STATEC

Shadow GAP: a new approach to gap estimation

Joint OGWG-ECFIN-JRC Conference:
“Assessment of output gaps and potential output in the context of the COVID-19 pandemic and its aftermath”

Vasja Sivec
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MOTIVATION & IDEA

HP filter:

- simple & works well most of the time
- can be inconsistent with third variables (unemployment, inflation or capacity utilization indicators)
- complexity in data-limited environment
- true DGP hard to identify

We take HP filter and apply ***inequality restrictions*** on the cycle:

Output GAP is restricted when it is “at odds” with the dynamics of the restricting variable (example: if inflation is above average and GAP is negative) and left unrestricted when it is in “accordance” with the dynamics of the restricting variable (example: when inflation is above average and GAP is positive).

MODEL and ESTIMATION

HP filter can be also estimated by using a trend-cycle decomposition model – introduction [EXAMPLE]

Following Harvey&Jaeger (1993) – estimation Kalman Smoother + maximum likelihood:

$$\lambda = \frac{\delta_\varepsilon^2}{\delta_\xi^2}$$

Signal to noise ratio - determines »smoothness« of the trend. λ =0 trend is equal to rGDP, λ=+inf trend is linear.

$$y_t = t_t + c_t$$

$$t_t = t_{t-1} + \beta_{t-1}$$

$$\beta_t = \beta_{t-1} + \xi_t$$

Trend - double random walk with time a varying slope.

$$\eta_t \sim N(0, \delta_\eta^2)$$
$$\xi_t \sim N(0, \delta_\xi^2)$$
$$\varepsilon_t \sim N(0, \delta_\varepsilon^2)$$

Shocks to level, slope and cycle.

$$c_t^R = \varepsilon_t$$

Cycle - just a residual. I also tried a cyclical model.

MODEL and ESTIMATION

Modification - intuition

RESHAPE

- HP filter is a »statistical« model, disregards valuable information in inflation, unemployment,…
- We estimate the cycle (=output gap) by imposing »Inequality Constraints«:

»If inflation is above average gap is less likely to be negative, if inflation is below average gap is less likely to be positive.« (for example)

Inequality Constraints – output gap is **greater or equal to zero** if inflation is above average. Constraint only binds if the condition is violated, “active set method” - translates non-linear constraints into linear constraints.

Soft Constraints – output gap is **more likely to be positive** if inflation is above average. (Why? Oil shock example. Can be too restrictive...)

MODEL MODIFICATION C… DE

Constraint only binds when Gap is at odds with the signaling variable (inflation).

Tightness of constraint is controlled for by the ratio of cycle to constraint equation variance: **κ=** $\frac{\delta_\xi^2}{\delta_\varepsilon^2}$

Implementation

$$\left.\begin{aligned} y &= t + c^R \\ t &= \cdots \\ \beta_t &= \cdots \\ c_t^R &= \varepsilon_t \end{aligned}\right\} \text{Same model as before...}$$

NEW - State augmentation (=add the following equation to the model):

$$D_t \times c_t^R = \xi_t; \quad where \;\; D_t = \begin{cases} 1 & if \;\; (\pi > \bar{\pi} \wedge c_t^{UR} < 0) \vee (\pi < \bar{\pi} \wedge c_t^{UR} > 0) \\ 0 & otherwise \end{cases}$$

$\xi_t \sim N(0, \delta_\xi^2)$

$\delta_\xi^2 = 0$ hard constraint (closes the gap if cond. is viol.)

$\delta_\xi^2 > 0$ soft constraint (makes gap less pos./neg.)

DISCLAIMER…

… for the ***ilustrative*** example that follows.

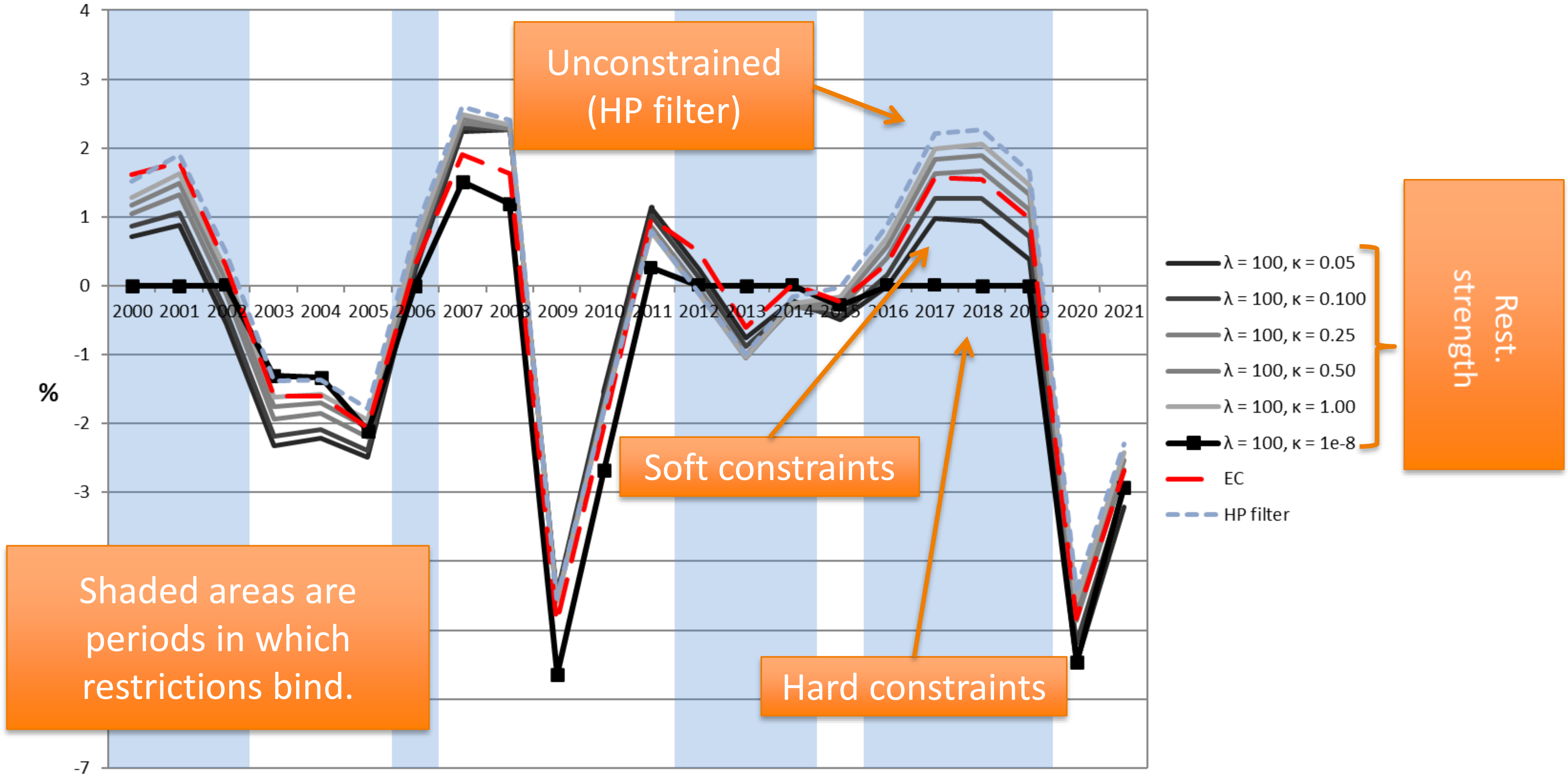
- Output GAP for DE, signal-to-noise set to $\lambda=100$, 1980-2021, Spring 2021 data vintage.

- Inflation cycle (CPI) will be used to derive sign restrictions. Other variables (CPI, wage inflation, unemployment cycle, capacity utilization) or de-trending methods (trend, moving average, differencing) could be more appropriate.

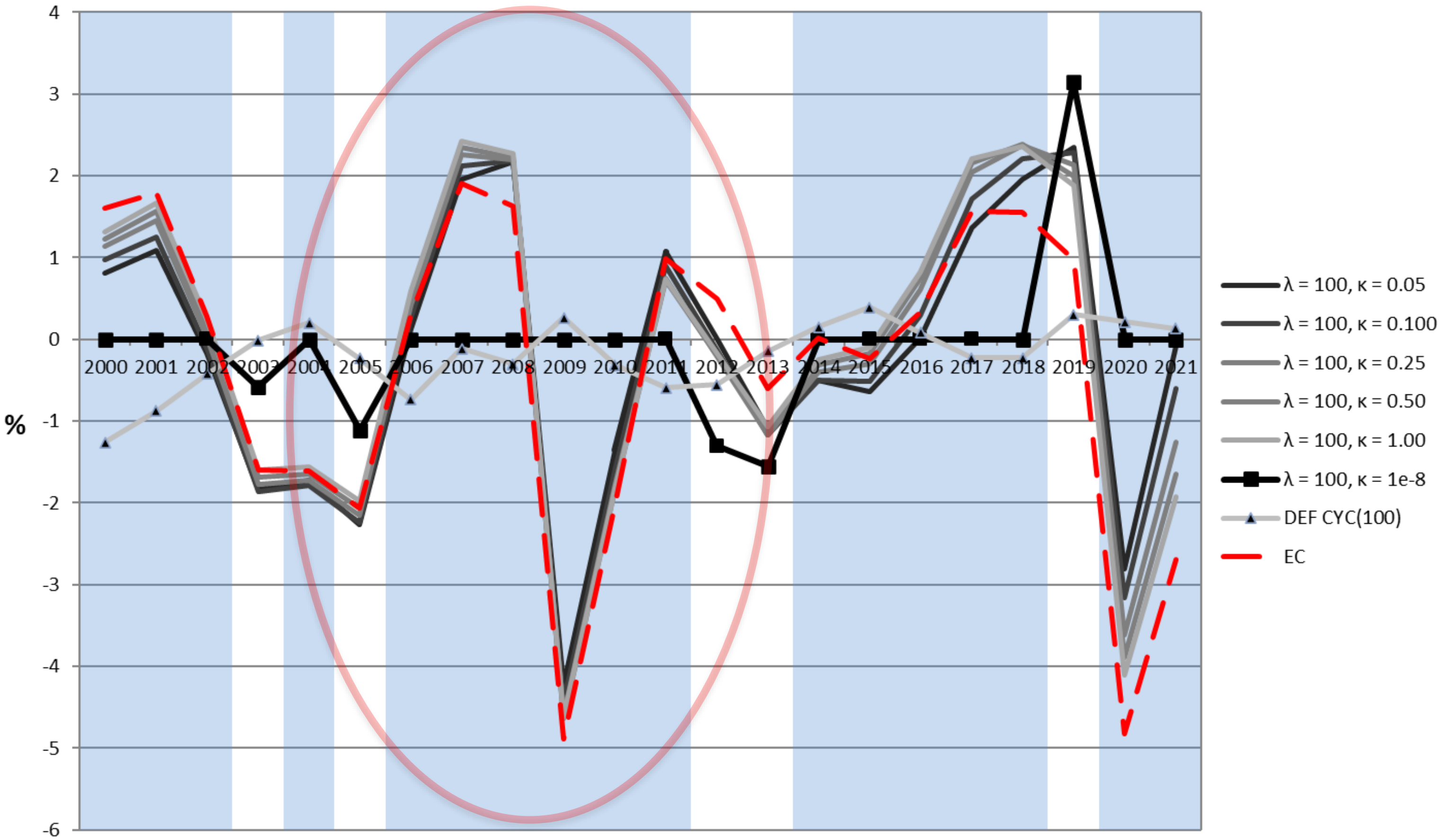
- This example only intends to illustrate how the method works.

- We investigate several other options in our empirical analysis.

Recent example: DE, CPI CYCLE 100, Spring 2021 data



CAUTION – it can also go wrong: DE, DEF CYCLE 100, Spring 2021 data



EMPIRICAL ANALYSIS

- Small Monte Carlo (not shown here)

Data

- Yearly data for 27 EU member states + UK.
- We use real time data (vintage data sets). Collected from the archive of the EC‘s Output Gap Working Group.
- Two types of data:
  - Data1, 34 vintages: Spring 2004-Spring 2021, forecasts up to T+2, benchmark model (only limited set of variables is available)
  - Data2, 15 vintages: Spring 2014 –Spring 2021, forecasts up to T+5, production function model

EMPIRICAL ANALYSIS cont.

Models

1. Benchmark model (Data1) – we decompose real GDP to trend and cycle

2. Production function model (Data2, EC model):

$$Y^{POT} = \overline{tfp} \times K^{\alpha} \times \left[popwa \times \overline{part} \times (1 - \overline{ur}) \times \overline{hours}\right]^{\alpha}$$

tfp - total factor productivity

popwa - population of working age,

part - labour market participation rate,

hours – average annual hours worked per employed person

other variables

CUBS

EMPIRICAL ANALYSIS cont.

Signaling variables, transformations, parameters and time

- ***5* candidate signalling variables**: CPI, GDP deflator, wage inflation index, unemployment rate, capacity utilization indicator (EC CUBS).
- ***3 de-trending methods*** for signalling variables: HP filtering, demeaning with 5 year moving averages, differencing
- ***6 tightness for the restrictions (κ)***: $10^{-7}\ (hard\ const.), 0.05, 0.1, 0.25, 0.5, 1\ (soft\ const.)$
- ***2 signal-to-noise ratios (λ)***: 10 or 100
- ***In-sample*** (1,…,T) and ***real-time*** (T+1) analysis

RESULTS – CORRELATIONS

… with the signalling variable

- **Example**: CPI cycle, λ = 10, average over **all** countries and vintages

In sample

	Rest. Tightness	Shadow Gap	HP gap	EC gap
Benchmark model	0	0.41	-0.01	-0.16
	0.05	0.25	-0.01	-0.16
	0.1	0.20	-0.02	-0.16
	0.25	0.13	-0.01	-0.16
	0.5	0.08	-0.01	-0.16
	1	0.04	-0.01	-0.16
Production function model	0	0.02	-0.09	-0.19
	0.05	0.00	-0.09	-0.19
	0.1	-0.01	-0.09	-0.19
	0.25	-0.03	-0.09	-0.19
	0.5	-0.05	-0.09	-0.19
	1	-0.07	-0.09	-0.19

Real-time

	Rest. Tight	Shadow Ga	HP gap	EC gap
Benchmark model	0	0.29	0.00	-0.01
	0.05	0.12	-0.01	-0.02
	0.1	0.08	-0.01	-0.03
	0.25	0.04	-0.01	-0.01
	0.5	0.03	0.00	-0.01
	1	0.02	0.01	0.00
Production function model	0	-0.04	-0.12	0.02
	0.05	-0.07	-0.12	0.02
	0.1	-0.08	-0.12	0.02
	0.25	-0.10	-0.12	0.02
	0.5	-0.11	-0.12	0.02
	1	-0.11	-0.12	0.02

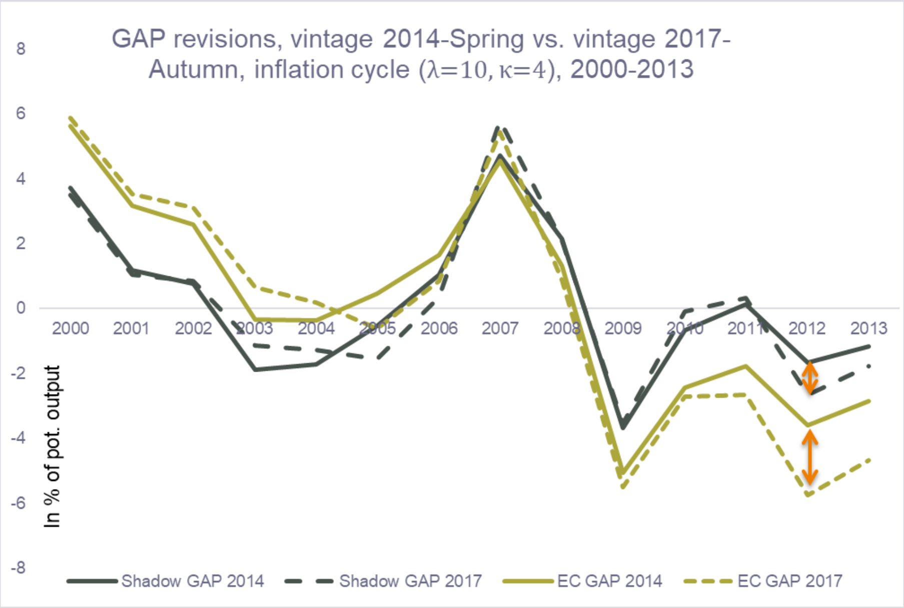
RESULTS: CORRELATIONS cont.

Averages over **all** signalling variables, tightness of restrictions and signal-to-noise ratios.

Since it's not «fine tuned» for each country this table shows a general tendency. Works quite well despite not being selective w.r.t. variables, parameters, etc…

	Type	basic model Shadow>HP	basic model Shadow>EC	production function model Shadow>HP	production function model Shadow>EC
In-sample	Cycle	98%	80%	100%	100%
	Demean	94%	60%	94%	58%
	Growth rates	81%	25%	39%	50%
Real-time	Cycle	73%	62%	47%	19%
	Demean	100%	60%	72%	6%
	Growth rates	13%	31%	36%	17%

RESULTS 3 – REVISIONS & FORECASTS



$$\Delta x_t = a + \sum_{p=1}^{q} \beta_p \Delta x_{t-1}\ \gamma GAP_{t-1} + u_t$$

REVISIONS

RMSE between current vintage GAP and final vintage GAP.

NOTE - RMSE is normalized to account for GAPs variance (constraints often reduce amplitude of GAPs).

FORECASTS

AR-X with GAP, BIC.
x_t is the of the signalling variable. To remain consistent with related literature **we forecast growth rates**.

RESULTS – REVISIONS & FORECASTING cont.

Averages over **all** signalling variables, tightness of restrictions and signal-to-noise ratios.

REVISIONS		basic models		basic models	
	Type	Shadow>HP	Shadow>EC	Shadow>HP	Shadow>EC
In-sample	Cycle	3%	88%	8%	75%
	Demean	27%	96%	11%	78%
	Differentiated	17%	83%	17%	75%
Real-time	Cycle	15%	0%	19%	0%
	Demean	4%	0%	75%	0%
	Differentiated	35%	0%	0%	0%

EC deals with Covid extremely well!

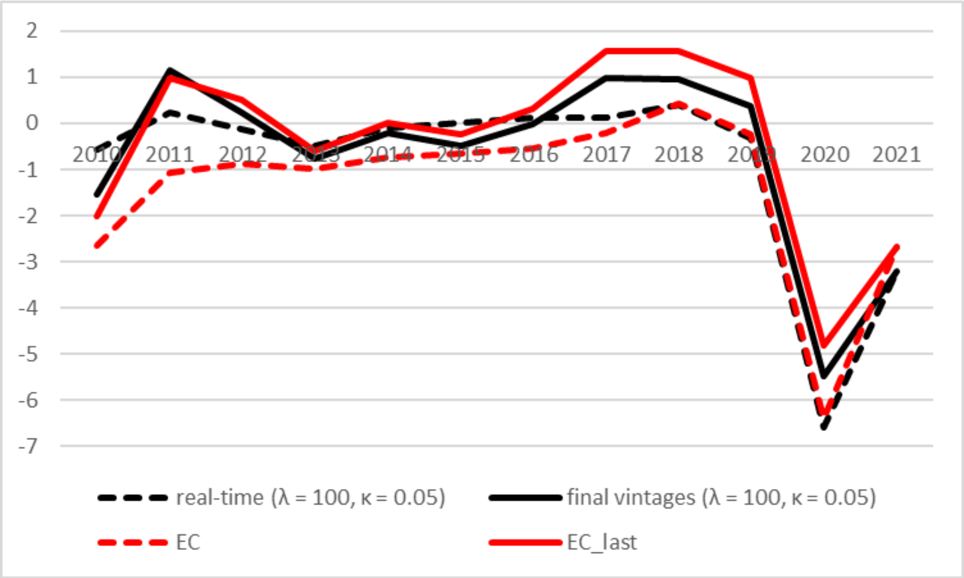
FORECASTS		basic models		basic models	
	Type	Shadow>HP	Shadow>EC	Shadow>HP	Shadow>EC
Real-time	Cycle	35%	57%	58%	94%
	Demean	50%	88%	44%	61%
	Differentiated	56%	96%	31%	78%

Unfortunately, we have not yet found a solution but…

Contemplating:

- Dummy for Covid in the cycle (already presented by Atanas), first results on benchmark models are encouraging.
- Dummy that loads on the series being smoothed ($y_t = t_t + c_t + D_{2020}$).
- Allow for a different variance of the cycle in 2020 and let the data speak.
- Does it make sense? Once in a generation type of shock?

Suggestions are welcome!



For some countries the method still performs well (example: Real Time T+1 OG (dashed), final vint. OG (solid)) for DE, CPI CYC 100. But for some it fails in Covid. We are still looking into this.

CONCLUSION

New approach to estimate Output GAP. Simple, almost no parameters to estimate, beneficial for short series or when the true DGP is uncertain.

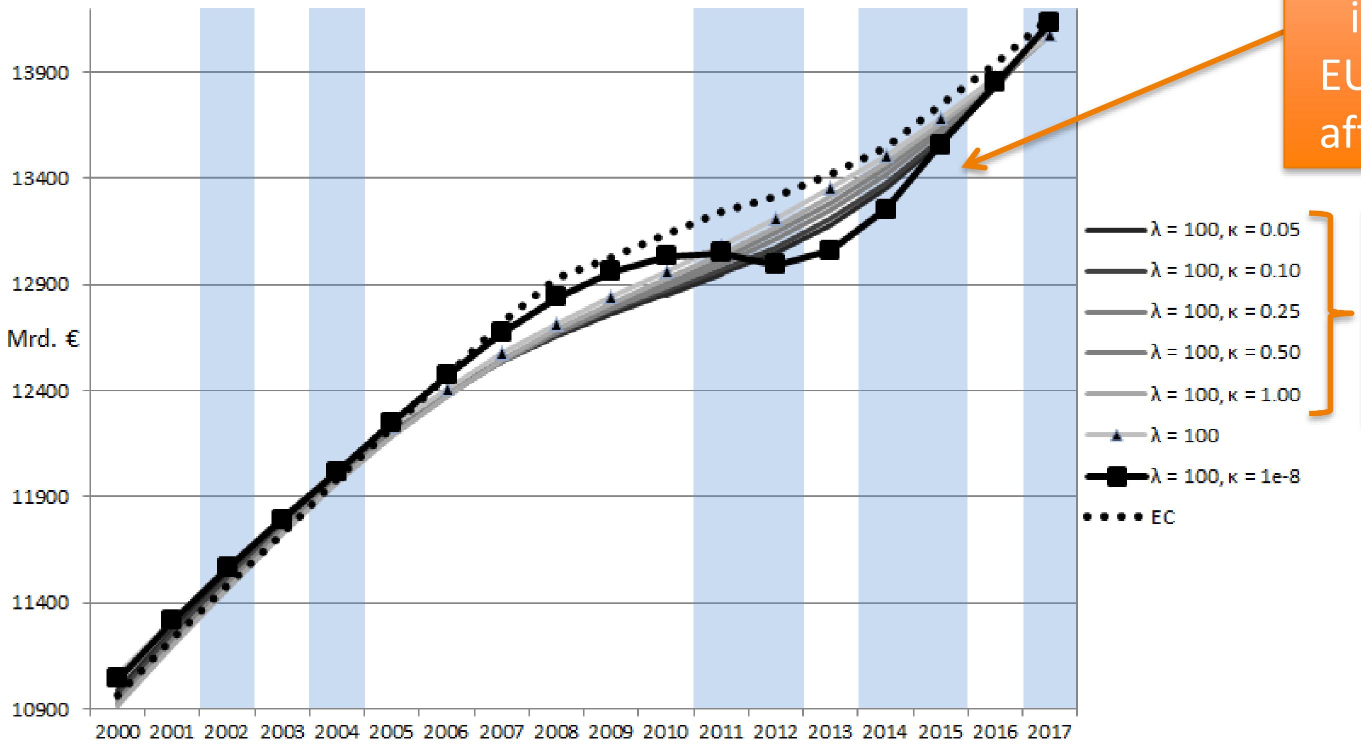
Inequality restrictions: restricted GAP when “at odds” with the dynamics of the restricting variable.

The resulting gap **is more highly correlated with the restricting variable** in 1-T. Before Covid also in T+1, but Covid hurt its performance.

Tends to perform well in forecasting.

Thank you for your attention!

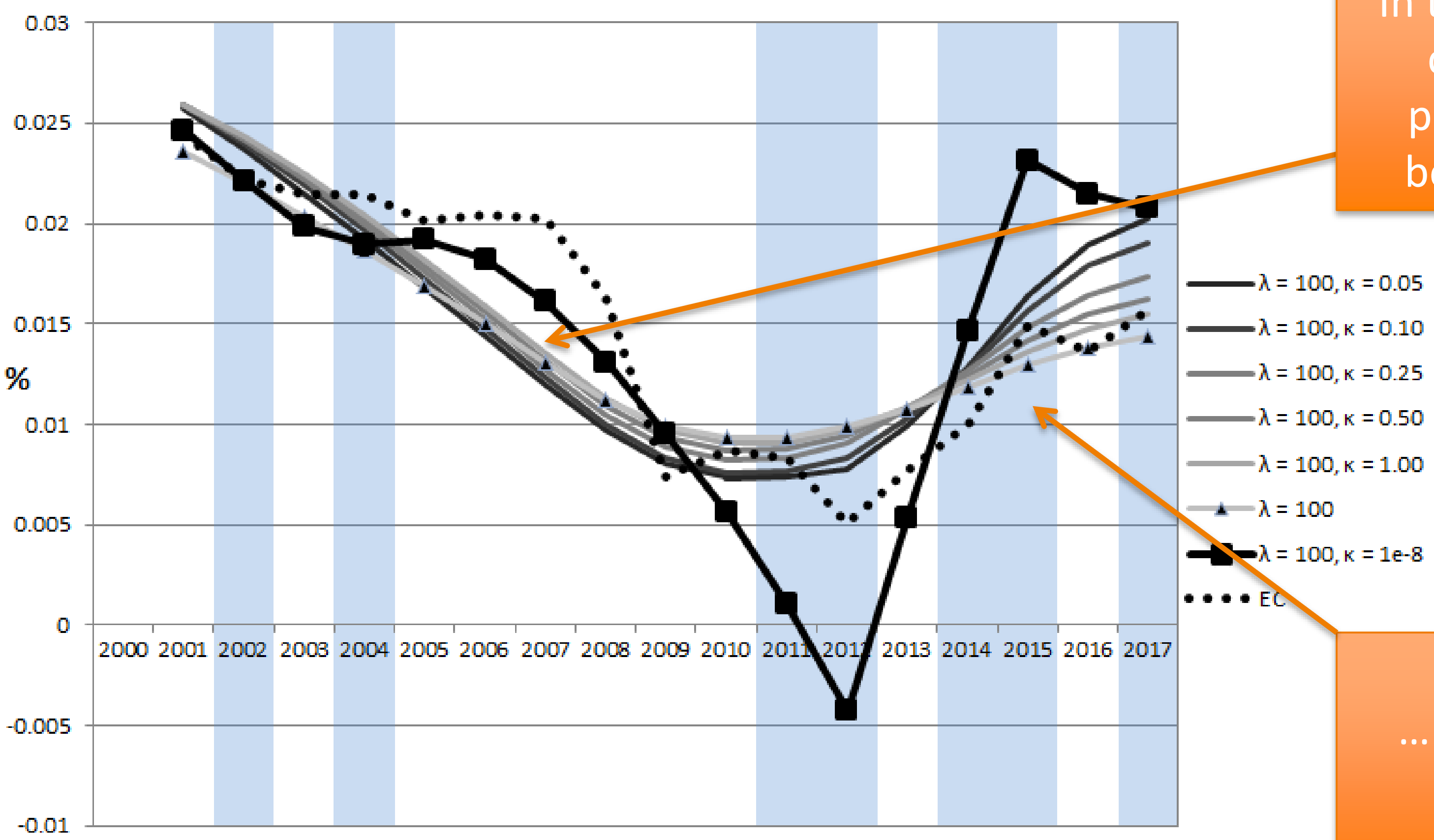
Example: Potential output, EU28, Autumn 2017 data, 2000-2017



In this example, restrictions imply lower EU28 potential after the crisis.

Tightness of restrictions

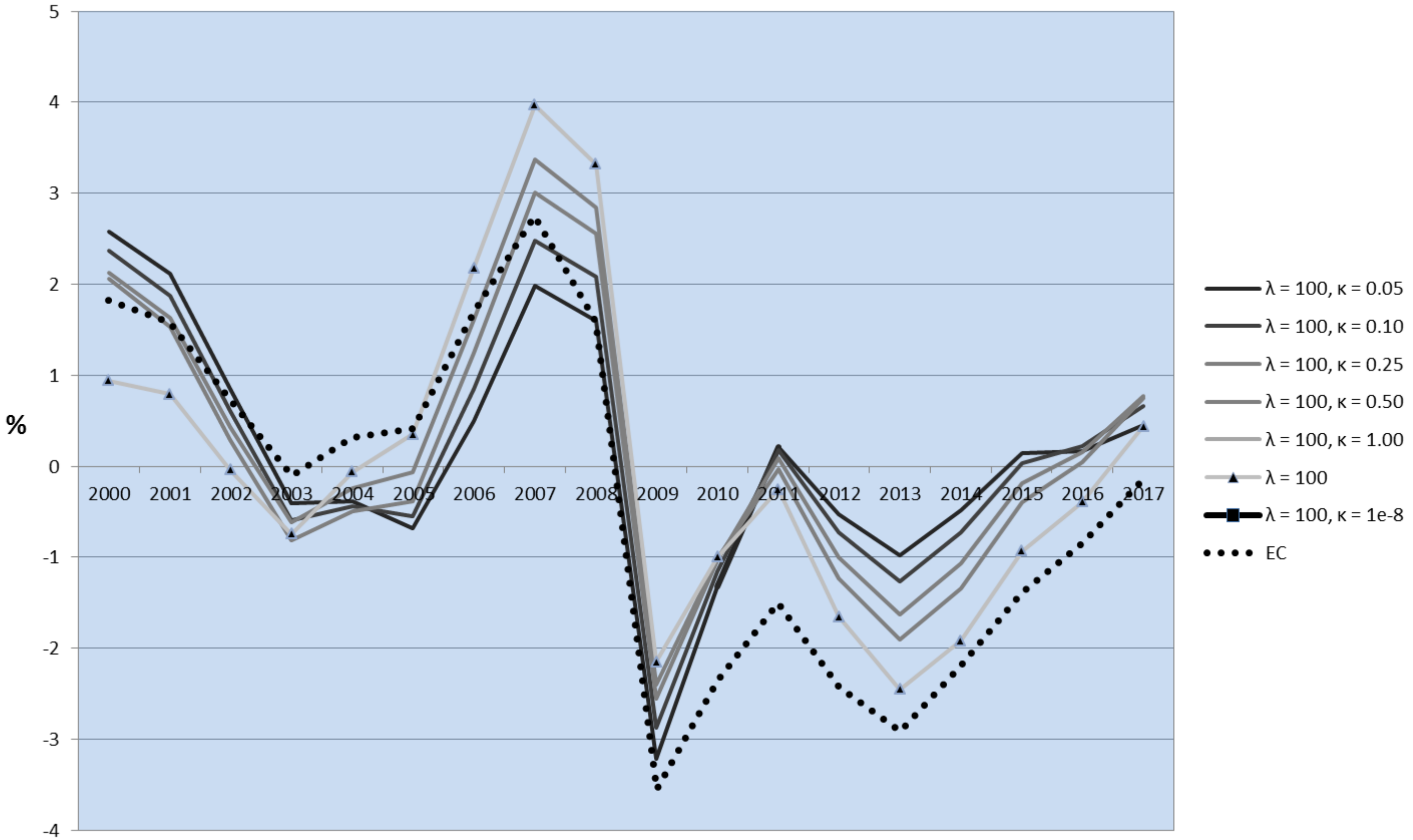
Example: Growth of potential output, EU28, Autumn 2017 data, 2000-2017



In this example, pre-crisis growth of potential is lower before the crisis…

… and higher after 2014.

Extensions – model for the cycle



Statistics - Correlations

In-sample

$$\overline{CORR} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{V}\sum_{v=1}^{V} r_{i,v} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{V}\sum_{v=1}^{V}\frac{\sum_{t=t_1}^{T}(OG_{i,t,v}-\overline{OG}_{i,v})(x_{i,t,v}-\overline{x}_{i,v})}{\sqrt{\sum_{t=t_1}^{T}(OG_{i,t,v}-\overline{OG}_{i,v})^2(x_{i,t,v}-\overline{x}_{i,v})^2}}$$

Real-time

$$\overline{CORR} = \frac{1}{N}\sum_{i=1}^{N} r_i = \frac{1}{N}\sum_{i=1}^{N}\frac{\sum_{v=1}^{V}(OG_{i,t=y,v}-\overline{OG}_i^{v})(x_{i,t=y(v),V}-\overline{x}_i^{V})}{\sqrt{\sum_{v=1}^{V}(OG_{i,t=y,v}-\overline{OG}_i^{v})^2(x_{i,t=y(v),V}-\overline{x}_i^{V})^2}}$$

Statistics - Revisions

In-sample

$$\overline{nRMSE} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{V}\sum_{v=1}^{V}\frac{RMSE_{i,v}}{\sigma_{OG_{i,t_1\ldots T,V}}} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{V}\sum_{v=1}^{V}\frac{\sqrt{\frac{1}{T-t_1}\sum_{t=t_1}^{T}\left(OG_{i,t,v} - OG_{i,t,V}\right)^2}}{\sigma_{OG_{i,t_1\ldots T,V}}} \quad (17)$$

Real-time

$$\overline{nRMSE} = \frac{1}{N}\sum_{i=1}^{N}\frac{RMSE_i}{\sigma_{OG_{i,V}}} = \frac{1}{N}\sum_{i=1}^{N}\frac{\sqrt{\frac{1}{V}\sum_{v=1}^{V}\left(OG_{i,t=y,v} - OG_{i,t=y(v),V}\right)^2}}{\sigma_{OG_{i,V}}}$$

EXAMPLE– Best performing models OLD UPDATE

	Benchmark	**Production function**
Signaling variable	CUBS/unemployment rate	gdp deflator/wage inflation
Transformation	differences	differences
Tightness of restrictions	medium (0.1, 0.25, 0.5)	medium/strong (0.25, 0.5, 1)
Signal to noise ratio	10	10/100

Benchamrk model: UR for signal, differenced I(1), kappa = 0.5, lambda = 10

	Shadow	HP	EC
CORR(x) in-sample	-0.60	-0.47	-0.54
CORR(x) real-time	-0.68	-0.63	-0.50
REVISIONS in-sample nRMSE	0.31	0.32	0.42
REVISIONS real-time nRMSE	0.70	0.78	0.77
FORECAST RMSE	2.32	2.45	2.78

Production function model: GDP deflator for signal, differenced I(2), kappa = 0.25, lambda = 100

	Shadow	HP	EC
CORR(x) in-sample	0.24	0.23	0.24
CORR(x) real-time	0.11	0.16	0.08
REVISIONS in-sample nRMSE	0.17	0.16	0.19
REVISIONS real-time nRMSE	0.16	0.18	0.21
FORECAST RMSE	0.73	0.75	0.75

Further work

- Mutliple restrictions (from 2 signaling variables). Easy to extend.
- Update with 2 new vintages. DONE
- Add stohastic level to get it closer to the EC's specification.
- DEAL with COVID.