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Daniel P. Monteiro

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Abstract

This paper presents a novel, integrated framework for the financial assessment of different types of common sovereign debt instruments in a currency union such as the euro area and provides results for their key credit risk properties at Member State and euro area level. The options under assessment include instruments involving full and partial mutualisation of sovereign risk, as well as instruments based on the pooling and tranching of national government debt without mutualisation. The results show that full risk mutualisation can lower financing costs for all participating countries, and that "eurobonds" would have weathered well the European sovereign debt crisis, even if partial mutualisation remains the most attractive option for the more creditworthy countries. Options involving just the tranching and pooling of Member State debt simply reallocate sovereign risk across instruments, although the "E-bonds" proposal can approximate the characteristics of mutualised "blue bonds" under certain conditions. The analytical framework can also be applied to assess the strength of sovereign risk interlinkages, the evolution of sovereign debt capacity over time and its decomposition into systemic and idiosyncratic factors.

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Keywords: Safe Assets, Eurobonds, Blue Bonds, E-bonds, ESBies.

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Contact: Daniel P. Monteiro, Directorate-General for Economic and Financial Affairs, daniel.monteiro@ec.europa.eu.

CONTENTS

1 Introduction

This paper introduces a methodology for the assessment of the credit risk properties of a wide range of common sovereign debt instruments, covering options with and without mutualisation. The approach relies on a structural credit risk model for the inference of sovereigns' debt capacity or "asset" value, from market behaviour. Given the debt capacity series of the participating sovereigns as well as key parameters such as their volatility and cross-country correlations, counterfactual simulations can be produced showing how different types of common debt instruments would have behaved over time.

The use of structural credit risk models in a sovereign context is scarce in the literature. The fact that sovereigns possess no directly observable asset value which, furthermore, cannot be traded, means that structural models are more easily implementable for the assessment of corporate credit risk, for which they were originally developed (see Merton (1974)). The more common approach of relying instead on reduced-form credit risk models is, however, not appropriate given the aim of the present analysis. While reduced-form models can generally offer a good explanatory power when assessing credit risk due to their data-driven structure and reliance on potentially many explanatory variables, they do not depend on the concept of underlying asset value (or sovereign debt capacity), which is the economically-meaningful variable forming the basis upon which scenarios involving the pooling together of debt capacities can be simulated.

Even though the scope of this paper relates to the credit risk properties of common debt instruments in a partial equilibrium setting, a number of broad macroeconomic and financial benefits have been ascribed to such instruments, including their ability to break the pernicious sovereign-bank loop whereby weak banks enfeeble their domestic governments which, in turn, contributes to further deteriorate the situation of the domestic banking sector. By not being directly linked to a particular sovereign, a common debt instrument can sever the direct channels of the sovereign-bank nexus, while offering credit institutions a safe asset for investment, risk and liquidity management purposes, as well as for use in monetary policy operations.[1] A large pool of safe common debt can also improve the macroeconomic and institutional functioning of the euro area through several other channels. It can bolster the international role of the euro by providing financial markets with a sizable euro-denominated safe asset,[2] while helping to deepen European capital markets.[3] Common European Union (EU) debt can also provide a powerful signal of commitment to the European project, generating positive confidence effects[4] and lowering perceived redenomination risks. In the case of instruments involving some form of debt tranching, common debt can also enhance the role of market discipline in an economic and monetary union by raising marginal sovereign financing costs without negatively impacting average interest rates.[5] More generally, common sovereign debt instruments can expand the supply of safe assets, the relative scarcity of which has been recognised in the literature as impairing proper macroeconomic functioning.[6]

Furthermore, from a financial viewpoint, the pooling together of imperfectly-correlated sovereign

[1]The sovereign-bank loop, which is thought to have played a large role in the euro area following the 2008 financial and economic crisis, is largely born of banks' home bias, whereby credit institutions tend to own disproportionately high levels of government debt issued by their domestic sovereign. On how a common European safe asset can help sever this loop see, e.g., Brunnermeier et al. (2017), Alogoskoufis and Langfield (2020) and Bellia et al. (2021).

[2]On how the constrained supply of euro-denominated safe assets has hindered the international role of the euro, see Ilzetzki et al. (2020).

[3]On the importance of safe assets for promoting the European Union's Capital Markets Union initiative see, e.g., Lannoo and Thomadakis (2019).

[4]These effects were arguably seen with the announcement of the NextGenerationEU (NGEU) proposal in the midst of the pandemic crisis in 2020, which implied the largest ever EU debt issuance, and which put sovereign risk on a firm downward trend across the euro area. For a sovereign risk timeline surrounding the announcement of NGEU, see Bellia et al. (2021).

[5]On the positive effect of higher marginal financing costs on fiscal prudence see, e.g., Meyermans (2019). The result that debt tranching raises marginal financing costs without negatively impacting average financing costs is discussed in this paper.

[6]See, for instance, Caballero and Farhi (2017).

risk can potentially lower overall credit risk premia and financing costs for participating governments, with potential positive spill-over effects for the private sector and the wider economy. A quantification of the credit risk premia on different forms of common debt instruments - as well as the extent and the conditions under which these instruments can achieve risk diversification in a manner not already available to market participants -, is one of the key focuses of this paper. In fact, while the potential savings or costs associated with different instruments is a matter of interest on its own, it is also intimately linked to the political economy surrounding the common EU issuance debate, given concerns of subsidisation of higher-yield Member States by more creditworthy countries.

While there can be various benefits to a common debt instrument in a currency union, potential drawbacks and implementability issues need also be recognised. Foremost among these is the fact that a sizable and permanent common debt instrument may require a higher level of political and economic integration, around which there may be no consensus. This is particularly true in the case of options that involve some form of mutualisation and risk sharing. In these cases, mechanisms should be in place that contain moral hazard issues whereby a Member State may unduly rise its debt issuance knowing that any increases in risk premia will be muted and shared by the participating countries. Mutualised options also require mechanisms that operationalise the joint guarantee in case of sovereign default so that, to the extent that it affects the mutualised instrument, the event remains an internal matter for the participating countries, without market implications. In addition, some of the benefits derived from the existence of a large pool of common safe assets are likely to partly cannibalise the benefits inherent to the status quo, as when this pool partly substitutes for existing safe national bonds, or when a lower liquidity premium on the common safe asset is associated with higher liquidity premia on national tranches. Finally, fully transitioning to a common debt instrument is an untested procedure which can involve a number of challenges, including the issue of how to deal with legacy debt.

Over the years a number of proposals have been made as to the design features of a common debt instrument in the euro area. One of the first was the blue bond and red bond proposal, as described in Delpla and von Weizsäcker (2010, 2011). According to this partial mutualisation proposal, Member States would issue a commonly guaranteed debt instrument (the blue bond) up until 60% of GDP. Any debt exceeding that level would be subordinated and exclusively guaranteed by the issuing Member State (the red bond). In European Commission (2011) three options were discussed. The first involved full sovereign risk mutualisation whereby national debt would be substituted by a single common debt instrument jointly guaranteed by all participating Member States. This fully mutualised instrument is sometimes called a eurobond. The second option is akin to the blue bond proposal while the third option involves splitting national debt into a senior and junior tranche, and subsequently pooling together the senior tranches into a single instrument. The latter option is akin to what has been called an E-bond in the literature, and has been analysed in, e.g., Leandro and Zettelmeyer (2018) and Giudice et al. (2019). It differs from the blue bond proposal notably because the pool of senior debt does not enjoy the joint guarantees of the blue bond and is, therefore, directly subject to the credit risk of the issuing sovereigns. Another proposal that has gathered significant interest is that of Brunnermeier et al. (2017), which is based on the pooling together of (part of) national sovereign debts by, e.g., a euro area agency, which would subsequently issue safe European Senior Bonds (ESBies) and riskier European Junior Bonds (EJBies). While neither E-bonds nor ESBies involve risk mutualisation, it should be noted that in the ESBies proposal debt tranching is done *a posteriori*, that is, as part of a securitisation process, after the debt securities have been issued and pooled together. This differs from the E-bond setup where tranching is done *a priori* by issuing debt securities in the primary market with different levels of seniority. In case a common European debt agency is involved in the E-bond construction, tranching can be achieved by having that institution collect funds in the market and lending them on to Member States with a preferred creditor status. Since 2020, the EU has embarked on its largest ever issuance programmes in connection with the NGEU and SURE initiatives, which have been set up as part of the EU's response to the covid-19 crisis. The resulting temporary debt instruments can be understood as involving a degree of partial mutualisation[7] and, although unprecedentedly large,[8] are nevertheless much smaller than some theoretical constructs involving permanent mutualisation, such as the blue bond and eurobond proposals.

The literature on common debt instruments lacks an integrated analytical framework that can be applied to all the different options in order to assess their key financial properties such as probabilities of default, loss given default and credit risk premia. In fact, this quantification is generally lacking as regards instruments involving mutualisation, although progress has been made regarding non-mutualised forms. A first attempt at quantifying the financial properties of a specific common debt instrument was made in Brunnermeier et al. (2017), where a two-stage Monte Carlo simulation is employed to study ESBies. However, such an approach is not extensible to cases involving mutualisation, does not offer a time series dimension allowing for counterfactual simulations over time (as it seeks consistency with a single historical data point), and the choice of probabilities of default and loss given default across countries and states of the world employed in the simulations involves a degree of arbitrariness. Since then, further progress has been made. For instance, ESRB High-Level Task Force on Safe Assets (2018) analyse ESBies (also called sovereign bond-backed securities or SBBS) extending the analysis in Brunnermeier et al. (2017) by relying on broader methodology and empirical evidence. ESBies are also the object of: i) Barucci et al. (2022), who analyse the riskiness of the senior and junior tranches, modeling default correlation across individual countries through a static Gaussian copula; ii) Frey et al. (2020), who employ an affine credit risk model with regime switching, extending it also to the case of E-bonds; and iii) Gottschalk (2022), who models default correlation using Gaussian and t copulas in a Bernoulli mixture.

Among the few instances where a structural credit risk model is applied to a sovereign context, Gapen et al. (2008) is the closest in spirit to the approach described in this paper. In that paper the authors present a framework for measuring sovereign risk based on the theory of contingent claims. Specifically, the authors rely on the Black and Scholes (1973) formulae for a call option and for the volatility of the underlying asset and proceed to tease out the unknown asset value and volatility from these two equations using standard methods. Besides pursuing different analytical goals and research questions, there are important methodological differences between the approach in Gapen et al. (2008) and the method presented in this present paper, as discussed in the following section. The approach followed in this paper also differs from the previously cited literature on common sovereign debt instruments by allowing for the analysis of instruments involving full and partial risk mutualisation, while offering a general framework that can be applied to analyse and draw comparisons across different options, including non-mutualised ones. Contrary to what has at times been the case in the literature, the present framework offers a dynamic risk analysis that is fully consistent with the evolution of market behaviour over time, and where key model parameters are empirically-derived, rather than assumed.

The remainder of this paper is organised as follows. Section 2 presents the main methodological aspects by discussing the underlying credit risk model, the estimation procedure and the data sources used. Section 3 provides an overview of the results by presenting the gains and losses in terms of risk premia changes at Member State and euro area level, for all the common debt instruments detailed in the following sections. Section 4 discusses the full mutualisation (eurobonds) option and Section 5 explains in detail the framework developed for assessing the partial mutualisation option (blue bonds and red bonds). Section 6 discusses common debt instruments that do not involve explicit mutualisation, namely a tranching and pooling option (akin to E-bonds), a pooling and tranching option (akin to ESBies and EJBies) and, for comparison sake, options simply involving either the pooling or the tranching of national sovereign debt. Section 7 discusses some extensions of the methodology, including an assessment of the strength of sov-

[7]Loans to Member States under the SURE programme benefit from a system of guarantees. As regards NGEU, safeguards have been established in the EU budget allowing for the increase in Member State contributions, which can be interpreted as *de facto* generating limited and temporary mutualisation.

[8]The combined issuance potential of NGEU and SURE can reach up to 6% of euro area GDP by 2026, gradually declining thereafter. See Monteiro (2022).

ereign risk interlinkages, a decomposition of the evolution of sovereign debt capacity over time into systemic and idiosyncratic risk components, and a robustness check based on a fat-tailed distribution. Section 8 points out avenues for future research and concludes.

2 Model, data sources and estimation

The approach followed in this paper for assessing the properties of common debt instruments can be illustrated with a full mutualisation (eurobonds) example, as shown in Figure 1. It starts off with the observation of market quotes for sovereign credit default swap (CDS) spreads, a measure of a sovereign's creditworthiness, for each i sovereign in the sample, in a given month t. By assuming a conventional loss given default (LGD), these spreads are then translated into probabilities of default (PD) through standard methods. Each PD thus inferred is then matched with the respective sovereign debt level expected for a future period $t + n$, where the chosen period is consistent with the time horizon of the CDS. In order to infer sovereign i's debt capacity in a given month t, a structural credit risk model is imposed on the time series of the estimated PDs and debt levels. In fact, these series represent either the output of the structural model (the PD) or one of its inputs (the debt level). The structure provided by the credit risk model is then used to tease out the remaining parameters of relevance: the level of a sovereign's debt capacity (or "asset" value), its trend growth rate and its volatility. Given the time series thus obtained for a sovereign's debt capacity, one can then proceed to simulate counterfactual or prospective scenarios where countries' debt capacities and debt levels are partly or fully pooled together. For instance, under full mutualisation, joint guarantees imply that countries' debt capacities can be added together and compared with the summation of their expected debt levels. The new inputs, i.e., the aggregate debt capacity of the participating sovereigns, the volatility and trend growth rate of this aggregate variable and the aggregate expected debt levels are then taken to the original structural credit risk to estimate the PD of the fully mutualised instrument. A new LGD is also derived that is consistent with both the initial LGD assumptions and the characteristics of the common debt instrument under assessment. The counterfactual CDS spreads are then obtained on the basis of the new PD and LGD, providing the credit risk premium on the new instrument. The remainder of this section discusses in more detail each of these steps, and how they are applied to evaluate common debt instruments in the euro area.

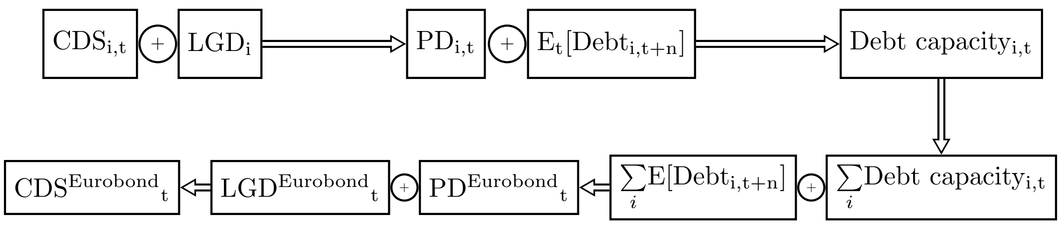

Figure 1: Deriving counterfactual credit risk premia for the case of full sovereign risk mutualisation (eurobonds)

The country sample considered throughout this paper comprises the founding Member States of the Euro (Austria, Belgium, Finland, France, Germany, Italy, Ireland, the Netherlands, Portugal and Spain), safe Luxembourg, and along with Greece, which adopted the Euro two years after its introduction. The time frame runs from January 2003 (i.e., one year after the physical introduction of the euro) to April 2016, for a total of 160 months. The country selection and time frame is partly motivated by data availability considerations and by a focus on the critical period of the sovereign debt crisis. In particular, the second wave of Euro adoption started in 2007, which means that the available time series under the Euro regime are, when available, significantly shorter for the seven Member States that have adopted the Euro since then. At the same time, their combined GDP is relatively small, adding only to approximately 2% of total euro area GDP by year-end 2015. This fact means that their inclusion in the sample would not significantly alter the qualitative and quantitative results shown in the following sections, while significantly restricting the starting date of our sample. The exclusion of Luxembourg from the analysis has to do with the absence of CDS data for that country. Again, Luxembourg's relatively small economic size (representing approximately 0.5% of euro area GDP in our sample period) and low debt levels mean that its exclusion should not noticeably alter the results. The beginning of the sample was set to January 2003 as, by that date, the CDS data for the 2-year horizon used in the estimations are available for all the Member States under analysis, with the exception of the Netherlands.[9] The chronological centrepiece of the analysis are the critical years of the European sovereign debt crisis, when risk premia reached their maximum levels, and which provide a natural stress test for the different constructions considered in this paper. The time sample stops in 2016, after the crisis has been resolved, and after the Public Sector Purchase Programme of the European Central Bank has been running for one year. In fact, as shown in the literature,[10] this programme exerted a significant downward pressure on sovereign yields, and may have entailed a degree of implicit mutualisation, which motivates focusing the analysis on a sample mostly unaffected by active policy intervention.

The inference of sovereign PDs from quoted sovereign CDS spreads is the first step in the procedure described in Figure 1. Euro-denominated CDS data based on pre-2014 ISDA definitions is taken from IHS Markit for a 2-year maturity. As discussed below, the choice of the maturity is such as to agree with the 2-year ahead debt forecasts used in the estimations. A monthly time frequency is considered throughout this paper. Although CDS spreads are available at a high frequency, debt data is of a lower frequency, motivating the use of monthly data as both a compromise between the frequencies of the two variables and as a safeguard against low trading volumes possibly affecting daily CDS data during some periods. As such, CDS data is based on the monthly average of daily CDS spreads. In some instances, 2-year CDS data is unavailable for short periods of time. In these cases, the dynamics of the 2-year spreads are made to emulate the dynamics of the corresponding 3-year CDS spreads, with any discrepancies in level between the two variables corrected for. PDs are inferred from CDS spreads based on a simple formula used by practitioners:[11]

$$PD = 1 - e^{\frac{-T \times CDS}{1-RR}}, \tag{1}$$

where T denotes the maturity in years (two, in the present case) and RR denotes the recovery rate, i.e., the percentage of the debt exposure that can be recovered by creditors in the event of default. RR is therefore the complement of the LGD, so that $1 - RR = LGD$. This formula is derived from a simplified no-arbitrage condition whereby the total income from the CDS spread accruing to the seller of the CDS contract, $T \times CDS$, should equal the expected loss on the reference bond faced by the seller of that CDS credit protection, $PD \times (1 - RR)$:[12]

$$PD \times (1 - RR) = T \times CDS. \tag{2}$$

It should be noted that both Equations (1) and (2) are simplified versions of more theoretically-correct formulae. The simplifications have essentially to do with the fact that i) both the payment of the CDS premia and the possibility of default are assumed to take place only at the maturity date of the reference bond and ii) cash flows are not discounted to their present value. The fact that a default is assumed to only possibly occur at the maturity date is consistent with the modeling framework discussed below and, in such a context, can be understood as a desirable

[9] The approach for dealing with data gaps for the Netherlands is described below.

[10] See, e.g., the results in Monteiro and Vasicek (2019) and the literature cited therein.

[11] For an application of the formula in an academic context see, e.g., Altman and Rijken (2011).

[12] Through simple algebraic manipulations we have that $PD \times (1 - RR) = T \times CDS \Leftrightarrow -PD = \frac{-T \times CDS}{(1-RR)} \Leftrightarrow e^{-PD} = e^{\frac{-T \times CDS}{1-RR}}$. Given that $e^{-PD} \approx 1 - PD$, we obtain that $PD \approx 1 - e^{\frac{-T \times CDS}{1-RR}}$, as per Equation (1). The logarithmic approximation error is negligible when PDs are low. For exceptionally high CDS spreads, Equation (1) ensures that PDs remain bounded between 0 and 1, which is the reason why a logarithmic approximation is employed. For instance, on the eve of Greece's default in March 2012, Equation (1) produces a PD of 99.9%, whereas in the absence of a logarithmic approximation Greece's inferred PD would be well above 100%.

property. The fact that the discounting and the time profile of cash-flows are not taken into account is of limited relevance given the short time horizon (two years) and the fact that discount rates are comparatively low for the period under consideration. In fact, as discussed below, the selection of an appropriate (risk-free) discount rate would be, in itself, problematic. For a more theoretically-correct approach for extracting PDs from CDS spreads, see Duffie (1999).

As discussed in Chan-Lau (2006), both bond yields and CDS spreads convey information on credit risk. We have relied essentially on CDS data given that it provides a direct measure of default risk[13] and does not strictly require the definition of a reference risk-free rate. In the case of the Netherlands, however, PDs were inferred from bond yields between 2003 and 2007 due to patchy CDS data during this period, using the following formula:

$$PD = \frac{1 - \frac{(1+r)^2}{(1+y)^2}}{(1 - RR)}, \tag{3}$$

where r denotes the (annualised) two-year risk free rate and y the (annualised) two-year yields on zero-coupon Dutch bonds.[14]

It should be noted that the sample covers a period when Greece was in default and no CDS spreads were observable. As such, Greece's inferred debt capacity for the period between April 2012 to May 2013 deviates from the procedure described hereunder and was linearly interpolated.[15]

The recovery rate RR is also taken from IHS Markit. Its modal value is 40% for each Member State, a conventional figure used in the quotation of many CDS contracts. The country-specific recovery rates are, however, allowed to deviate somewhat from their modal values for some periods in the IHS Markit database. As recovery rates and LGDs are subject to uncertainty, other assumptions would, nevertheless, be possible. When dealing with LGDs, the approach followed in this paper was guided by two criteria:

1. Consistency with usual assumptions made by market participants so that inferred PDs are not distorted and can reflect a representative market assessment;

2. Consistent treatment of LGDs across the different common debt instruments, and between these instruments and the initial LGD assumption used to infer historical PDs, so as to prevent the emergence of spurious results.

As regards Criterion 1, the choice of a conventional 40% figure for the RR (equivalently, of a 60% figure for the LGD) and the use of commercial RR series available to market participants is consistent with the intention of inferring debt capacities from actual market assessment and behaviour. Besides being the market convention for the quotation of sovereign CDS contracts,

[13] All financial instruments are potentially subject to pricing inefficiencies, including CDS quotes. In the present exercise, we take historical pricing as given and use it as a basis to simulate counterfactual performances, without seeking to "correct" for possible pricing inefficiencies in historical series. It is also worth mentioned that the crucial global financial crisis period seems to have deepened and increased interest in sovereign CDS markets, as observable in the offering of new maturities and denominations, as well as in the reduction of stale quotes in high-frequency data.

[14] This formula is derived from a no-arbitrage condition whereby the market price B of a zero-coupon bond is equal to its discounted expected payout at maturity. As such, we have that $B = \frac{(1-PD)\times M + PD \times RR \times M}{(1+r)^2} \Leftrightarrow \frac{1}{(1+y)^2} = \frac{(1-PD)+PD\times RR}{(1+r)^2} \Leftrightarrow PD = \frac{1-\frac{(1+r)^2}{(1+y)^2}}{(1-RR)}$, where M denotes the face value of the bond at maturity. The use of a risk-free discount rate means that the PDs thus inferred are understood to be risk-neutral. Yields on two-year zero-coupon German bonds were taken as the risk-free rate and, for consistency sake, Dutch PDs obtained from Equation (3) are bounded downwards by German PDs inferred from CDS swaps, a restriction that is only active in some months.

[15] An alternative procedure would be for the interpolation to be informed by the dynamics of other southern European countries. However, the correlation between Greece and other countries is particularly low in our sample, as confirmed by a regression of changes in Greece's debt capacity on other Member States'.

the use of a 40% RR can also be observed in, e.g., Altman and Rijken (2011).[16] Additionally, while lower sovereign LGD figures may sometimes be found in the literature, there is a case for relying on a more prudent and conservative 60% benchmark when simulating the counterfactual properties of common debt instruments as low LGDs could easily mean that, in some cases, senior debt tranches would be mechanistically assessed as risk-free.[17]

As regards Criterion 2, the detailed description in Sections 4 to 6 explains how overall consistency is achieved. It should be noted that, while a 60% LGD provides a unifying assumption at country-level, specific LGDs emerge endogenously from that assumption in the instruments involving the pooling and tranching of sovereign debt.

The inference of a sovereign's debt capacity (which can also be understood as their "asset value", broadly defined) relies on a structural credit risk model. In particular, sovereigns are assumed to possess a latent debt capacity that evolves stochastically, with default happening if this variable falls below actual debt levels. Let $A_{t,i}$ denote sovereign i's debt capacity in month t. As in a standard Merton model, the value process for $A_{t,i}$ is assumed to follow a geometric Brownian motion which, in discrete time, can be written as:

$$A_{t+1,i} = A_{t,i}e^{\mu_i+\sigma_i(W_{t+1,i}-W_{t,i})}, \tag{4}$$

where μ_i represents a trend growth rate,[18] σ_i is a volatility parameter and $W_{t+1,i} - W_{t,i}$ are i.i.d and $\sim N(0,1)$.[19] Sovereign i defaults if its debt capacity drops below its debt level. As such, its two-year ahead probability of default is given by

$$PD_{t,i} = P(A_{t+24,i} < D_{t+24,i}|\mathcal{F}_t), \tag{5}$$

where $D_{t+24,i}$ denotes the (expected) debt level (in billions of euro) of sovereign i in two years time and $\mathcal{F}_t$ the filtration at time t.[20] From Equation (4), it follows that $ln\left(A_{t+24,i}\right) \sim N\left(ln\left(A_{t,i}\right) + 24\mu_i, 24\sigma_i^2\right)$. As such,

$$PD_{t,i} = P\left(ln\left(A_{t,i}\right) + 24\mu_i + \sigma_i\sum_{n=1}^{24}\left(W_{t+n,i} - W_{t+n-1,i}\right) < ln(D_{t+24,i})\right)$$

$$= P\left(\sum_{n=1}^{24}\left(W_{n,i} - W_{n-1,i}\right) < \frac{ln(D_{t+24,i})-ln(A_{t,i})-24\mu_i}{\sigma_i}\right) = \Phi\left(\frac{ln(D_{t+24,i})-ln(A_{t,i})-24\mu_i}{\sqrt{24}\sigma_i}\right),$$

where Φ denotes the cumulative distribution function of a standard normal random variable. Let us denote $\Phi^{-1}\left(PD_{t,i}\right)$ as $dd_{t,i}$, the distance to default of sovereign i at time t. It follows that

$$\Phi^{-1}\left(PD_{t,i}\right) = \frac{ln(D_{t+24,i}) - ln\left(A_{t,i}\right) - 24\mu_i}{\sqrt{24}\sigma_i},$$

[16] For an in-depth discussion of the appropriateness of a 60% LGD assumption, as representing both a conservative and representative figure, see Subsection 3.1 of Giudice et al. (2019).

[17] For instance, with a 40% LGD, a country with debt worth 120% of GDP would not, in the event of default, impose losses on a senior tranche worth as much as 70% of GDP.

[18] For simplicity of notation, μ_i refers what is usually denoted as $\mu_i - \frac{1}{2}\sigma_i^2$ in a geometric Brownian motion. Because σ_i is not stochastic in our specification, the trend does not vary over time as a function of σ_i, motivating the present simplification.

[19] Subsection 7.2 departs from the normality assumption and conducts a robustness check based on a fat-tailed t distribution.

[20] The filtration represents the amount of information available at a given moment t, which is increasing over time. For simplicity, the conditioning on the $\mathcal{F}_t$ filtration is dropped in subsequent notation.

i.e.

$$ln(D_{t+24,i}) = ln(A_{t,i}) + 24\mu_i + \sqrt{24}\sigma_i \times dd_{t,i}. \tag{6}$$

The above equation provides a linear relationship between the logarithm of debt two years ahead, the logarithm of sovereign debt capacity and the distance to default. The logarithm of the expected two-year ahead debt is derivable from contemporaneous forecasts while the distance to default can be obtained from the PDs inferred via Equation (1). The remaining variable is unobservable and can be estimated with a Kalman filter by casting the model in state-space form. This is done by defining Equation (6) as the signal equation and the logarithm of Equation (4) as the state equation:

$$\text{Signal equation: } ln(D_{t+24,i}) = ln(A_{t,i}) + 24\mu_i + \sqrt{24}\sigma_i dd_{t,i} \tag{7}$$

$$\text{State equation: } ln(A_{t,i}) = ln(A_{t-1,i}) + \mu_i + \sigma_i \left(W_{t,i} - W_{t-1,i}\right) \tag{8}$$

There are two approaches for obtaining the series for the debt variable $D_{t+24,i}$. One possibility would be to rely on historical general government debt data, which is available at quarterly frequency for euro area Member States, and to interpolate the monthly figures via, e.g., a cubic spline. However, this approach assumes perfect foresight on the part of market participants. In order to sidestep this assumption, one can alternatively rely on contemporaneous debt forecasts. This is the approach followed in this paper, which uses nominal general government debt forecasts obtained from the European Commission's European Economic Forecast series. While this means taking these forecasts as representing the market consensus, there are no superior options for the concerned sample period in terms of country coverage, frequency of publication and cross-country consistency. In addition, given the European Commission's role as enforcer of the Stability and Growth Pact, its public finance forecasts carry non-negligible market weight as far as euro area countries are concerned. It is also worth noting that existing assessments show that European Commission economic forecasts tend to have an accuracy that is comparable to those of other international institutions and consensus-type forecasts (see Fioramanti et al. (2016)). European Commission forecasts were published twice a year between 2003 and 2008, as well as between 2010 and 2012. For 2009 and from 2013 onward three forecasts are available per year. For a given month t let $t^{preceding}$ denote the date of the forecast publication that immediately precedes t. Similarly, let $t^{succeeding}$ denote the publication date of the succeeding forecast document. We have therefore that $t^{preceding} \leq t \leq t^{succeeding}$, with $t^{preceding} < t^{succeeding}$. The two-year ahead forecast debt level in month t was calculated as a weighted average of the debt forecasts of the two neighbouring forecasts publications ($D^{preceding}_{t+24,i}$ and $D^{succeeding}_{t+24,i}$), with the weights adding up to one and reflecting the relative proximity of month t to the forecast document dates:

$$D_{t+24,i} = \left(\frac{t^{succeeding} - t}{t^{succeeding} - t^{preceding}}\right) \times D^{preceding}_{t+24,i} + \left(\frac{t - t^{preceding}}{t^{succeeding} - t^{preceding}}\right) \times D^{succeeding}_{t+24,i} \tag{9}$$

The nominal debt levels D_i are not explicitly shown in the European Commission's forecast publications and are obtained based on the projections for the debt ratio of general government, for real GDP and for the GDP deflator.[21] Also, given that any two-year ahead forecast in year y always refers to December of year $y+2$ in the forecast documents, D_{t+24} are, when necessary, linearly extrapolated or interpolated by a few months from the forecast debt value for December of year $y+2$ on the basis of the trend growth rate implicit in the respective forecast document.

[21] By definition, $D_{t+24,i} = (General\ Government\ Debt\ Ratio_{t+24,i}) \times GDP^{Real}_{t+24,i} \times \frac{GDP\ Deflator_{t+24,i}}{GDP\ Deflator_{t,i}}$.

This is necessary so that D_{t+24} indeed matches the same calendar month as t. Both Equation (9) and the aforementioned intra- and extrapolations are based on the underlying assumption that investors do not abruptly revise their two-year ahead forecasts on a monthly basis.

The reasons for choosing the previously-described model formulation and some of the underlying model assumptions deserve further consideration. While there are several competing structural credit risk models available in the literature,[22] the present framework offers two fundamental advantages. First, it relies on a fairly standard formulation, based on the Merton model's building blocks. As such, it offers a simple benchmark against which more complex approaches can be judged. Second, the model structure is sufficiently simple for it to be cast in an exact linear state-space form, thus allowing for the inference of the $A_{t,i}$ variables and related parameters. However, it should be noted that its simplicity also carries significant implications as to the underlying assumptions. For instance, we assume that default can only occur at a certain maturity horizon, as in the Merton model. This differs from the more complex Black and Cox (1976) model where default can happen at any time between t and maturity. While this would allow for a more theoretically-correct assumption in the present context, the added complexity of the Black-Cox model means that it cannot be cast into a linear state-space form that can be estimated with the standard Kalman filter. Overall, the possibility of default at any point in time might mean higher inferred levels for the $A_{t,i}$ variables, although the dynamics and covariances among sovereigns, which represent more fundamental aspects of the analysis, need not change materially. Other features that could potentially be part of more complex formulations include jump-diffusion dynamics, stochastic volatility and other forms of time-variability in the parameters.

It is also worth noting that structural credit risk models are usually applied in contexts where the composition and value of the assets of the entity under assessment can be determined with some degree of accuracy, namely by relying on market valuations and balance sheet statements. In a sovereign context, the notion of asset is fuzzier and the market-based measure $A_{t,i}$ includes therefore all the hard and soft factors that make up a sovereign's debt capacity. Besides classic elements such as physical and financial assets (including reserves) which play a modest role in a sovereign context when compared with canonical corporate applications, $A_{t,i}$ also comprises a market-based assessment of the government's willingness and capacity to raise revenue and cut expenses as necessary to ensure solvency. It also includes factors common across sovereigns, such as the monetary policy stance.

The sovereign-specific standard deviations σ_i represent another area where the Merton-like framework relies on comparatively simple assumptions. While the estimation procedure considers these standard deviations constant over time and known to investors, more complex formulations could include stochastic volatilities, jump-diffusion processes, other forms of time-variability in the parameters, as well as formulating future debt levels as a stochastic process. As before, however, more complex formulations would not lend themselves to an exact state-space estimation. In addition, should more complex features be empirically relevant, the present formulation will indirectly incorporate them by duly adjusting the inferred $A_{t,i}$.[23] Table 1 shows the set of standard deviation parameters σ_i as estimated via the state-space procedure, as well as the same standard deviations empirically calculated *ex post,* on the basis of the obtained $A_{t,i}$ series. As can be observed, space-state standard deviations are highly congruent with their *ex post* empirical counterparts, with the possible exception of Greece where the empirical standard deviation is somewhat larger than the state-space version. The set of trend growth rates μ_i is identical in the state-space and *ex post* empirical version for all countries.

The empirically-calculated correlation matrix for changes in the logarithm of sovereign debt capacity, $\Delta ln(A_{t,i})$, is shown as a heat map in Figure 2. This correlation matrix is a key component of the analysis of common debt instruments carried out in Sections 4 to 6. In

[22]For a review, see e.g., Bielecki and Rutkowski (2010).

[23]For instance, an increase in asset volatility would be captured in the model as a drop in the estimated $A_{t,i}$. In particular, because there are no error terms in Equations 7 and 8, an increase in asset volatility would, *ceteris paribus*, lower dd and lead to an equivalent decrease in A.

Table 1: Space-state and *ex post* empirical estimates of σ_i

	Space-state	Empirical
AT	0.009	0.010
BE	0.005	0.005
DE	0.007	0.008
EL	0.013	0.018
ES	0.013	0.014
FI	0.014	0.015
FR	0.006	0.007
IE	0.021	0.021
IT	0.003	0.004
NL	0.012	0.013
PT	0.008	0.009

particular, the analysis in those sections specialises the log-normality assumption embedded in Equation (4) to the stronger assumption of joint log-normality on the basis of the correlations shown in Figure 2. Subsection 7.2 shows how to depart from the joint (log-)normality assumption in order to test for the robustness of the results under a fat-tailed distribution. Alternatively, the analysis in the following sections could also have been carried out on the basis of a joint non-parametric empirical distribution of $\Delta ln(A_{t,i})$.

Increasing average correlation

	FR	ES	NL	FI	BE	AT	DE	PT	IE	IT	EL
FR		0.71	0.74	0.67	0.63	0.59	0.65	0.57	0.58	0.52	0.18
ES	0.71		0.73	0.69	0.79	0.56	0.54	0.53	0.72	0.35	0.10
NL	0.74	0.73		0.72	0.65	0.49	0.65	0.49	0.57	0.42	0.10
FI	0.67	0.69	0.72		0.59	0.54	0.61	0.58	0.38	0.49	0.20
BE	0.63	0.79	0.65	0.59		0.57	0.37	0.42	0.68	0.37	0.14
AT	0.59	0.56	0.49	0.54	0.57		0.50	0.64	0.38	0.44	0.18
DE	0.65	0.54	0.65	0.61	0.37	0.50		0.60	0.46	0.46	0.05
PT	0.57	0.53	0.49	0.58	0.42	0.64	0.60		0.28	0.45	0.22
IE	0.58	0.72	0.57	0.38	0.68	0.38	0.46	0.28		0.34	0.15
IT	0.52	0.35	0.42	0.49	0.37	0.44	0.46	0.45	0.34		0.27
EL	0.18	0.10	0.10	0.20	0.14	0.18	0.05	0.22	0.15	0.27	

Figure 2: Correlation matrix of $\Delta ln(A_{t,i})$

Before concluding the present section, a comparison with Gapen et al. (2008), who derive a measure of sovereign assets based on the theory of contingent claims, can serve to highlight some further methodological aspects. In concrete, Gapen et al. (2008) rely on the Black-Scholes formula for contingent claims, while this paper relies instead on the stochastic differential equation for modeling the asset value process that underpins both the Black-Scholes and the Merton model. This means that in the contingent claims approach only the asset value and volatility can be inferred, while the estimation of the trend growth rate of the asset value is sidestepped thanks to the change of measure embedded in the Black-Scholes option pricing formula, as allowed for by the Girsanov (1960) theorem. As such, while the contingent claims approach relies on numerical methods for extracting two unknowns (the level and volatility of the sovereign asset) from two non-linear equations (i.e., the Black-Scholes formulae for a call option and the volatility of an asset), the present approach relies on an exact state-space estimation method using information from market-based probabilities of default. One implication is that the approach described in this section allows for perfect conformity at all times with market-based spreads and PDs, which is not the case in the contingent claims approach. Another important implication stems from the fact that the contingent claims approach requires the input of an observable risk-free interest rate for its inference procedure. Determining the appropriate risk-free rate is, however, challenging in our sample. While interbank lending rates have often been taken as proxies for the risk-free rate, the interbank market stress experienced in connection with 2008 crisis meant that this rate would not be appropriate for part of our sample (see, for instance, European Central Bank (2014)). The alternative approach of considering the yields on the most creditworthy euro area Member States as the risk-free rate is equally problematic as it would impose *a priori* a zero probability of default on any such Member State (or even negative probabilities, in the case of negative spreads), a supposition that is not confirmed by an analysis of CDS spreads and which would partly thwart the objectives of a comparative and counterfactual analysis, as any instrument assumed to be risk-free cannot be improved upon from a credit risk perspective. There are also other differences worth noting between the contingent claims approach of Gapen et al. (2008) and the approach described in this section. While domestic currency liabilities are taken as the equivalent of the notion of equity in the contingent claims approach, as applied to emerging markets, this would not be appropriate for euro area Member States which, as a rule, issue all of their debt in the domestic euro currency. In the present approach, sovereign equity could be defined as $A_{t,i} - D_{t,i}$. It therefore represents an amount in excess of total debt, rather than a particular type of debt, and can be understood as a measure of fiscal space. In this connection, it is worth observing that sovereign debt capacity is measured with respect to total (projected) debt outstanding, which constitutes the sovereign's distress barrier, as per Equation (5). This contrasts with the aforementioned contingent claims approach, where the distress barrier is defined by assumption as equal to short-term external debt plus one half of long-term external debt. Finally, the call option on sovereign assets underlying the contingent claims approach is, of course, of a hypothetical nature, as a sovereign's asset is non-observable, non-tradable and cannot be seized compulsorily in the event of default.

3 Counterfactual funding cost gains and losses

This section presents upfront some key comparative results for the different common debt instruments discussed in this paper. With this overview in mind, the details of each option are then explored in Sections 4 to 6. For each common debt instrument and for each Member State, a measure of counterfactual funding cost gains or losses is constructed based on the estimated average decrease or increase in credit risk premia offered by that instrument when compared with historical premia. This measure is labeled the instantaneous steady-state (ISS) gain or loss and is calculated according to the following formula:

$$ISS_i = \frac{\sum_{t=1}^{T} CDS_{t,i}^{Counterfactual} - CDS_{t,i}^{Historical}}{T} \tag{10}$$

The ISS measure reflects the average risk premium gain or loss per month between January 2003 and April 2016, is measured in interest rate basis points and abstracts from the country-specific profile of debt rollover by assuming that changes in a country's credit risk are immediately reflected on the risk premium paid on the entire debt stock. It also abstracts from the country-specific maturity structure of government debt by applying the same differential gain across all maturities (i.e., by assuming in effect a level shift in the sovereign yield curve). In the instruments involving tranching, $CDS^{Counterfactual}$ reflects the average spread and is calculated as the weighted average of the estimated CDS spreads on each tranche, with the weights reflecting the relative size of the tranches at that point in time. In order to purge approximation errors from the ISS measure and to make the counterfactual perfectly comparable, the $CDS^{Historical}$ is a reconstructed historical spread that is similar, but not perfectly identical to the spread taken from our data sources and used in Equation (1). Three changes are introduced in the reconstructed historical spread. Firstly, it assumes a constant RR of 40%, rather than the rates taken from IHS Markit, which are mostly equal to 40%, but allow for deviations in some periods. The reason for this is that a 40% RR (equivalently, a 60% LGD) is a unifying assumption across different debt instruments[24] and, as such, is embedded in the $CDS^{Counterfactual}$. Secondly, historical CDS spreads are recalculated on the basis of the more theoretically-correct Equation (2) rather than Equation (1). This is due to the fact that the logarithmic approximation is no longer necessary when calculating $CDS^{Counterfactual}$ from counterfactual PDs, as the methodology described in Section 2 implies that these are necessarily bounded between 0 and 1. Finally, reconstructed historical PDs rather than the original historical PDs are used when deriving $CDS^{Historical}$ from Equation (2). The reconstructed PDs are similar to the historical PDs, but may diverge slightly as they are based on the empirical standard deviations shown in Table 1 rather than on the space-state standard variations. The reason for relying on the empirical standard deviations and covariances springs from the fact that these form the basis for the analysis of common debt instruments. In fact, a reliance on space-state covariances would be problematic for two reasons. Firstly, while the procedure can extract variances, it is not set in a manner that allows for the extraction of the full covariance matrix. The extraction of this matrix would require estimating $\frac{n\times(n+1)}{2} = \frac{11\times(11+1)}{2} = 66$ parameters for a set of 11 countries while relying on a parsimonious input data structure, something which is neither statistically nor computationally advisable. Secondly, instruments involving mutualisation rely on the summation of log-normally-distributed sovereign debt capacity, for which no exact statistical distribution is known. In those cases, the new variables resulting from the summation of log-normal variables are assumed to be log-normal, in line with standard rule-of-thumb approximations and with the value process assumptions of the Merton model. For the instruments involving mutualisation, the moments of the new log-normal distributions are thus made to match the underlying empirical moments, which can be calculated on the basis of the previously inferred $A_{t,i}$.[25]

Table 2 presents the ISS gains and losses for the different options under assessment for the period running from January 2003 to April 2016. For the sake of comparability, in all options involving some form of debt tranching the cut-off point for the senior tranche was set at 60% of GDP, the reference debt level set out in the European Union's Stability and Growth Pact. The last row of Table 2 shows the average (reconstructed) CDS spread for our sample period, allowing for a comparison of the historical creditworthiness of Member States and providing an upper bound for their respective ISS gains.

It can be observed that full mutualisation delivers the largest ISS gains for the euro area. These gains are particularly large for Member States that underwent market stress in the crisis period while being much more modest, though still positive overall, for the most creditworthy Member States. Partial mutualisation delivers the second highest ISS gains which, when compared with full mutualisation, tend to be larger for the most creditworthy Member States and significantly lower for Member States that underwent market stress. It should be noted that partial mutualisation and the other two options that involve *a priori* tranching[26] (tranching and pooling, and simple tranching) deliver rather similar gains both at euro area and Member State level. As discussed in the following sections, this is a consequence of the choice for the cut-off point of the senior tranche,[27] and is not a universal result. Overall, the first four options in Table 2 have in common the fact that they would have explicitly allowed for a Pareto movement as far as funding costs are concerned, whereby all eleven Member States would have been better off during the period under analysis when compared with the historical performance.

[24]Although a 60% LGD acts as a unifying assumption, this does not mean that all instruments and tranches are assumed to have a 60% LGD. The details of the LGD calculations are described in the following sections.

[25]Approximating a sum of lognormal variables with a lognormal distribution by matching the first and second moments is known as the Fenton-Wilkinson approximation (see Fenton (1960)). While this procedure generally entails a degree of approximation error, it is particularly justified in our case by being fully consistent with the modeling assumptions of the Merton model, whereby future "asset" values are lognormally distributed.

[26]By *a priori* tranching we mean a situation where Member States themselves issue two different types of debt securities with different levels of seniority. This contrasts with *a posteriori* tranching where an agency acquires un-tranched debt instruments and repackages them into tranches, as in the case of the pooling and tranching approach.

[27]It is also the consequence of a "sequential default" assumption, which will be discussed in Subsection 6.1.

Table 2: ISS gain (+) / loss (-) (basis points, January 2003 to April 2016)

	Euro Area	AT	BE	DE	EL	ES	FI	FR	IE	IT	NL	PT
Full mutualisation (Eurobonds)	70	26	25	4	707	75	5	14	121	94	10	209
Partial mutualisation (Blue bond and red bond)	19	27	12	11	17	49	17	13	33	16	19	55
Tranching and pooling ("E-bonds")	18	26	11	10	17	48	15	11	33	16	17	55
Simple tranching	18	24	13	11	18	42	1	13	30	17	12	55
Pooling and tranching ("ESBies" and "EJBies")[28]	0	-8	-61	-35	572	26	12	-52	35	-17	-8	121
Simple pooling[28]	0	-43	-45	-66	637	6	-65	-55	51	24	-60	139
Average CDS spread (2003-2016)	82	38	37	16	719	88	17	26	133	106	22	221

Note: the cut-off level of the senior tranches was set at 60% of GDP, where applicable. The euro area figures refer to the total gains accruing to the euro area aggregate composed of the 11 Member States considered in the simulations. Results for i) tranching and pooling and for ii) simple tranching are based on a sequential default assumption (see Subsection 6.1).

[28] The figures shown under i) pooling and tranching and ii) simple pooling represent the gains and losses of a hypothetical common debt agency associated with transacting different sovereign bonds in the market, or the gains and losses that would accrue to Member States under a scheme of mandatory issuance via a common debt agency.

The pooling and tranching and simple pooling options are neutral in terms of ISS gains at euro area level due to their pure financial engineering nature. This means that if a euro area debt agency were to issue these instruments by acquiring the underlying government bonds in the market and by securitising them, it should break even. Likewise, Member States would be placing their debt on the market under similar conditions as before, and would therefore be indifferent with respect to these schemes. However, if they were to become the compulsory form through which Member States would issue their debt, bypassing placement in primary debt markets by individual sovereigns, the schemes would benefit the least creditworthy Member States and impose ISS costs on the most creditworthy countries. While this paper reserves the term mutualisation for the cases where this happens explicitly through joint guarantees, as in the eurobond and blue bond instruments, the fact that the simple pooling and the pooling and tranching options imply either ISS costs or ISS benefits depending on the concerned Member States could be understood as a form of implicit mutualisation, if Member States were obliged to issue their debt directly through these common debt instruments. Even in this case, however, it would not mean that these two options could not allow for a Pareto movement in a broader macro-financial sense. In fact, while not delivering overall gains from a strict risk premia perspective and imposing risk premia costs on some Member States, the pooled debt instruments underlying the two options could still deliver positive institutional and stability dividends along the lines discussed in Section 1.

Some additional caveats should be borne in mind when considering the ISS results. Firstly, the counterfactual results refer to a particular sample running from January 2003 to April 2016, which confers a large weight to the sovereign debt crisis period. The inclusion of longer periods of stability would imply tamer CDS dynamics and further increase the ISS gains of the more creditworthy Member States. The analysis can thus be considered conservative from the viewpoint of this group of countries. Secondly, the counterfactual simulations assume unchanged debt issuance and unchanged commitment to debt service on the part of Member States. This implies, for instance, that under full mutualisation the proper institutional mechanisms would have been in place preventing higher or excessive issuance on the back of lower funding costs and joint guarantees. Likewise, under partial mutualisation and in the options involving *a priori* tranching, higher market discipline imposed on the junior tranche could result in lower marginal issuance, which however is not assumed in the counterfactual. Indeed, the underlying assumption for the mutualised instruments is unchanged moral hazard, and the results should be read in a *ceteris paribus* spirit. Thirdly, the sample covers a period when Greece was in default and no Greek CDS spreads were observable. In order not to shorten the sample size, the assumed historical figures for Greece from April 2012 to May 2013 are consistent with a scenario where Greece progressively retreats from the default edge after having approached a PD of 100%. Finally, the debt weights used to calculate the euro area aggregate ISS and the country-specific ISS for the options involving tranching are based on two-year ahead debt projections in order to ensure consistency with expected LGDs, which are also based on projected debt levels. This leads to zero euro area ISS gains for the pooling and tranching and simple pooling options, highlighting the neutrality of pure financial engineering for the overall euro area ISS. However, the aggregation of ISS gains and losses across Member States and tranches could also have been calculated on the basis of current, nowcast debt levels. Such time inconsistency between the nowcast debt levels used in the ISS aggregation and the forecast two-year ahead debt levels used for calculating LGDs (and, therefore risk premia) would generally lead to small changes in the ISS measure, whose sign and magnitude is sample-specific. The alternative ISS results for an aggregation based on nowcast debt shares are shown in Appendix A.

4 Full sovereign risk mutualisation

Under full mutualisation, euro area Member States issue their debt under a single instrument commonly guaranteed by all (i.e., a eurobond). In this case, the euro area defaults if its aggregate debt capacity falls below its aggregate debt level:

$$PD_{t,EA} = P\left(\sum_i A_{t+24,i} < \sum_i D_{t+24,i}\right). \quad (11)$$

In the event of default, a standard 60% LGD on aggregate euro area debt is assumed. It can be observed in Table 2 that this option provides, by far, the highest potential gains as reckoned by the ISS measure. All Member States are seen to benefit from a counterfactual reduction in credit risk premia in average terms over the period running from 2003 to 2016. This reduction is very high for sovereigns that have experienced market stress while being comparatively modest for some of the most creditworthy Member States. Furthermore, the counterfactual simulations suggest that full mutualisation would have kept the euro area PD contained throughout even the most acute phases of the crisis period (Figure 3). For a limited period of time concentrated in 2011 and 2012, the full mutualisation counterfactual somewhat underperforms the historical PDs and risk premia of DE, FI and NL, although these ISS losses are more than offset when the full sample is considered (Figure 4).

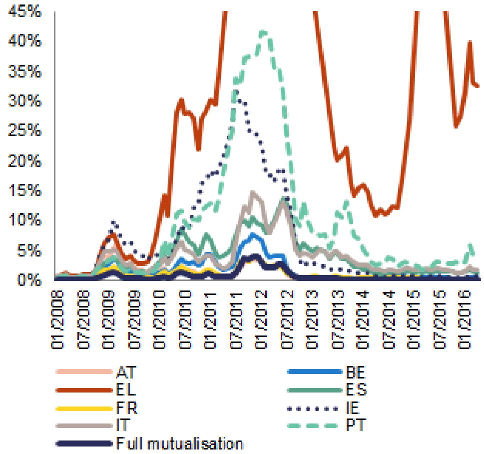

Figure 3: PDs in the crisis period: full mutualisation vs historical (selected Member States)[29]

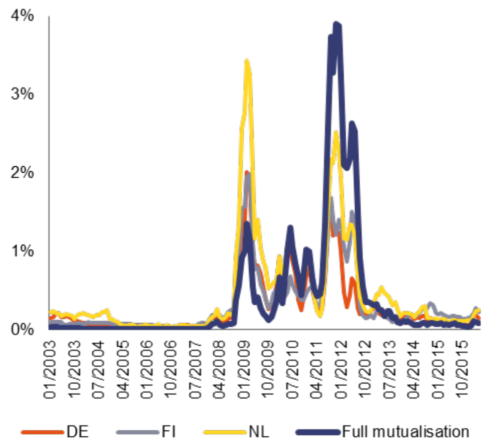

Figure 4: PDs of most creditworthy Member States: full mutualisation vs historical

The strongly positive results observed for the euro area ISS reflect the gains from the diversification of idiosyncratic sovereign risk and from the summation of national fiscal spaces, both

[29] Vertical axis truncated for visualisation purposes, given the outlier dynamics of EL.

of which can only be achieved through mutualisation. When debt capacity is pooled together, it becomes less volatile due to a diversification effect from adding up imperfectly correlated national debt capacities. Lower volatility, in turn, decreases the probability of hitting a debt-based distress barrier at a future point in time. At the same time, the combined fiscal space of the euro area is necessarily larger than that of any individual Member State, allowing for a bigger "safety buffer". These types of gains cannot be emulated by a market participant holding a diversified portfolio of debt securities, as can be observed by comparing the euro area results for full mutualisation with those for pure pooling in Table 2. In a corporate context, these gains are akin to the lower financing costs achieved by conglomerates encompassing within a single company a diversified set of imperfectly correlated assets. In effect, the full mutualisation counterfactual is similar in this respect to a corporate merger, which brings together imperfectly correlated assets and different equity buffers under a single corporate structure.[30]

While this section considers the theoretical financial gains from full mutualisation, it does not explore the real-world setting under which eurobonds would operate. It should be noted that while eurobonds diversify and reduce the credit risk of the euro area aggregate, so that a credible eurobond is essentially risk-free, this does not mean that individual Member States would never find themselves in a situation of insolvency. In fact, insolvency remains possible whenever a Member State's debt is too high when compared to its debt capacity. In order for eurobonds to be implementable it is thus necessary that internal euro area mechanisms exist so that market-facing common debt remains unaffected even in the event of an idiosyncratic default[31] of a Member State. These mechanisms can involve capital transfers, debt restructuring, financial assistance in the form of concessional loans and structural adjustment programmes that raise debt capacity. Some mechanisms, such as capital transfers, could imply non-negligible costs for participating Member States in the event of sovereign default. These costs are not reckoned in the ISS measure, which only captures changes in funding costs. However, a sovereign default generally implies significant negative spill-overs for the other Member States in the currency union also in the absence of mutualised instruments, as attested by the macrofinancial turbulence surrounding Greece's default during the European sovereign debt crisis. As such, the quantification of the relative costs of an actual sovereign default, with and without mutualised debt, requires a general equilibrium approach, with the results being dependent on the assumed governing institutional framework, an exercise that is beyond the scope of this paper. It can be noted, however, that contrary to the historical experience with Greece, mechanisms for handling sovereign default should be set out *ex ante*, for credibility sake. Also, the mere existence of eurobonds may prevent the emergence of idiosyncratic default, as the faltering Member State remains insulated from high interest rates and self-fulling prophecies of default, a potential benefit that is also not captured in the ISS measure.[32] These considerations and the need for an internal mechanism for handling sovereign defaults extends *mutatis mutandis* to the case of partial mutualisation, as analysed in the next section.

Another relevant consideration on the implementability of eurobonds relates to the issue of moral hazard. By lowering marginal financing costs and rendering them less reactive to national debt levels, eurobonds can incentivise Member States to increase their debt to levels that may be sub-optimal from the viewpoint of the euro area aggregate. As such, eurobonds require a proper institutional architecture preventing excessive debt buildup as well as corrective mechanisms for returning excessive debt to prudent levels. The discussion of these mechanisms is beyond the scope of this work, which assumes unchanged moral hazard. In fact, depending on the strength of such (possibly novel) mechanisms and on the particular form of mutualisation to be implemented, it is both conceivable that moral hazard increases or that is lessens, thus preventing taking an *a priori* view on the matter. The partial mutualisation case discussed in the next section can, in

[30] For an early discussion of how conglomerate mergers can increase debt capacity, see Lewellen (1971).

[31] By idiosyncratic default, we mean a situation where default is driven by idiosyncratic rather than systemic, euro area-wide factors. See also Subsection 7.1.

[32] Concretely, the inferred sovereign debt capacities were treated as exogenous with respect to risk premia. However, for a given debt level, inferred debt capacity can itself decline in response to high risk premia. This effect can lead to an underestimation of the benefits of full mutualisation.

particular, provide effective incentives for reducing excessive debt taking.

Before concluding the present section, it is worth noting that while the PDs under full mutualisation that can be observed, e.g., in Figure 4 refer to a hypothetical eurobond-like debt instrument, they can also be interpreted as a summary measure of actual systemic stress. This is due to the fact that PDs under full mutualisation capture the aggregate riskiness of the euro area when country-specific risk has been diversified to the maximum possible extent.

5 Partial mutualisation

Partial mutualisation involves, in the present context, a euro area senior debt tranche that is commonly guaranteed by all participating Member States, and which coexists with national junior tranches that are guaranteed only by the issuing Member State. This is the setting of the blue bond and red bond proposal whereby Member States would issue a jointly guaranteed senior debt instrument up until a debt level worth 60% of its GDP (the blue bond). Any debt issuance exceeding this amount would not be guaranteed and would be issued as a separate junior instrument (the red bond), thus inviting market discipline to bear on the issuing sovereign whenever it exceeds a 60% debt-to-GDP ratio.

The analytical framework for assessing the case of partial mutualisation requires particular consideration. Generally speaking, the framework must capture the fact that a sovereign must be ready and willing to default on its junior tranche in order to safeguard the creditworthiness of the commonly-issued blue bonds. This can be tackled analytically by considering that a sovereign's asset value or debt capacity is primarily pledged to honouring the blue bond ($A^P_{t,i}$), while the residual debt capacity ($A^R_{t,i}$) supports the red bond. As such, we have that total debt capacity is decomposed as follows:

$$A_{t,i} = A^P_{t,i} + A^R_{t,i}. \tag{12}$$

While $A_{t,i}$ is an estimated value and $A^R_{t,i}$ is a residual, a rule is needed that can specify $A^P_{t,i}$ in an appropriate manner. The rule underpinning the analysis in this section is such that a sovereign pledges to honour both its share of the blue bond as well as any aggregate debt capacity shortcomings arising in other Member States, up to its maximum available debt capacity $A_{t,i}$. Analytically, this can be written as:[33]

$$A^P_{t,i} = \begin{cases} D^S_{t,i} + \left[\sum_{j \neq i} \left(D^S_{t,j} - A_{t,j}\right)\right]^+ & \text{if } D^S_{t,i} + \left[\sum_{j \neq i} \left(D^S_{t,j} - A_{t,j}\right)\right]^+ < A_{t,i} \\ A_{t,i} & \text{if } D^S_{t,i} + \left[\sum_{j \neq i} \left(D^S_{t,j} - A_{t,j}\right)\right]^+ \geq A_{t,i} \end{cases} \tag{13}$$

where D^S_i represents the level of senior debt issued by sovereign i under the blue bond instrument. It can be shown that this formulation encapsulates the desirable result that Member States' debt capacity is entirely pledged to the commonly-issued blue bonds as the euro area aggregate approaches default. In particular, it can be shown that

$$\sum_i A_{t,i} \leq \sum_i D^S_{t,i} \Rightarrow \left(A^P_{t,i} = A_{t,i} \wedge A^R_{t,i} = 0\right) \ \forall i. \tag{14}$$

[33] An equivalent specification would be $A^P_{t,i} = min\left(D^S_{t,i} + \left[\sum_{j \neq i}\left(D^S_{t,j} - A_{t,j}\right)\right]^+,\ A_{t,i}\right)$. However, representing the function in terms of branches arguably makes for an easier presentation of the proofs in Appendix B.

Implication (14) tells us that at the critical point where $\sum_i A_{t,i} = \sum_i D^S_{t,i}$, Member States will have defaulted on their junior tranches and will be pledging all of their debt capacity to the blue bonds, as demanded by a credible blue bond scheme. As such, under mutualisation of Member States' senior debt $D^S_{t,i}$, the euro area will only default on the blue bonds if its aggregate debt capacity falls below the total amount of outstanding blue bond debt:

$$PD_{t,EA} = P\left(\sum_i A_{t+24,i} < \sum_i D^S_{t+24,i}\right). \tag{15}$$

It can be shown additionally that the framework provided by Equations (12) and (13) satisfies the following set of desirable properties:

1. Analytical tractability: bivariate normal distributions can be derived allowing for the calculation of PD^J_i, the probability of default on the junior tranches (the red bonds);

2. As long as the rest of the euro area is solvent on their blue bonds, sovereigns are the residual beneficiary of their own debt capacity: $\left(\sum_{j\neq i} A_{t,j} > \sum_{j\neq i} D^S_{t,j}\right) \Rightarrow \frac{\partial A^P_{t,i}}{\partial A_{t,i}} = 0 \ \forall i$;

3. Sovereign debt capacity is fully pooled as euro area debt approaches default: $A^P_{t,i} \uparrow A_{t,i} \ \forall i$ as $\sum_i A_{t,i} \downarrow \sum_i D^S_{t,i}$;

4. The probability of default on the junior tranches is higher under partial mutualisation than under simple tranching without mutualisation: $PD^J_i > P\left(A_{t+24,i} < D_{t+24,i}\right)$.

Appendix B proves these properties as well as Implication (14). Property 2 means that marginal improvements in a sovereign's debt capacity are generally to their own direct benefit, rather than to the euro area's. Property 3 and the related Implication (14) show that the present formalisation works as intended as regards credible common debt: a sovereign will first default on its red bond before permitting the euro area to default on the common blue bond. Property 4 means that the present formalisation works as intended as regards junior national debt: default on the red bonds can happen not only due to a drop in a sovereign's debt capacity, as is the case in the absence of mutualisation,[34] but also due to systemic failure in partner countries. In fact, under partial mutualisation, default on junior national debt can occur under two circumstances:

1. Idiosyncratic country default;

2. Systemic debt capacity insufficiency of partner countries conducive to default on the blue bond.

Let D^J_i denote the level of junior debt issued by sovereign i under the red bond instrument. We have that the probabilities of default on the junior tranches (the red bond) are given by:

$$PD^J_{t,i} = P\left(A^R_{t+24,i} < D^J_{t+24,i}\right). \tag{16}$$

Appendix B shows how $PD^J_{t,i}$ can be calculated based on the law of total probability.

In the event of a euro area default on the senior blue bond, the LGD is assumed to be a standard 60%. The LGD on junior red bonds is given by

[34] See Subsection 6.1 for the calculation of the probabilities of default on the junior tranche in the absence of mutualisation.

$$LGD^J_{t,i} = min\left(100\%, \frac{60\% \times \left(D^S_{t,i} + D^J_{t,i}\right)}{D^J_{t,i}}\right). \tag{17}$$

Formula (17) means that the junior tranche is wiped out in the event of default (a 100% LGD), except in the cases where the junior tranche is so large as to represent more than 60% of debt outstanding. As a 60% LGD is the unifying assumption, the LGD on the junior tranche is therefore capped at a maximum representing 60% of total debt outstanding.[35]

Partial mutualisation delivers the second largest ISS gain for the euro area. All Member States are seen to benefit in ISS terms, but the gain is much smaller for sovereigns that have experienced market stress when compared to the full mutualisation scenario. This is due to the fact that these countries would have paid a high risk premium on their junior tranches, partly offsetting the lower premium paid on the senior tranche (see Figure (5)). Conversely, the ISS gains are somewhat higher for some of the most creditworthy Member States when compared with the full mutualisation case as the level of the joint guarantees that the latter option implies would have caused a non-negligible deterioration in their risk profile during the peak crisis period, in 2012 (see Figure (4)). At the same time, the risk premia on the junior tranches of the most creditworthy Member States remains an order of magnitude lower than that of countries that have experienced market stress, in case of partial mutualisation (see Figure (6)).

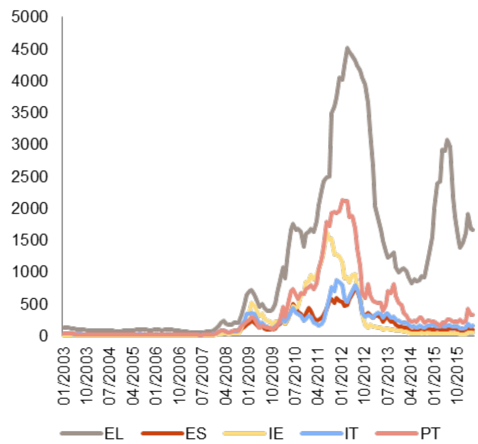

Figure 5: Risk premia on junior tranches under partial mutualisation and other options involving *a priori* tranching for EL, ES, IE, IT and PT (in basis points, 60% of GDP cut-off point)

It can also be observed that the ISS gain is only slightly higher than under the tranching and pooling, and tranching options. This is due to the chosen cut off point which, at 60% of GDP, produces senior tranches characterised by very low risk of default.[36] Due to the very low risk inherent in the senior tranches, mutualisation cannot provide a significant enhancement. For higher cut-off points, however, the additional euro area gains provided by partial mutualisation become more evident. This point is illustrated in Table 3 which compares the results for a 60%

[35]There is a moot theoretical situation not captured by Equation (17) related to a type two default (systemic default of partner countries). Suppose that $D^J_{t,i}$ is sufficiently large so that Equation (17) yields $LGD^J_{t,i} < 100\%$. In this case, full theoretical correctness would require splitting the default space into two mutually exclusive events: i) the case where the debt capacity insufficiency of partner countries is less severe so that country i can impose in effect an $LGD^J_{t,i} < 100\%$ and still ensure that the blue bond is honoured and ii) the case where the debt capacity insufficiency of partner countries is more severe so that default on the blue bond is unavoidable, or at least an LGD higher that the one resulting from Formula (17) is required in order to honour the blue bond, in which case one could impose $LGD^J_{t,i} = 100\%$. While Equation (17) will be re-used in the next section, this situation is specific to the partial mutualisation option and does concern other instruments.

[36]This quantitative result is consistent with the historical experience whereby defaults in advanced economies experiencing a sovereign debt-to-GDP ratio of only 60% are virtually unknown.

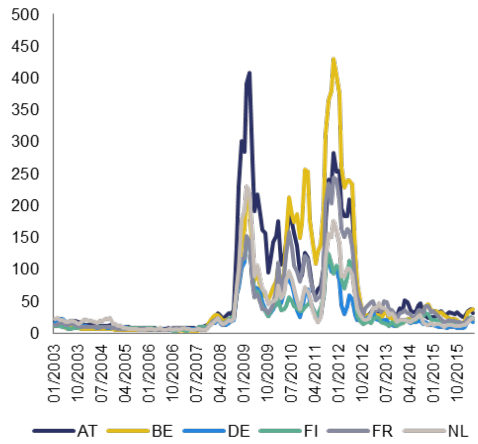

Figure 6: Risk premia on junior tranches under partial mutualisation and other options involving *a priori* tranching for AT, BE, DE, FI, FR and NL (in basis points, 60% of GDP cut-off point)

and an 80% cut-off point. It can also be observed how partial mutualisation sits between the full mutualisation and the tranching and pooling options, approaching the former for high cut-off points, and the latter for low cut-off points. In fact, the PD on the blue bonds converges to that of the eurobonds as the cut-off point increases. In such a case, $\sum_i D^S_{t+24,i} \rightarrow \sum_i D_{t+24,i}$ and, given the continuity of the normal distribution function, convergence of the blue bond's PD follows readily from Equations (11) and (15). It is also worth noting how different Member States show different optimal mutualisation levels. While increasing mutualisation from 60% of GDP to 80% increases ISS gains for all Member States, going from 80% of GDP to full debt mutualisation decreases the ISS gains for the most creditworthy countries. Full mutualisation is thus ISS-optimal only for a subset of Member States.

Finally, it should be noted that for a cut-off point of 60% of GDP, the ISS gains observed under the partial mutualisation, pooling and tranching, and tranching options are essentially driven by the fact that tranching allows for sequential default whereby a sovereign can default on the junior tranche and still preserve the senior tranche, unless faced with the (unlikely) event of its debt capacity falling so much as to render the senior debt unserviceable. The sequential default assumption is implicit in the analysis of blue bonds and follows from the analytical framework previously presented. In the case of other instruments involving *a priori* tranching, sequential default is an explicit assumption, as will be discussed in Subsection 6.1. Partial mutualisation can thus prove also to be a superior option from an ISS perspective when compared with other *a priori* tranching options if it can facilitate sequential default.

Table 3: ISS gain (+) / loss (-) for 60% and 80% cut-off points (basis points, January 2003 to April 2016)

	Euro Area	AT	BE	DE	EL	ES	FI	FR	IE	IT	NL	PT
Full mutualisation (no cut-off point)	70	26	25	4	707	75	5	14	121	94	10	209
Partial mutualisation (80% cut-off point)	38	38	25	16	99	76	17	22	71	45	22	120
Tranching and pooling (80% cut-off point)	27	25	14	3	91	63	4	10	61	37	9	110
Partial mutualisation (60% cut-off point)	19	27	12	11	17	49	17	13	33	16	19	55
Tranching and pooling (60% cut-off point)	18	26	11	10	17	48	15	11	33	16	17	55

Note: The cut-off level of the senior tranches was set at 60% and 80% of GDP, where applicable. The euro area figures refer to the total gains accruing to the euro area aggregate composed of the 11 Member States considered in the simulations. Results for tranching and pooling are based on a sequential default assumption (see Subsection 6.1).

6 Common debt instruments without mutualisation

This section considers a set of debt instruments that do not involve any form of explicit mutualisation.

6.1 Tranching and pooling

Under the tranching and pooling approach, euro area Member States issue senior and junior bonds, with the former being pooled together into a single euro area debt instrument. The tranching and pooling approach can also be understood as a situation where participating countries confer a preferred creditor status to a euro area debt agency, which funds itself in the market through a common bond and lends on the proceeds to Member States. The debt issued by such an agency is, in this case, the senior tranche, and is akin to "E-bonds", as first proposed in Monti (2010), and again in Juncker and Tremonti (2010).

Given that the junior tranche is the first tranche to incur losses in the event of default, its PD equals the historical PD inferred via Equation (1). Alternatively, $PD^J_{t,i}$ can be calculated as

$$PD^J_{t,i} = P\left(A_{t+24,i} < D_{t+24,i}\right) = \Phi\left(\frac{ln(D_{t+24,i}) - ln\left(A_{t,i}\right) - 24\mu_i}{\sqrt{24}\sigma_i}\right), \tag{18}$$

where $D_{t,i} = D^S_{t,i} + D^J_{t,i}$ represents the total debt level of sovereign i (i.e., the sum of the junior and senior tranches). Either approach yields the same PD as the signal Equation (7) ensures that the model-based PDs of Equation (18) equal the CDS-based PDs at all times. The LGD on the junior tranche is given by Equation (17) and follows the same rationale of the previous section. The PD on the senior tranche is the probability of the debt capacity $A_{t+24,i}$ falling so much as to render the senior debt unserviceable:

$$PD^S_{t,i} = P\left(A_{t+24,i} < D^S_{t+24,i}\right) = \Phi\left(\frac{ln(D^S_{t+24,i}) - ln\left(A_{t,i}\right) - 24\mu_i}{\sqrt{24}\sigma_i}\right). \tag{19}$$

Equation (19) embodies the principle of sequential default allowed for by the existence of two tranches. In effect, Equation (19) is based on the assumption that a sovereign will not default simultaneously on both tranches, unless forced to by a large drop in $A_{t,i}$. Under the alternative assumption of simultaneous default, the probability of default on the senior tranche equals $PD^J_{t,i}$ as long as $\frac{D^J_{t,i}}{D_{t,i}} < 60\%$ (i.e., as long as $D^J_{t,i}$ is not large enough to absorb a standard LGD of 60% on total debt), and zero otherwise. Under sequential default, sovereign i only defaults on $D^S_{t,i}$ after having defaulted first on $D^J_{t,i}$ and finding $A_{t+24,i} < D^S_{t+24,i}$. Moving from a simultaneous default assumption to a sequential default assumption tends to imply a decrease in expected losses on total debt, which is the driver of the ISS gains shown in Table 2.[37] For an illustration of this mechanism, see Appendix (C).

A sovereign's LGD in the event of default on its senior tranche is a standard 60%. However, given that senior bonds are pooled together, the LGD on this pooled senior instrument must be calculated from the individual sovereign LGDs and the respective debt amounts involved. As the LGD on the pooled instrument depends, inter alia, on the particular combination of countries defaulting, this implies considering all possible, mutually-exclusive default events. With 11 countries, there are 2047 default events depending on the number of countries defaulting and, for that number, on the possible country combinations:

[37] In fact, ISS gains from tranching and pooling can be shown to be zero for the euro area aggregate under a simultaneous default assumption, while being either positive or negative at Member State level, depending on the riskiness of the concerned sovereign.

$$\text{Number of possible default events} = \sum_{k=1}^{11} \binom{11}{k} = 2047. \tag{20}$$

The set of PDs associated with each of the 2047 default events j that are possible at time t, $\{PD_{t,j}^{2047}\}$, can be calculated based on the multivariate log-normal distribution of $A_{t,i}$ taking into account the assessed debt capacity and projected debt levels at time t, as well as the correlation matrix in Figure (2). The expected LGD on the senior tranche is then given by

$$LGD_t^S = \sum_{j=1}^{2047} PD_{t,j}^{2047} \times LGD_{t,j}^{2047},$$

where $\{LGD_{t,j}^{2047}\}$ represents the set of all possible 2047 LGDs on the senior tranche. A particular $LGD_{t,j}^{2047}$ will depend on the number of countries defaulting in event j, their respective senior debt levels at time t and the size of the pool of senior debt at time t. Say that for a particular default event j the set of countries defaulting is given by I_j. Then

$$LGD_{t,j}^{2047} = \frac{0.6 \times \sum_{k \in I_j} D_{t,k}^S}{\sum_i D_{t,i}^S}. \tag{21}$$

At a cut-off point equal to 60% of GDP, the PDs and expected losses on the senior tranches will be very low, implying negligible risk premia. The risk premia on junior tranches will be comparatively high. Not only are junior tranches exposed to the historical PDs inferred from CDS data, which reached high levels in some cases, but also the LGDs will be 100% if the junior tranches are not particularly large, meaning that relatively thin junior tranches are wiped out in the event of default. Figures (5) and (6) display the counterfactual risk premia on the junior tranches.

6.2 Simple tranching

The pure tranching option is presented for illustration purposes as it does not represent a type of common debt instrument. It is similar to the tranching and pooling option in terms of design and ISS results, with the difference lying in the fact that senior tranches are not pooled into a single debt instrument by a euro area agency. The case of Finland merits, however, closer consideration as the one instance showing significant ISS differences between the two options, as per the results in Table 2. In fact, Finland's case is illustrative of how tranching and pooling can provide incentives for keeping government debt levels low. Projected government debt in Finland remained below 60% of GDP during most of our time sample meaning that, during such periods, Finland would not have incurred any risk premia penalty via a junior tranche under a tranching and pooling approach while, at the same time, benefiting from the possibility of issuing all of its debt via a very safe senior common debt instrument.

It should be noted that, in order for the aforementioned benefits to materialise, an hypothetical euro area debt agency with a preferred creditor status would need to have issued a senior common debt instrument on behalf of Member States. Otherwise, if this agency were to simply acquire existing securities in the market, tranche and repackage them, any ISS gains for Finland under the tranching and pooling option would spring exclusively from the existence of tranching and would thus be identical to those shown in Table (2) for the simple tranching option.[38] The

[38] If, furthermore, the sequential default assumption is abandoned in favour of the simultaneous default assumption, simple tranching becomes a pure financial engineering operation, producing zero ISS gains and losses at both Member State and euro area level.

observation that Member States' ISS gains and losses under the options involving pooling will vary depending on whether the hypothetical euro area debt agency simply acquires and pools existing government debt securities, or is the vehicle through which governments issue their debt (or a tranche thereof), will be further discussed in the following subsection.

6.3 Pooling and tranching

Pooling and tranching can describe a situation where a euro area agency acquires government bonds of participating Member States, pools these bonds together and securitises the bond pool by issuing senior and junior debt tranches. The instruments discussed in this subsection are similar in nature, though not identical in detail, to the ESBies and EJBies proposal of Brunnermeier et al. (2017). For direct comparability with the other options, the ESB instrument considered in this subsection pools together national debt worth up to 60% of GDP and the EJB instrument pools together the remaining national debt in excess of 60% of GDP.[39] As such, the euro area-wide senior tranche (i.e., the ESB) is given by $D_t^{ESB} = \sum_i D_{t,i}^S$ while the euro area-wide junior tranche (i.e., the EJB) is given by $D_t^{EJB} = \sum_i D_{t,i}^J$. The ESB is senior to EJB and will only take losses after the EJB has been wiped out.

The PD on the EJB is driven by the worst country performers as it suffices that a Member State defaults for the EJB to be affected. With 11 countries in sample, default can occur in 2047 ways, as per Equation (20). To each of these mutually exclusive events will correspond a particular LGD on the EJB. Let I_j denote the set of countries defaulting in a particular default event j. Then

$$LGD_{t,j}^{EJB} = max\left(100\%, \frac{0.6 \times \sum\limits_{k \in I_j} D_{t,k}}{D_t^{EJB}}\right). \tag{22}$$

Equation (22) says that a defaulting country will impose a 60% loss on its debt outstanding, in line with the standard LGD assumption. The LGD on the EJB will then depend on the number of countries defaulting, on their debt levels and on the total size of the EJB pool. Depending on the combination of these factors, the LGD on the EJB may reach 100%, in which case any remaining losses will pass on to the ESB. Consequently, the LGD on the ESB for an event j with a set I_j of defaulting countries is given by

$$LGD_{t,j}^{ESB} = max\left(0\%, \frac{0.6 \times \sum\limits_{k \in I_j} D_{t,k} - D_t^{EJB}}{D_t^{ESB}}\right). \tag{23}$$

As in Subsection (6.1), the set of PDs on the EJB for each of the 2047 default events, $\left\{PD_{t,j}^{2047}\right\}$, can be calculated based on the multivariate log-normal distribution of $A_{t,i}$ taking into account the assessed debt capacity and projected debt levels at time t, as well as the correlation matrix in Figure (2). The expected LGD on the EJB is then given by

$$LGD_t^{EJB} = \sum_{j=1}^{2047} PD_{t,j}^{2047} \times LGD_{t,j}^{EJB}.$$

[39] In the original ESBies/EJBies proposal, the level of outstanding debt eligible for purchase by the euro area agency is simultaneously capped at 60% of GDP per country and by a portfolio key based on that country's GDP weight vis-à-vis the euro area. The cut-off (or "subordination") levels for the senior tranche are then set as a percentage of the total debt acquired by the agency.

The probability of default on the senior tranche in event j equals $PD^{2047}_{t,j}$ as long as $LGD^{ESB}_{t,j} > 0\%$ (i.e., as long as the EJB is not large enough to entirely absorb the losses from default), and zero otherwise. The expected LGD on the ESB can be calculated as

$$LGD^{ESB}_{t} = \sum_{j=1}^{2047} PD^{2047}_{t,j} \times LGD^{ESB}_{t,j}.$$

As can be observed in Table (2), ESBies and EJBies do not change overall expected losses and are thus ISS-neutral for the euro area. A euro area agency implementing the scheme by acquiring government debt securities in the market will break even from an ISS perspective, making gains on the comparatively cheap securities of the least creditworthy Member States and incurring offsetting losses on the comparatively expensive securities of the most creditworthy countries. These gains and losses would accrue to the respective Member States in case they were forced to issue their debt directly via ESBies and EJBies, and required to pay the interest on these instruments. This would imply a form of implicit mutualisation as some of the most creditworthy countries would be forced to issue part of their debt via an EJB whose risk profile would be contaminated by the worst performers. Conversely, the latter would benefit from the credit enhancement provided by a large debt pool, which lowers the LGD. The hypothetical ISS losses would affect, in particular, countries simultaneously displaying a comparatively favourable risk profile and debt levels in excess of 60%. Finland provides an example where the first condition is met, but not the second for most of the sample period. As such, Finland would stand to gain from an ISS perspective, thereby illustrating how mandatory direct issuance via ESBies and EJBies can provide incentives for containing debt levels.

As regards risk profiles, the EJB makes for a risky instrument, particularly in times of crisis. Given that the worst country performers drive the default risk, it is exposed to PDs on the high side even in normal times and can reach very high PDs during crisis periods (see Figure (7) for the counterfactual dynamics of PDs and LGDs). The counterfactual LGD^{EJB}_{t} are relatively low and decreased over time as the size of the EJB pool increased. For a cut-off point as high as 60% of GDP, the ESB is a safe asset showing low counterfactual PDs even in peak crisis periods, as well as low counterfactual LGDs (Figure (8)). The findings that i) ESBies are safe instruments, that ii) EJBies can be quite risky, and that iii) pooling and tranching does not change aggregate risk levels align with those of previous studies.

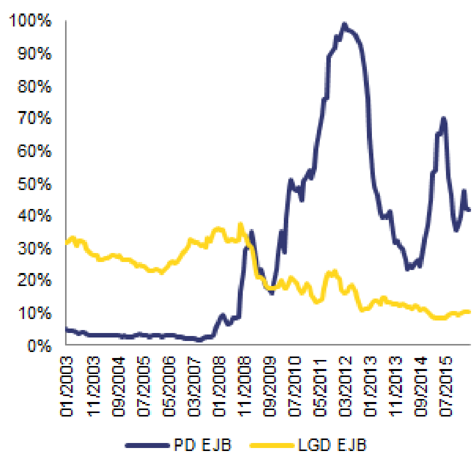

Figure 7: Counterfactual PDs and LGDs on European Junior Bonds (EJBies)

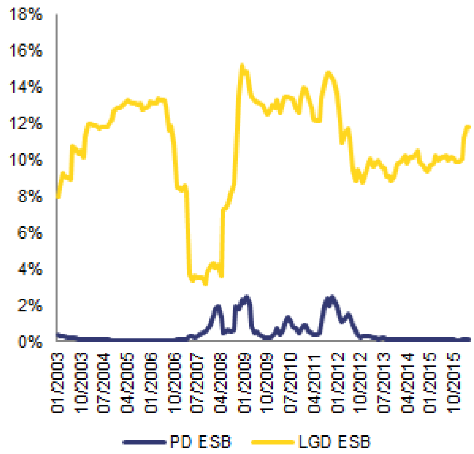

Figure 8: Counterfactual PDs and LGDs on European Senior Bonds (ESBies)

6.4 Simple pooling

The simple pooling approach is presented for illustration purposes. It differs from the previous option in the sense that the agency acquiring the debt stock of euro area Member States issues a single debt instrument without any type of tranching. The PD on the pool equals the PD on an EJB, as shown in Figure (7). The LGD on the pool is given by the following two equations:

$$LGD_{t,j}^{Pool} = \frac{0.6 \times \sum\limits_{k \in I_j} D_{t,k}}{\sum\limits_{i} D_{t,i}}$$

and

$$LGD_t^{Pool} = \sum_{j=1}^{2047} PD_{t,j}^{2047} \times LGD_{t,j}^{Pool},$$

where j and I_j have the same meaning as before. As expected, pooling presents no ISS gains or losses for the euro area aggregate. Compared with the ESB/EJB option, simple pooling widens the divergence between the hypothetical ISS gains and losses of the least and most creditworthy countries in case of mandatory issuance via a common debt agency.

7 Extensions

7.1 Systemic euro area risk and idiosyncratic national risk

The framework presented in Section 2 allows for the derivation of a euro area factor measuring systemic sovereign debt capacity and for the decomposition of national dynamics into systemic and idiosyncratic components. In particular, say X_t is the first principal component of the set of debt capacity variables $\{\Delta ln\,(A_{t,i})\}$. Then X_t is a weighted average of the normalised variations in the logarithm of debt capacity, $\Delta ln\,(A_{t,i})^N$, across sovereigns. In our application, normalisation implies that the $\Delta ln\,(A_{t,i})^N$ have mean zero and variance one, and that X_t has therefore mean zero. The loadings corresponding to the first principal component are shown in Table 4. Let X_t^N denote the re-scaled, or normalised, X_t, so that X_t^N has variance one. The respective normalised loadings are also shown in Table 4.

Table 4: Loadings of X_t

	Loadings	Normalised loadings
AT	0.30	0.09
BE	0.32	0.10
DE	0.31	0.09
EL	0.10	0.03
ES	0.35	0.11
FI	0.33	0.10
FR	0.35	0.11
IE	0.28	0.09
IT	0.25	0.08
NL	0.34	0.11
PT	0.29	0.09

For a given sovereign i, $\Delta ln\,(A_{t,i})^N$ can be decomposed as

$$\Delta ln\,(A_{t,i})^N = \frac{\Delta ln\,(A_{t,i}) - \mu_i}{\sigma_i} = \rho_i X_t^N + \sqrt{1-\rho_i^2}\,\varepsilon_{t,i}, \tag{24}$$

where

$$\begin{pmatrix} \Delta ln\,(A_{t,i})^N \\ X_t^N \\ \varepsilon_{t,i} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & \rho_i & \sqrt{1-\rho_i^2} \\ \rho_i & 1 & 0 \\ \sqrt{1-\rho_i^2} & 0 & 1 \end{bmatrix}\right).$$

X_t^N can be understood as the systemic euro area component and $\varepsilon_{t,i}$ as the idiosyncratic component driving the dynamics of the (normalised, logged) debt capacity of sovereign i. Given that X_t is the first principal component, it is uncorrelated with the $\varepsilon_{t,i}$ residual by construction. Also, given that both $\Delta ln\,(A_{t,i})^N$ and X_t^N have variance one, ρ_i represents the correlation between the evolution of the (normalised) sovereign debt capacity and the systemic euro area factor. Likewise, $\sqrt{1-\rho_i^2}$ represents the correlation between a sovereign's debt capacity and its idiosyncratic component. From Equation (24) it follows that

$$\Delta ln\,(A_{t,i}) = \mu_i + \sigma_i \rho_i X_t^N + \sigma_i \sqrt{1-\rho_i^2}\,\varepsilon_{t,i}. \tag{25}$$

The correlation parameter ρ_i can be obtained from an OLS estimation of Equation (24). Given ρ_i, the $\varepsilon_{t,i}$ can be inferred from the OLS residuals. Given that μ_i and σ_i are known from Section 2, the parameters in Equation (25) are fully known. Table 5 presents the parameters of relevance. It is worth noting that Equation (25) implies that the bilateral correlations between sovereign debt capacities shown in Figure 2 can be decomposed into two elements: an indirect correlation intermediated by the systemic factor X_t^N and a direct correlation via the $\varepsilon_{t,i}$ residuals. For sovereigns i and j the former is given by $\rho_i \times \rho_j$ while the latter is given by $\sqrt{1-\rho_i^2} \times \sqrt{1-\rho_j^2} \times \rho_{\varepsilon_i,\varepsilon_j}$, where $\rho_{\varepsilon_i,\varepsilon_j}$ denotes the correlation between the residuals of sovereign i and j. This type of decomposition can be applied to simulate the debt capacity responses to different types of shocks, as illustrated in Appendix D for two historical periods. For example, by negatively shocking the systemic risk factor it is possible to observe how the deterioration in creditworthiness varies across countries. Generally speaking, this deterioration will be higher for Member States that are more dependent on the systemic factor X_t^N, as well as for those countries with a worse starting position. Alternatively, it is also possible to negatively shock the idiosyncratic component, as exemplified for Italy in Appendix D. The transmission of this type of shock is assumed to be based on the correlation matrix of $\varepsilon_{t,i}$, and tends to be muted. Also, a negative, purely idiosyncratic shock in one country can lead to positive spill-overs in another country, which can be the result of a flight-to-safety effect. Finally, “non-discriminated” country shocks can be simulated by relying simply on the correlation matrix of debt capacities as shown in Figure 2, without further consideration as to their systemic or idiosyncratic nature. This is exemplified for comparison purposes in the last column of the tables in Appendix D.

Table 5: Decomposition parameters for $\Delta ln\left(A_{t,i}\right)$

	μ_i	σ_i	ρ_i	$\sqrt{1-\rho_i^2}$	$\sigma_i\rho_i$	$\sigma_i\sqrt{1-\rho_i^2}$
AT	0.004	0.010	0.74	0.68	0.008	0.007
BE	0.003	0.005	0.79	0.61	0.004	0.003
DE	0.003	0.008	0.75	0.66	0.006	0.005
EL	0.004	0.018	0.23	0.97	0.004	0.017
ES	0.007	0.014	0.86	0.50	0.012	0.007
FI	0.005	0.015	0.82	0.58	0.012	0.009
FR	0.005	0.007	0.87	0.50	0.006	0.003
IE	0.008	0.021	0.70	0.71	0.015	0.015
IT	0.003	0.004	0.61	0.79	0.002	0.003
NL	0.004	0.013	0.84	0.54	0.011	0.007
PT	0.006	0.009	0.72	0.70	0.006	0.006

Appendix E shows for each Member State the decomposition of the evolution of $\Delta ln\left(A_{t,i}\right)$ into a systemic and idiosyncratic component. It can be noticed that debt capacities increased markedly during the second half of 2008 and the first half of 2009 for most Member States. This reflects a market-based assessment of sovereigns’ determination to increase their debt capacity in order to cope with higher expected debt levels following the onset of the financial crisis. This can be achieved by, for example, increasing projected tax-raising capacity and revenue, decreasing projected expenditure and investment, or by embarking on structural reforms. Due to the simultaneous nature of the shock, the systemic component tends to be the main driver of the dynamics during this period. The observed increase in debt capacity does not mean, however, that sovereigns were able to fully rise to the challenges posed by the financial crisis. This point is illustrated in Appendix F which compares, for each Member State, how debt capacity, as assessed by market participants, evolved vis-à-vis expected debt levels. In particular, Appendix F shows for each sovereign the cumulated difference between the two variables from the beginning of the sample period until a given month $t^{current}$:

$$\text{Cumulated financial gap}_{i,t^{current}} = \sum_{t=1}^{t^{current}} \left(\Delta ln\left(A_{t,i}\right) - \Delta ln\left(D_{t+24,i}\right)\right). \tag{26}$$

As can be observed, a negative cumulated financial gap opened up for all euro area Member States in either 2007 or 2008, although its size and subsequent closure processes differed markedly across countries. While for the stronger euro area economies the gap was mostly or fully closed by 2016, for the more vulnerable countries a significant negative gap still persisted by the end of the sample period. An increasing trend in the cumulated financial gap measure requires $\Delta ln\left(A_{t,i}\right) > \Delta ln\left(D_{t+24,i}\right)$ and reflects therefore a period of improving creditworthiness. In this connection, it can also be interesting to focus on a sovereign’s idiosyncratic efforts towards improving its credit standing. These can be reckoned as:

$$\begin{aligned}\text{Idiosyncratic sovereign efforts}_{i,t} &= \left(\Delta ln\left(A_{t,i}\right) - \sigma_i\rho_i X_t - \Delta ln\left(D_{t+24,i}\right)\right)\\ &= \mu_i + \sigma_i\sqrt{1-\rho_i^2}\varepsilon_{t,i} - \Delta ln\left(D_{t+24,i}\right).\end{aligned} \tag{27}$$

By subtracting the zero-meaned systemic component $\sigma_i \rho_i X_t$ in Equation (27) one can focus on sovereign-specific elements. The idiosyncratic sovereign effort series are shown in Appendix F as centred three-month moving averages, for a smoother depiction.

7.2 Assessing robustness under a fat-tailed distribution

The structural model underpinning the analysis in the preceding sections relies on a log-normality assumption for the evolution of a sovereign's financial debt capacity or "asset" value over time. As such, the evolution of the logarithm of a sovereign's debt capacity is governed by the quintessential mesokurtic distribution, the normal distribution. The literature suggests, however, that the dynamics of (logged) asset prices may be better approximated by distributions with fat tails (see, e.g., Campbell et al. (1996)), which place higher probability weight on extreme movements. This subsection shows how the analysis in the preceding sections can be adapted to a fat-tails framework and how the robustness of some results can be checked by comparing them with results obtained under a fat-tailed multivariate t distribution with 4 degrees of freedom.

In order to account for fat tails, a multivariate log-t distribution can be derived that matches the baseline multivariate log-normal distribution in the sense of being based on the same first and second moments. This matching (log-)t distribution is obtained as follows. Let $ln\left(A_t\right)$ denote the vector composed of elements $ln\left(A_{t,i}\right)$. Given that $ln\left(A_{t+24}\right) \sim N\left(ln\left(A_t\right) + 24\mu, 24\Sigma\right)$, then

$$\sqrt{2} \times ln\left(A_{t+24}\right) \sim N\left(\sqrt{2}\left(ln\left(A_t\right) + 24\mu\right), 2 \times 24\Sigma\right), \quad (28)$$

where Σ denotes the covariance matrix of $\Delta ln\left(A_t\right)$. A matching t distribution possessing the same mean and covariance matrix as $\sqrt{2} \times ln\left(A_{t+24}\right)$ can then be defined has having 4 degrees of freedom ν and covariance parameter $24\mathit{\Sigma}$. Let us denote this distribution as $t_4\left(ln\left(A_t\right) + 24\mu, 24\Sigma\right)$. Then, the covariance matrix of t_4 is $\frac{\nu}{\nu-2} \times 24 \times \Sigma = 2 \times 24 \times \Sigma = covar\left(\sqrt{2} \times ln\left(A_{t+24}\right)\right)$.

As an example, the matching probability under t_4 that $ln\left(A_{t+24}\right) < ln\left(D_{t+24}\right)$ is given by the cumulative distribution function of t_4 assessed at $\frac{ln(D_{t+24})}{\sqrt{2}}$, where the normalisation of the debt level by the square root of two is done so as to ensure consistency of the critical levels with the original normal distribution. The matching t distribution with 4 degrees of freedom is thus based on the same mean and correlation matrix as the original normal distribution, with critical levels adjusted to reflect the multiplication by $\sqrt{2}$ in Equation (28). This probability calculated under a fat-tailed t distribution will be higher than $PD\left(ln\left(A_{t+24}\right) < ln\left(D_{t+24}\right)\right)$ calculated under the normal distribution for extreme events (i.e., in the cases where the PD calculated under the normal distribution is very low).[40]

Allowing for fat-tails shifts the probability mass towards extreme events, but does not necessarily increase the probability of events that are merely deemed unlikely. Figures (9) and (10) illustrate this point for the PDs of ESBies and EJBies. Given that default on an ESB is an extreme event, a fat-tailed distribution assumption yields higher PDs, thus producing a more conservative assessment of the ESB instrument. In contrast, default on an EJB may be unlikely most of the time, but it is not an extreme event. As such, the fat-tailed distribution assumption produces PDs that are often lower than the baseline PDs obtained under the normal distribution. In this case, relying on a fat-tailed distribution does not make for a more conservative assessment.

[40] In a situation where the multivariate t distribution is normalised to a zero mean and defined in terms of a correlation matrix, as is the case of some software implementations, an equivalent normalisation can be implemented. If $ln\left(A_{t+24}\right) \sim N\left(ln\left(A_t\right) + 24\mu, 24\Sigma\right)$, then

$$\frac{\sqrt{2}}{\sqrt{24}} \times diag\left(\Sigma\right)^{-\frac{1}{2}} \times \left[ln\left(A_{t+24}\right) - ln\left(A_t\right) - 24\mu\right] \sim N\left(0, 2R\right),$$

where $diag\left(\mathit{\Sigma}\right)$ is a diagonal matrix with the variances of A in the diagonal, and R is the correlation matrix of A. The matching t distribution with 4 degrees of freedom will then have zero mean and covariance matrix R.

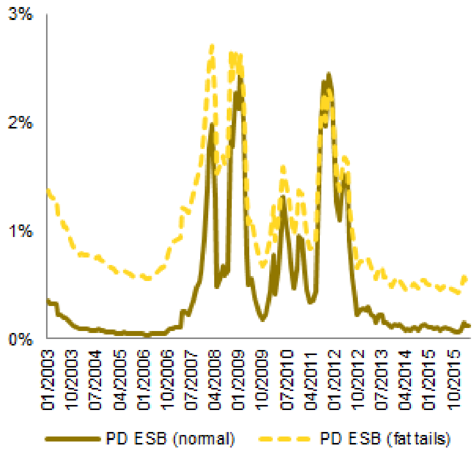

Figure 9: PDs on European Senior Bonds (ESBies) under normal and fat-tailed distributions

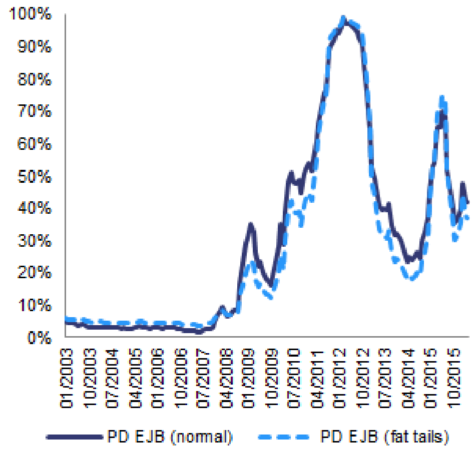

Figure 10: PDs on European Junior Bonds (EJBies) under normal and fat-tailed distributions

Given that expected LGDs are calculated, inter alia, on the basis of PDs, they also vary under a fat-tailed distribution.

8 Conclusions and future research

This paper introduced a novel, integrated framework for the assessment of the key financial properties of common sovereign debt instruments in a currency union such as the euro area. By relying on market-based information extracted from CDS spreads or bond yields, on projected debt levels and on the structure provided by a structural credit risk model, a measure of a sovereign's debt capacity or "asset" value can be extracted using state-space methods. By taking into account the empirical properties of the debt capacity series thus derived, different types of common debt instruments can be simulated and their performance assessed based on a simple measure of counterfactual credit risk premia gains and losses. The approach is suitable for the assessment of instruments involving full or partial mutualisation of sovereign risk, whose quantitative analysis had been lacking in the literature.

Of the different types of common debt instruments considered in this paper, fully mutualised "eurobonds" provide, by far, the largest counterfactual benefit in terms of a reduction in funding costs, as far as the euro area aggregate is concerned. This is due to the diversification of idiosyncratic risks afforded by mutualisation and to the pooling together of national fiscal spaces under a single instrument. These benefits are largest for euro area Member States that underwent market stress following the 2008 financial crisis, but also remain in evidence, albeit modestly, for the most creditworthy countries. In fact, while the counterfactual simulations show that eurobonds could result in higher risk premia for specific countries during peak crisis periods, these losses are more than offset when our full sample period is considered.

Another type of instrument simulated in this paper involves partial mutualisation of sovereign risk through the issuance of commonly-guaranteed "blue bonds" and of national "red bonds". The analytical approach for assessing partial mutualisation requires particular consideration. This paper developed a framework which operationalises the partial mutualisation concept and that is shown to fulfill a number of desirable properties. Partial mutualisation delivers the second highest euro area gains for the sample period under analysis. As before, all Member States are seen to gain from a blue bond/red bond scheme, although the gains are larger for the more creditworthy Member States and smaller for the less creditworthy countries when compared with the case of full mutualisation. This is due to the market discipline imposed on Member States via the red bond which, for a mutualisation threshold representing 60% of GDP, can command a significant risk premium. Overall, different Member States show different optimal levels of mutualisation. While full mutualisation is optimal from funding costs viewpoint for the less creditworthy Member States, the more creditworthy countries stand to gain the most from high, but incomplete, levels of mutualisation.

Two options were then simulated that do not involve mutualisation but rather the *a priori* tranching of national debt by Member States into a junior and a senior instrument. This can be achieved by issuing debt instruments with different seniority levels in primary markets, or by conferring a preferred creditor status to a common European debt agency. For a cut-off point of 60% of GDP for the senior tranche, these instruments can present gains which are akin to those achieved by partial mutualisation. This result rests, however, upon the assumption that Member States default only sequentially under *a priori* tranching (i.e., that they avoid defaulting on the senior tranche unless forced to by a precipitous drop in their debt capacity). In such cases, the "firewall" effect provided by the junior tranches lowers the expected loss on total debt outstanding, leading to lower overall financing costs. The fact that mutualisation of the senior tranches can hardly improve upon simple *a priori* tranching is also linked to the cut-off point chosen in the simulations which, at 60% of GDP, renders the senior tranches very low risk. For such a cut-off point, mutualisation provides only a modest credit enhancement, although its design offers the advantage that the sequential default assumption is necessarily met.

Alternatively, a common debt agency may acquire in the market debt securities of participating Member States and repackage them into senior and junior instruments, in line with the "ESBies" and "EJBies" proposal. This operation does not change overall financing costs for the euro area aggregate, nor for participating Member States, reflecting its pure financial engineering nature. In case Member States were compelled to issue directly via ESBies and EJBies, this would represent an implicit form of mutualisation producing no aggregate gains while imposing higher counterfactual risk premia on the more creditworthy Member States due to the risk contamination of the common debt instruments as a result of the participation of the riskiest Member States in the scheme. Conversely, the least creditworthy Member States would benefit from a lower risk premium due to the relatively small loss given default on the common instruments afforded by the large pool of underlying debt. The ESBies are assessed as safe instruments even at a cut-off point as high as 60% of GDP, while the EJBies can be highly risky, particularly in times of stress.

As an extension of the analysis, it was shown how sovereign debt capacity dynamics can be decomposed into systemic and idiosyncratic factors allowing for, inter alia, the simulation of shock responses. As expected, countries most affected by systemic shocks are those starting from a more vulnerable position, or which are more dependent on the systemic risk factor.

A purely idiosyncratic negative shock to one country tends to transmit only mutely to other sovereigns and may generate positive spill-overs, as in the case of flight-to-safety effects. The aforementioned decomposition also allows for the reckoning of a sovereign's idiosyncratic efforts in maintaining its creditworthiness over time.

Finally, it was shown how the robustness of the results can be tested by relying on a "matching" log-t distribution. This fat-tailed distribution is based on the same first and second moments of the baseline log-normal distribution and allows for a more conservative assessment when considering low probability events.

Two broad avenues open up as regards future research, which can be classified as either of an intensive or of an extensive nature. The intensive avenue would involve exploring the financial properties of common debt instruments under variations of the previously-introduced framework. This could include the employment of structural credit risk models other than the Merton-like model, the use of market information for maturities other than the two-year horizon or the consideration of different sample periods. While the main intent of this paper was to assess the credit risk properties of various instruments under a benchmark framework, there is a case for considering other structural models capturing more complex features of sovereign risk dynamics such as the possibility of instantaneous default, stochastic volatility or time variation in the parameters. It should be noted, however, that more sophisticated models are not, in general, amenable to the exact linear state-space representation forming the basis for the estimation strategy developed in this paper.

The extensive avenue for future research would involve quantifying the gains and losses from common debt instruments under a broader framework encompassing macroeconomic aspects. Such a framework can incorporate the partial-equilibrium financial assessment developed in this paper, which can be understood as providing a lower bound for overall net gains should the broader stability and institutional benefits mentioned in Section 1 materialise. At the same, the macroeconomic implication and cross-border spill-overs of an actual sovereign default will be different depending on whether the euro area economy is endowed with a mutualised instrument or not, meaning that a full reckoning of the gains and losses at Member State level go beyond the funding cost calculations of this paper in that event. Another relevant question relates to tackling the implications for moral hazard of some of these instruments, which could be addressed through reference to the conclusions of the fiscal reaction function literature on how marginal financing costs affect fiscal prudence, or through the incorporation of fully-fledged strategic default models into the analysis. At the limit, a full consideration of the effects of introducing a common sovereign debt instrument requires a general equilibrium framework, with Monteiro (2023) providing an instance of such a type of analysis by employing a dynamic stochastic general equilibrium model to simulate the role of eurobonds in the context of a capital flight within the euro area.

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A ISS gains/losses based on nowcast debt shares

Table 6: ISS gain (+) / loss (-) based on nowcast debt shares (basis points, January 2003 to April 2016)

	Euro Area	AT	BE	DE	EL	ES	FI	FR	IE	IT	NL	PT
Full mutualisation (Eurobonds)	70	26	25	4	707	75	5	14	121	94	10	209
Partial mutualisation (Blue bond and red bond)	22	30	13	11	42	64	17	15	54	17	20	72
Tranching and pooling ("E-bonds")	21	29	13	10	42	63	15	14	53	17	19	71
Simple tranching	21	27	14	11	43	57	2	15	50	17	14	72
Pooling and tranching ("ESBies" and "EJBies")[41]	5	-3	-59	-37	578	48	13	-41	50	-17	1	132
Simple pooling[41]	0	-43	-45	-66	637	6	-65	-55	51	24	-60	139

Note: the cut-off level of the senior tranches was set at 60% of GDP, where applicable. The euro area figures refer to the total gains accruing to the euro area aggregate composed of the 11 Member States considered in the simulations. Results for i) tranching and pooling and for ii) simple tranching are based on a sequential default assumption (see Subsection 6.1).

[41] The figures shown under i) pooling and tranching and ii) simple pooling represent the gains and losses of a hypothetical common debt agency associated with transacting different sovereign bonds in the market, or the gains and losses that would accrue to Member States under a scheme of mandatory issuance via a common debt agency.

B A framework for analysing partial sovereign risk mutualisation

This appendix proves the properties of the framework for analysing partial mutualisation introduced in Section 5.

In order to prove **Implication (14)** it suffices to show that $\sum_i A_{t,i} \leq \sum_i D^S_{t,i} \Rightarrow A^P_{t,i} = A_{t,i}\ \forall i$, given that $A^R_{t,i} = 0$ whenever $A^P_{t,i} = A_{t,i}$, according to Equation (12). For a given time t let the set I of participating countries be partitioned into $S_< = \left\{i \in I : A_{t,i} < D^S_{t,i}\right\}$ and $S_\geq = \left\{i \in I : A_{t,i} \geqslant D^S_{t,i}\right\}$. If $i \in S_<$, Equation (13) implies that $A^P_{t,i} = A_{t,i}$. If $i \in S_\geq$ observe that

$$\sum_i A_{t,i} \leq \sum_i D^S_{ti} \Leftrightarrow D^S_{t,i} - A_{t,i} + \sum_{j \neq i} \left(D^S_{t,j} - A_{t,j}\right) \geqslant 0. \tag{29}$$

Given that $i \in S_\geq$, we have that $D^S_{t,i} - A_{t,i} \leq 0$ and therefore Inequality (29) implies that $\sum_{j\neq i} \left(D^S_{t,j} - A_{t,j}\right) \geqslant 0$. It then follows that $D^S_{t,i} - A_{t,i} + \sum_{j\neq i} \left(D^S_{t,j} - A_{t,j}\right) \geqslant 0 \Leftrightarrow D^S_{t,i} - A_{t,i} + \left[\sum_{j\neq i} \left(D^S_{t,j} - A_{t,j}\right)\right]^+ \geqslant 0 \Leftrightarrow D^S_{t,i} + \left[\sum_{j\neq i} \left(D^S_{t,j} - A_{t,j}\right)\right]^+ \geqslant A_{t,i}$. Per Equation (13) this condition implies that $A^P_{t,i} = A_{t,i}$.

Let us now see how the probabilities of default on the junior tranche, PD^J, can be calculated from **Equation (16)**. We have that

$$PD^J_{t,i} = P\left(A^R_{t+24,i} < D^J_{t+24,i}\right)$$

$$= P\left(\sum_{j\neq i} A_{t+24,j} > \sum_{j\neq i} D^S_{t+24,j}\right) \times P\left(A^R_{t+24,i} < D^J_{t+24,i} | \sum_{j\neq i} A_{t+24,j} > \sum_{j\neq i} D^S_{t+24,j}\right)$$

$$+P\left(\sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D^S_{t+24,j}\right) \times P\left(A^R_{t+24,i} < D^J_{t+24,i} | \sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D^S_{t+24,j}\right),$$

where the second equality follows from the law of total probability whereby the default space is split into a idiosyncratic country default (the first term) and a systemic default setting (the second term). By incorporating Equations (12) and (13), we have that

$$PD^J_{t,i} = P\left(\sum_{j\neq i} A_{t+24,j} > \sum_{j\neq i} D^S_{t+24,j}\right) \times P\left(A_{t+24,i} < D_{t+24,i} | \sum_{j\neq i} A_{t+24,j} > \sum_{j\neq i} D^S_{t+24,j}\right)$$

$$+P\left(\sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D^S_{t+24,j}\right) \times P\left(\sum_i A_{t+24,i} < \sum_i D^S_{t+24,i} + D^J_{t+24,i} | \sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D^S_{t+24,j}\right),$$

i.e.

$$PD_{t,i}^{J}=P\left(A_{t+24,i}<D_{t+24,i}\bigcap\sum_{j\neq i}A_{t+24,j}>\sum_{j\neq i}D_{t+24,j}^{S}\right) \tag{30}$$

$$+P\left(\sum_{j}A_{t+24,j}<\sum_{j}D_{t+24,j}^{S}+D_{t+24,i}^{J}\bigcap\sum_{j\neq i}A_{t+24,j}<\sum_{j\neq i}D_{t+24,j}^{S}\right).$$

The first term in Equation (30) can be calculated from the bivariate distribution function of $\left(A_{t+24,i},\sum\limits_{j\neq i}A_{t+24,j}\right)$ while the second term can be calculated based on the bivariate distribution of $\left(\sum\limits_{i}A_{t+24,i},\sum\limits_{j\neq i}A_{t+24,j}\right)$.[42] Under the assumption of joint normality of $\{\triangle ln(A_{t,i}\}$ and taking into account the empirical covariance matrix of $\triangle ln(A_t)$, the required bivariate distributions can be obtained as shown below, allowing for the calculation of the probabilities of idiosyncratic country default (the first term) and systemic default of partner countries for each sovereign and for each point in time (see below). $PD_{t,i}^{J}$ is then the sum of these two probabilities.

We now proceed to prove Properties 1 to 3, as listed in Section 5.

Property 1 is based on the fact that a distribution is multivariate normal if and only if every linear combination of its component variables is normally distributed. Therefore, under the assumption of joint normality for $\{\triangle ln\left(A_{t,i}\right)\}$, we have that for a given sovereign i

$$\begin{bmatrix} 0 & ... & 1 & ... & 0 \end{bmatrix}\times\begin{bmatrix}\triangle ln\left(A_{t,1}\right)\\ ...\\ \triangle ln\left(A_{t,i}\right)\\ ...\\ \triangle ln\left(A_{t,n}\right)\end{bmatrix}=\triangle ln\left(A_{t,i}\right)\sim N\left(\mu_i,\sigma_i^2\right)$$

and

$$\begin{bmatrix} 1 & ... & 0 & ... & 1 \end{bmatrix}\times\begin{bmatrix}\triangle ln\left(A_{t,1}\right)\\ ...\\ \triangle ln\left(A_{t,i}\right)\\ ...\\ \triangle ln\left(A_{t,n}\right)\end{bmatrix}=\sum_{j\neq i}\triangle ln\left(A_{t,j}\right)\sim N\left(\sum_{j\neq i}\mu_j,\sigma^2_{\sum\limits_{j\neq i}\triangle ln(A_{t,j})}\right),$$

where $\sigma^2_{\sum\limits_{j\neq i}\triangle ln(A_{t,j})}$ denotes the variance of the variable $\sum\limits_{j\neq i}\triangle ln\left(A_{t,j}\right)$. In order to show that $\left(\triangle ln\left(A_{t,i}\right),\sum\limits_{j\neq i}\triangle ln\left(A_{t,j}\right)\right)$ are joint normally distributed observe that any linear combination of the two variables is normally distributed:

$$a\times\triangle ln\left(A_{t,i}\right)+b\times\sum_{j\neq i}\triangle ln\left(A_{t,j}\right)=$$

[42] As before, the Fenton-Wilkinson approximation is used when dealing with the sum of log-normal variables.

$$= \begin{bmatrix} b & ... & a & ... & b \end{bmatrix} \times \begin{bmatrix} \triangle ln\left(A_{t,1}\right) \\ ... \\ \triangle ln\left(A_{t,i}\right) \\ ... \\ \triangle ln\left(A_{t,n}\right) \end{bmatrix} \sim N\left(a \times \mu_i + b \times \sum_{j\neq i} \mu_j, \sigma^2_{a\times \triangle ln(A_{t,i}) + b \times \sum_{j\neq i} \triangle ln(A_{t,j})}\right).$$

A similar reasoning can be applied to show that $\left(\sum_i \triangle ln\left(A_{t,i}\right), \sum_{j\neq i} \triangle ln\left(A_{t,j}\right)\right)$ are also joint normally distributed variables. Given the bivariate normal distributions of $\left(\triangle ln\left(A_{t,i}\right), \sum_{j\neq i} \triangle ln\left(A_{t,j}\right)\right)$ and $\left(\sum_i \triangle ln\left(A_{t,i}\right), \sum_{j\neq i} \triangle ln\left(A_{t,j}\right)\right)$, the probability of default on the red bonds, $PD^J_{t,i}$, can be calculated from Equation (30).

Property 2 follows from the fact that if $\left(\sum_{j\neq i} A_{t,i} > \sum_{j\neq i} D^S_{ti}\right)$, then the first branch of Equation (13) implies that $A^P_{t,j} = D^S_{t,i}$. As $A_{t,j}$ does not enter the previous equality, we have that $\frac{\partial A^P_{t,j}}{\partial A_{t,j}} = 0$. It should be noted that this conclusion does not apply when both $\left(\sum_{j\neq i} A_{t,i} > \sum_{j\neq i} D^S_{ti}\right)$ and $A_{t,j} < D^S_{t,i}$. In this case, the second branch of Equation (13) yields $A^P_{t,j} = A_{t,j}$. However, this is a case where sovereign i is in idiosyncratic default and any debt capacity gains are immediately pledged to that sovereign's share of the blue bond, $D^S_{t,i}$. As such, sovereign i is arguably the residual beneficiary of its own debt capacity also in this case.

Property 3 says that $A^P_{t,i} \uparrow A_{t,i}\ \forall i$ as $\sum_i A_{t,i} \downarrow \sum_i D^S_{t,i}$. In continuous time, $A_{t,i}$ is a geometric brownian motion and, therefore, a continuous process. Let us assume that $D^S_{t,i}$ is also a continuous process. The property can be verified by proving each of the following statements:

1. $\sum_i A_{t,i} \geq \sum_i D^S_{t,i}. \Rightarrow A^P_{t,i} \leq A_{t,i}\ \forall i.$

2. $A^P_{t,i} - A_{t,i}\ = f\left(\sum_i \left(A_{t,i} - D^S_{t,i}\right)\right)$ is continuous (where f represents a functional relationship).

3. $\sum_i A_{t,i} = \sum_i D^S_{t,i} \Rightarrow A^P_{t,i} = A_{t,i}\ \forall i.$

In order to show Statement 1., partition the set I of participating countries into sets $S_<$ and $S_\geq$, as previously defined. Given that $\sum_i A_{t,i} \geq \sum_i D^S_{t,i}$, we have that $S_\geq$ will be non-empty, while $S_<$ may or may not be empty. If $i \in S_<$, Equation (13) implies that $A^P_{t,i} = A_{t,i}$ and Statement 1. holds. If $i \in S_\geq$, consider the following two cases. If $\left[\sum_{j\neq i}\left(D^S_{t,j} - A_{t,j}\right)\right]^+ = 0$ then Equation (13) implies that $A^P_{t,i} = D^S_{t,i} \leq A_{t,i}$ and statement 1 holds. If $\left[\sum_{j\neq i}\left(D^S_{t,j} - A_{t,j}\right)\right]^+ > 0$ then $\sum_{j\neq i}\left(D^S_{t,j} - A_{t,j}\right) + D^S_{t,i} - A_{t,i} = \sum_i D^S_{t,i} - \sum_i A_{t,i} \leq 0 \Leftrightarrow D^S_{t,i} + \left[\sum_{j\neq i}\left(D^S_{t,j} - A_{t,j}\right)\right]^+ \leq A_{t,i}$. The last inequality together with Equation (13) mean that $A^P_{t,i} \leq A_{t,i}$, implying that Statement 1. also holds in this case.

In order to show Statement 2., let us consider in turn each branch of Equation (13). Regarding the first branch of the equation, if $D_{t,i}^S + \left[\sum_{j\neq i}\left(D_{t,j}^S - A_{t,j}\right)\right]^+ < A_{t,i}$, then $A_{t,i}^P - A_{t,i} = D_{t,i}^S - A_{t,i} + \left[\sum_{j\neq i}\left(D_{t,j}^S - A_{t,j}\right)\right]^+$. If, in turn, $\sum_{j\neq i}\left(D_{t,j}^S - A_{t,j}\right) > 0$, then $A_{t,i}^P - A_{t,i} = -\sum_i \left(A_{t,i} - D_{t,i}^S\right)$, which is continuous. Otherwise, $A_{t,i}^P - A_{t,i} = -\left(A_{t,i} - D_{t,i}^S\right)$, which is also continuous. Finally, $A_{t,i}^P - A_{t,i}$ is continuous at the critical point where $\sum_{j\neq i}\left(D_{t,j}^S - A_{t,j}\right) = 0$ given the continuity of the maximum function. Regarding the second branch of the equation, if $D_{t,i}^S + \left[\sum_{j\neq i}\left(D_{t,j}^S - A_{t,j}\right)\right]^+ \geq A_{t,i}$, then $A_{t,j}^P - A_{t,i} = 0$, which is trivially continuous. It remains to establish continuity at the threshold where $A_{t,i}^P$ switches branches, i.e., at $D_{t,i}^S + \left[\sum_{j\neq i}\left(D_{t,j}^S - A_{t,j}\right)\right]^+ = A_{t,i}$. Continuity in this case follows from the fact that $A_{t,i}^P$ can be written as the minimum between $D_{t,i}^S + \left[\sum_{j\neq i}\left(D_{t,j}^S - A_{t,j}\right)\right]^+$ and $A_{t,i}$,[43] which is a continuous function of two functions we have shown to be continuous.

Finally, as regards Statement 3., we have seen from Implication (14) that when $\sum_i A_{t,i} = \sum_i D_{t,i}^S$, we have that $A_{t,i}^P = A_{t,i}\ \forall i$.

In order to show **Property 4,** let us denote the probability of default on the blue bond as $PD^{J,Blue}$ and the probability of default on the junior bond under a tranching approach as $PD^{J,Tranche}$. We have seen in this appendix that

$$PD_{t,i}^{J,Blue} = P\left(\sum_{j\neq i} A_{t+24,j} > \sum_{j\neq i} D_{t+24,j}^S\right) \times P\left(A_{t+24,i} < D_{t+24,i} | \sum_{j\neq i} A_{t+24,j} > \sum_{j\neq i} D_{t+24,j}^S\right)$$

$$+P\left(\sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D_{t+24,j}^S\right) \times P\left(\sum_i A_{t+24,i} < \sum_i D_{t+24,i}^S + D_{t+24,i}^J | \sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D_{t+24,j}^S\right).$$

At the same time, we have seen in Subsection 6.1 that

$$PD^{J,Tranche} = P\left(A_{t+24,i} < D_{t+24,i}\right)$$

$$= P\left(\sum_{j\neq i} A_{t+24,j} > \sum_{j\neq i} D_{t+24,j}^S\right) \times P\left(A_{t+24,i} < D_{t+24,i} | \sum_{j\neq i} A_{t+24,j} > \sum_{j\neq i} D_{t+24,j}^S\right)$$

$$+P\left(\sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D_{t+24,j}^S\right) \times P\left(A_{t+24,i} < D_{t+24,i} | \sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D_{t+24,j}^S\right).$$

[43] See footnote 33.

Therefore, in order to show that $PD^{J,Blue} > PD^{J,Tranche}$, it suffices to show that

$$P\left(\sum_i A_{t+24,i} < \sum_i D^S_{t+24,i} + D^J_{t+24,i} | \sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D^S_{t+24,j}\right) > P\left(A_{t+24,i} < D_{t+24,i} | \sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D^S_{t+24,j}\right).$$

This is true, as

$$P\left(\sum_i A_{t+24,i} < \sum_i D^S_{t+24,i} + D^J_{t+24,i} | \sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D^S_{t+24,j}\right)$$

$$= P\left(A_{t+24,i} < D_{t+24,i} + \left(\sum_{j\neq i} D^S_{t+24,j} - \sum_{j\neq i} A_{t+24,j}\right) | \sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D^S_{t+24,j}\right)$$

$$> P\left(A_{t+24,i} < D_{t+24,i} | \sum_{j\neq i} A_{t+24,j} < \sum_{j\neq i} D^S_{t+24,j}\right).$$

C Simultaneous default versus sequential default: an example

The following example illustrates the gains in terms of expected loss on total debt obtained from moving from a situation where an issuer defaults simultaneously on its junior and senior tranches to a situation where it defaults only sequentially. Take the case of sovereign i with debt D_i worth 100% of GDP, a PD_i of 10% and a standard LGD of 60%. The expected loss for this sovereign before tranching is $PD_i \times LGD = 10\% \times 60\% = 6\%$. Assume now that sovereign i's debt has been split 60% into a senior tranche D_i^S and 40% into a junior tranche D_i^J. We now have two possibilities:

1. **Simultaneous default**: irrespective of tranching, in the event of default, sovereign i defaults simultaneously on both tranches imposing an LGD of 60% on total debt outstanding. This is the type of behaviour underlying the pooling and tranching option of Subsection (6.3) where sequential default is not possible due to the fact that there is only one type of debt security issued in the primary market. The expected loss on total debt outstanding is the weighted average of the expected loss on each tranche: $PD_i^J \times LGD_i^J \times \frac{D_i^J}{D_i} + PD_i^S \times LGD_i^S \times \frac{D_i^S}{D_i} = 10\% \times 100\% \times 40\% + 10\% \times \frac{(60\% - 40\%)}{60\%} \times 60\% = 6\%$. There are no credit risk premia gains from tranching as the expected loss is the same.

2. **Sequential default**: sovereign i only defaults on the senior tranche after having defaulted first on the junior tranche. This is the assumption underlying the options with *a priori* tranching, which open up this possibility for the sovereign. The expected loss on overall debt outstanding is $PD_i^J \times LGD_i^J \times \frac{D_i^J}{D_i} + PD_i^S \times LGD_i^S \times \frac{D_i^S}{D_i} = 10\% \times 100\% \times 40\% + 0\% \times 60\% \times 60\% = 4\%$ where PD_i^S is assumed to be zero, for illustration sake,[44] and LGD_i^S is a conventional 60%. The gains from tranching result from this lower expected loss.

[44] A PD of approximately zero for a senior tranche worth 60% of GDP is consistent with model-based results for several countries and sample periods.

D Systemic euro area shocks and idiosyncratic country shocks: an illustration

	Situation in Apr-08		Systemic EA shock (2 std dev)				Idiosyncratic shock to IT (2 std dev)				Combined systemic (2 std dev) and idiosync. shock (2 std dev) to IT				Non-discriminated shock to IT (2 std dev)			
	CDS	PD	CDS	PD	Δ CDS	Δ PD	CDS	PD	Δ CDS	Δ PD	CDS	PD	Δ CDS	Δ PD	CDS	PD	Δ CDS	Δ PD
AT	15	0.5%	36	1.2%	20	0.7%	15	0.5%	0	0.0%	35	1.2%	20	0.7%	26	0.9%	11	0.4%
BE	14	0.5%	34	1.1%	21	0.7%	12	0.4%	-2	-0.1%	29	1.0%	16	0.5%	22	0.7%	8	0.3%
DE	8	0.3%	20	0.7%	12	0.4%	8	0.3%	0	0.0%	20	0.7%	12	0.4%	14	0.5%	6	0.2%
EL	111	3.7%	138	4.6%	27	0.9%	129	4.3%	18	0.6%	158	5.3%	47	1.6%	142	4.7%	31	1.0%
ES	23	0.8%	58	1.9%	35	1.2%	18	0.6%	-5	-0.2%	46	1.5%	23	0.8%	34	1.1%	11	0.4%
FI	10	0.3%	27	0.9%	17	0.6%	10	0.3%	0	0.0%	27	0.9%	17	0.6%	19	0.6%	9	0.3%
FR	10	0.3%	27	0.9%	18	0.6%	10	0.3%	0	0.0%	27	0.9%	17	0.6%	18	0.6%	9	0.3%
IE	16	0.5%	37	1.2%	20	0.7%	15	0.5%	-2	-0.1%	33	1.1%	16	0.5%	25	0.8%	8	0.3%
IT	33	1.1%	64	2.1%	30	1.0%	76	2.5%	42	1.4%	132	4.4%	99	3.3%	92	3.1%	59	2.0%
NL	9	0.3%	24	0.8%	16	0.5%	8	0.3%	-1	0.0%	21	0.7%	13	0.4%	15	0.5%	6	0.2%
PT	36	1.2%	75	2.5%	39	1.3%	37	1.2%	1	0.0%	76	2.5%	40	1.3%	58	1.9%	22	0.7%

	Situation in Apr-16		Systemic EA shock (2 std dev)				Idiosyncratic shock to IT (2 std dev)				Combined systemic (2 std dev) and idiosync. shock (2 std dev) to IT				Non-discriminated shock to IT (2 std dev)			
	CDS	PD	CDS	PD	Δ CDS	Δ PD	CDS	PD	Δ CDS	Δ PD	CDS	PD	Δ CDS	Δ PD	CDS	PD	Δ CDS	Δ PD
AT	19	0.6%	43	1.4%	24	0.8%	19	0.6%	0	0.0%	42	1.4%	24	0.8%	31	1.0%	13	0.4%
BE	22	0.7%	53	1.8%	31	1.0%	19	0.6%	-3	-0.1%	46	1.5%	24	0.8%	34	1.1%	12	0.4%
DE	10	0.3%	24	0.8%	14	0.5%	10	0.3%	0	0.0%	24	0.8%	14	0.5%	17	0.6%	8	0.3%
EL	1103	36.8%	1215	40.5%	111	3.7%	1178	39.3%	75	2.5%	1290	43.0%	187	6.2%	1231	41.0%	127	4.2%
ES	53	1.8%	122	4.1%	69	2.3%	43	1.4%	-11	-0.4%	99	3.3%	46	1.5%	76	2.5%	23	0.8%
FI	13	0.4%	34	1.1%	21	0.7%	13	0.4%	0	0.0%	33	1.1%	20	0.7%	24	0.8%	10	0.3%
FR	14	0.5%	38	1.3%	24	0.8%	14	0.5%	0	0.0%	37	1.2%	23	0.8%	26	0.9%	12	0.4%
IE	25	0.8%	54	1.8%	29	1.0%	23	0.8%	-2	-0.1%	48	1.6%	23	0.8%	37	1.2%	12	0.4%
IT	92	3.1%	160	5.3%	68	2.3%	185	6.2%	93	3.1%	295	9.8%	203	6.8%	218	7.3%	126	4.2%
NL	14	0.5%	37	1.2%	23	0.8%	12	0.4%	-2	-0.1%	33	1.1%	19	0.6%	23	0.8%	9	0.3%
PT	197	6.6%	338	11.3%	141	4.7%	200	6.7%	3	0.1%	340	11.3%	144	4.8%	279	9.3%	82	2.7%

Figure 11: Systemic euro area shock and idiosyncratic shock to Italy

Note: CDS denotes the 2-year sovereign CDS spreads in basis points and PD the 2-year sovereign probability of default. For an explanation of the shocks, see the discussion in Subsection 7.1.

E Decomposition of the evolution of sovereign debt capacity into systemic and idiosyncratic components

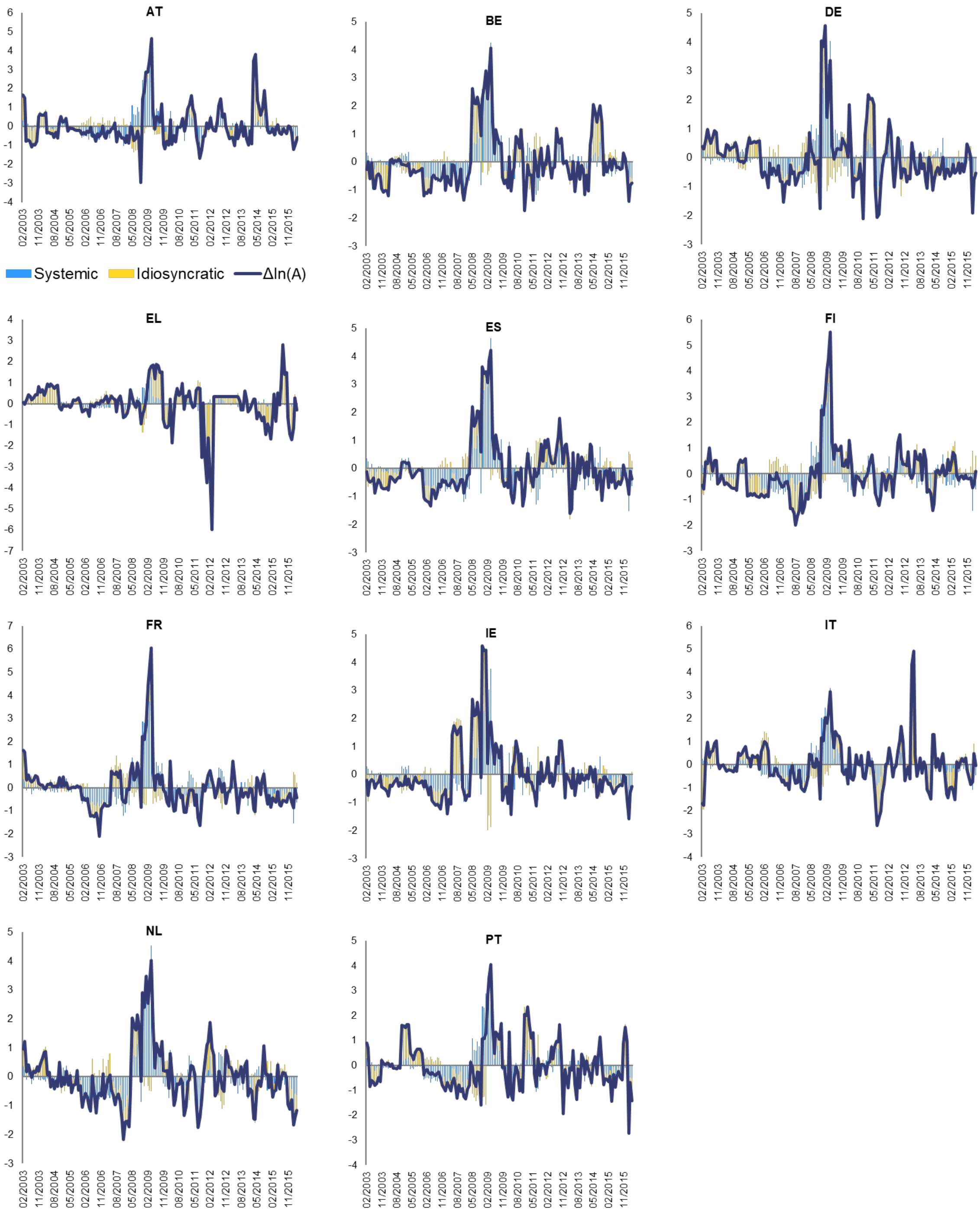

Figure 12: Decomposition of $\Delta ln\,(A_{t,i})^N$: systemic vs. idiosyncratic components

F Cumulated financial gap and idiosyncratic sovereign efforts

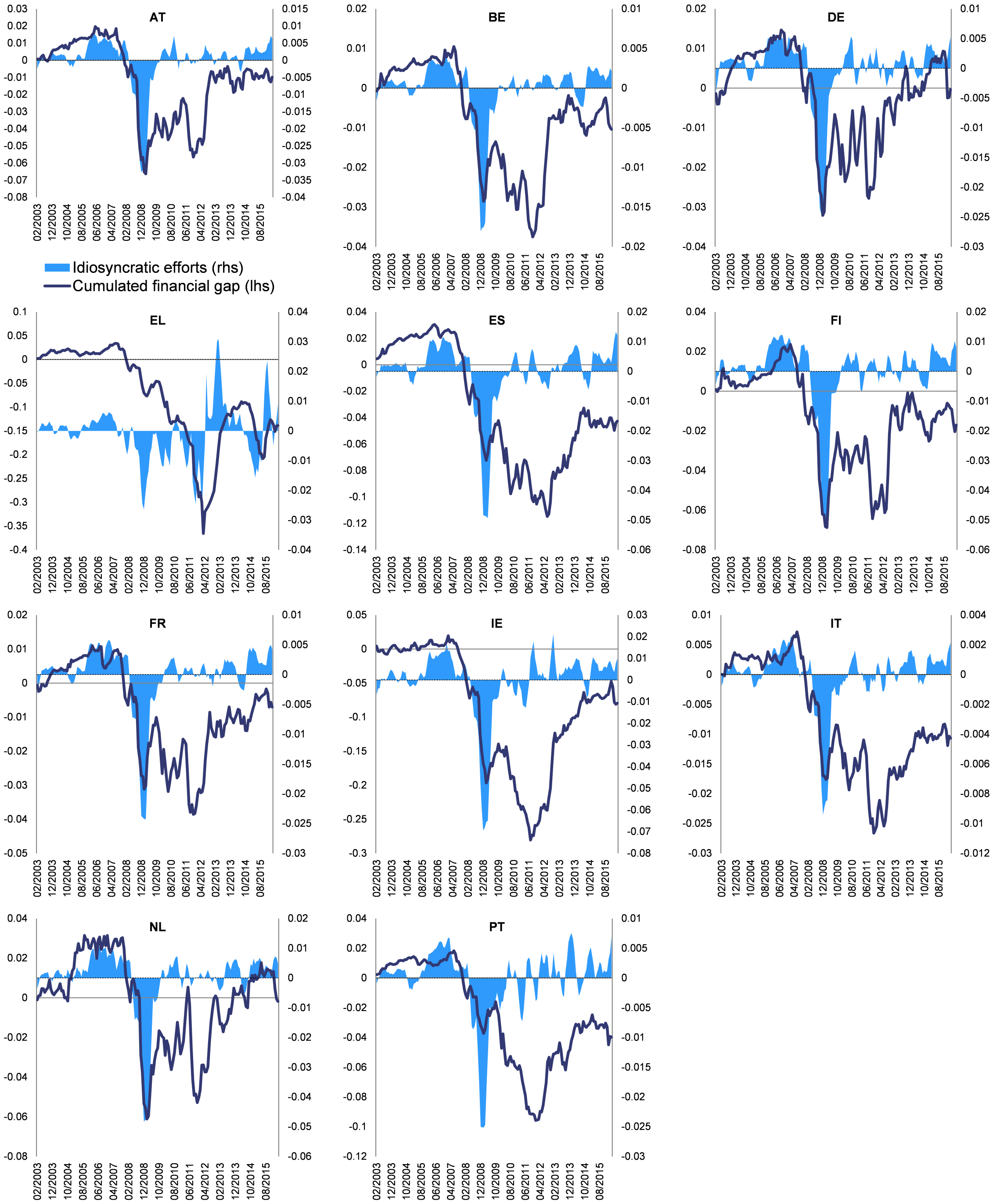

Figure 13: Cumulated financial gap and idiosyncratic sovereign efforts

Note: for the definitions of cumulated financial gap and idiosyncratic sovereign efforts see Equations (26) and (27) in Subsection 7.1. Idiosyncratic sovereign effort is shown as a centred three-month moving average.

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