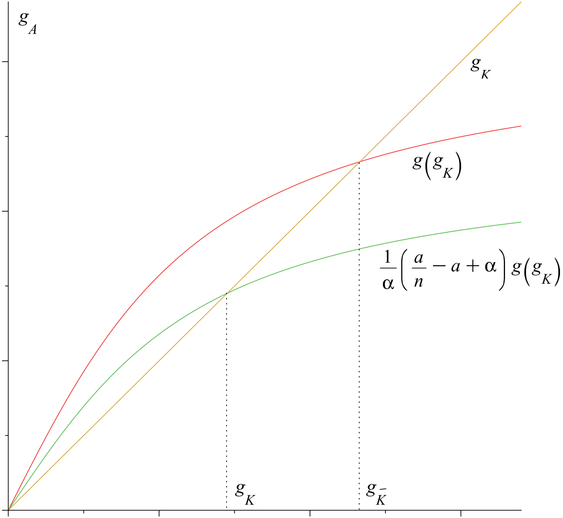


Figure: The fixed point of function g

(III) Dynamics of the system: investment in natural capital (1)

Division of production

$$Y = I + V + C$$

$$I = sY, \quad V = \nu Y$$

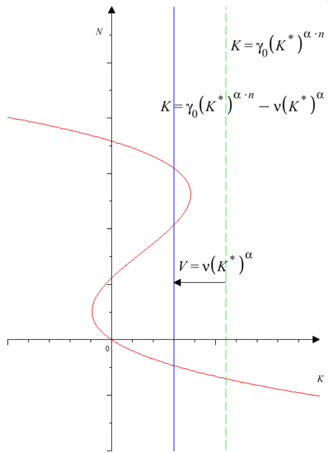
- Dynamics:

$$\begin{aligned} \dot{K} &= Y - C - V - \delta K, \\ \dot{N} &= rN\left(\frac{N}{CT} - 1\right)\left(1 - \frac{N}{CC}\right) - P + V. \end{aligned}$$

- Stationary points (under assumption of constant rates of savings)

$$\begin{aligned} \dot{K} &= sK^{\alpha} - \delta K, \\ \dot{N} &= rN\left(\frac{N}{CT} - 1\right)\left(1 - \frac{N}{CC}\right) - \gamma_0 K^{\alpha n} + \nu K^{\alpha}. \end{aligned}$$

(III) Dynamics of the system: investment in natural capital (2)



Internal positive capital: $K^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$; choice of $\nu \in (0, 1-s)$

Figure: The effect of investment in natural capital.

$$\gamma_0 \cdot \left(\frac{s}{\delta}\right)^{\frac{\alpha(n-1)}{1-\alpha}} - r\tilde{N}\left(\frac{\tilde{N}}{CT}-1\right)\left(1-\frac{\tilde{N}}{CC}\right)\cdot\left(\frac{s}{\delta}\right)^{-\frac{\alpha}{1-\alpha}} \leq \nu \leq 1-s.$$

Main conclusions

1. Extension of the standard neoclassical model of economic growth by the requirement to preserve natural capital significantly enriches the dynamic behaviour of the economy.

Main conclusions

1. Extension of the standard neoclassical model of economic growth by the requirement to preserve natural capital significantly enriches the dynamic behaviour of the economy.
2. In the model with a fixed rate of savings, we provide the condition for existence of a stable equilibrium with a positive stock of natural capital. Requirements:

$$\Delta < 0 \iff \begin{array}{l} \left[-\frac{2}{27}(CT+CC)^3 + \frac{E}{r}CT\cdot CC + \frac{1}{3}(CT+CC)\cdot CT\cdot CC\right]^2 \\ +\frac{4}{27}\left[CT\cdot CC - \frac{1}{3}(CT+CC)^2\right]^3 < 0 \end{array}$$

(nonempty set of (CT, CC) when r and E are fixed)

(K_0, N_0) belongs to the bassin of attraction of the stationary point

Main conclusions

1. Extension of the standard neoclassical model of economic growth by the requirement to preserve natural capital significantly enriches the dynamic behaviour of the economy.
2. In the model with a fixed rate of savings, we provide the condition for existence of a stable equilibrium with a positive stock of natural capital. Requirements:

$$\Delta < 0 \iff \begin{array}{l} \left[-\frac{2}{27}(CT+CC)^3 + \frac{E}{r}CT\cdot CC + \frac{1}{3}(CT+CC)\cdot CT\cdot CC\right]^2 \\ +\frac{4}{27}\left[CT\cdot CC - \frac{1}{3}(CT+CC)^2\right]^3 < 0 \end{array}$$

   (nonempty set of (CT, CC) when r and E are fixed)

   (K_0, N_0) belongs to the bassin of attraction of the stationary point

3. Keeping the natural capital N on constant level requires capital accumulation to slow down.

Main conclusions

1. Extension of the standard neoclassical model of economic growth by the requirement to preserve natural capital significantly enriches the dynamic behaviour of the economy.
2. In the model with a fixed rate of savings, we provide the condition for existence of a stable equilibrium with a positive stock of natural capital. Requirements:

$$\Delta < 0 \iff \begin{array}{l} \left[-\frac{2}{27}(CT+CC)^3 + \frac{E}{r}CT\cdot CC + \frac{1}{3}(CT+CC)\cdot CT\cdot CC\right]^2 \\ +\frac{4}{27}\left[CT\cdot CC - \frac{1}{3}(CT+CC)^2\right]^3 < 0 \end{array}$$

   (nonempty set of (CT, CC) when r and E are fixed)

   (K_0, N_0) belongs to the bassin of attraction of the stationary point

3. Keeping the natural capital N on constant level requires capital accumulation to slow down.
4. Investments in natural capital are an additional factor allowing to obtain an internal stable equilibrium.

Literature (1)

- S. Alessandro, *Non-linear dynamics of population and natural resources: The emergence of different patterns of development*, Ecological Economics 2007, vol. 62, s. 473-481. L. Bretschger, *Is the Environment Compatible with Growth? Adopting an Integrated Framework*, CER-ETH – Center of Economic Research at ETH Zurich, Working Paper no 16/260.
- J. A. Brander, M. S. Taylor *The Simple Economics of Easter Island: A Ricardo-Malthus Model of Renewable Resource Use*, The American Economic Review, Vol. 88, No. 1 (Mar., 1998), pp. 119-138
- P. Comolli, *Sustainability and growth when manufactured capital and natural capital are not substitutable*, Ecological Economics 2006, vol. 60.
- P. Dasgupta, G. Heal, *The optimal depletion of exhaustible resources*, Review of Economic Studies 41, Symposium on The Economics of Exhaustible Resources, 1974, s. 3-28.
- J. M. Hartwick, *Intergenerational Equity and the Investing of Rents from Exhaustible Resources*, The American Economic Review, 67(5), 1977, s. 972-974.

Literature (2)

- D. H. Meadows, D. L. Meadows, J. Randers, W. W. Behrens, *Granice Wzrostu*, Państwowe Wydawnictwo Ekonomiczne, Warszawa 1973.
- J. Rodrigues et. al., *Constraints on dematerialisation and allocation of natural capital alon a sustainable growth path*, Ecological Economics 2005, vol. 54, s. 382-396. S. Bringezu, H. Schutz, S. Moll, *Rationale for and Interpretation of Economy-Wide Materials Flow Analysis and Derived Indicators*, Journal of Industrial Ecology 2003, Vol. 7, No. 2
- R. Solow, *Intergenerational equity and exhaustible resources* , Review of Economic Studies, 41, Symposium on The Economics of Exhaustible Resources, 1974, s. 29-46
- J. Stiglitz, *Growth with exhaustible natural resources: efficient and optimal growth paths*, Review of Economic Studies, 41, Symposium on The Economics of Exhaustible Resources, 1974, s. 123-137.

THANK YOU FOR YOUR ATTENTION