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# Should We Use Linearised Models to Calculate Fiscal Multipliers?

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# Should We Use Linearised Models to Calculate Fiscal Multipliers?

Jesper Lindé and Mathias Trabandt

## Abstract

We calculate the magnitude of the fiscal spending multiplier in linearised and nonlinear solutions of a New Keynesian model at the zero lower bound. Importantly, the model is amended with real rigidities to simultaneously account for the *macroeconomic evidence* of a low Phillips curve slope and the *microeconomic evidence* of frequent price re-optimisation. We show that the nonlinear solution is associated with a much smaller multiplier than the linearised solution in long-lived liquidity traps, and pin down the key features in the model which account for the difference. Our results caution against the common practice of using linearised models to calculate fiscal multipliers in long-lived liquidity traps.

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## 1. Introduction

The magnitude of the fiscal spending multiplier is a classic subject in macroeconomics. To calculate the magnitude of the multiplier, economists typically employ a linearized version of their actual nonlinear model. Does linearizing the nonlinear model matter for the conclusions about the multiplier? We document this may be the case, especially in long-lived liquidity traps. When interest rates are expected to be constrained by the zero (or effective) lower bound for a protracted time period, the nonlinear solution suggests a much smaller multiplier than the linearized solution of the same model.

The financial crisis and “Great Recession” have revived interest in the magnitude of the fiscal spending multiplier. A quickly growing literature suggests that the fiscal spending multiplier can be very large when nominal interest rates are expected to be constrained by the zero (or effective) lower bound (ZLB) for a prolonged period, see e.g. Eggertsson (2010), Davig and Leeper (2011), Christiano, Eichenbaum and Rebelo (2011), Woodford (2011), Coenen et al. (2012) and Leeper, Traum and Walker (2015). Erceg and Linde (2014) show that in a long-lived liquidity trap fiscal stimulus can come at low cost to the treasury and even be a “fiscal free lunch”. Conversely, the results of the above literature suggest that it is difficult to reduce government debt in the short-run through aggressive government spending cuts in long-lived liquidity traps: fiscal consolidation can in fact be self-defeating in such a situation.

Importantly, the bulk of the existing literature analyzes fiscal multipliers in models where all equilibrium equation have been linearized around the steady state, except for the ZLB constraint on the monetary policy rule. Implicit in the linearization procedure is the assumption that the linearized solution is accurate even far away from the steady state. However, recent work by Boneva, Braun, and Waki (2016) suggests that linearization produces severely misleading results at the zero lower bound. Essentially, Boneva et al. argue that extrapolating decision rules far away from the steady state is invalid.

Our paper provides a positive analysis of the effect of spending-based fiscal stimulus on output and government debt using a fully nonlinear model. We compare the fiscal spending multipliers for output and government debt of the nonlinear and linearized solution as function of the liquidity trap duration. Moreover, our framework allows us to pin down the key features which account for the difference between the multiplier schedule for the nonlinear and linearized solutions of the model.

The model environment is a variant of the canonical New Keynesian DSGE model of Woodford (2003). This model features monopolistic competition and Calvo sticky prices and the central bank follows a Taylor rule subject to the ZLB constraint on nominal rates.<sup>1</sup> A distinct difference to the existing literature which uses variations of the standard New Keynesian model – for instance the recent work by Boneva, Braun and Waki (2016), Christiano, Eichenbaum and Johannsen (2016), Eggertsson and Singh (2016), Fernandez-Villaverde et al. (2015) and Nakata (2015) – is that we consider a framework with real rigidities. Specifically, we introduce real rigidities through the Kimball (1995) state-dependent demand elasticity which allows our model to simultaneously account for the *macroeconomic evidence* of a low linearized Phillips curve slope (0.01) and the *microeconomic evidence* of frequent price re-optimization (3-4 quarters).<sup>2</sup> Due to its ability to ease the tensions between the macro- and microeconomic evidence on price setting, the Kimball aggregator has gained traction in New Keynesian models and it is for example used in the workhorse Smets and Wouters (2007) model.

Our main results are as follows. First, we show that the nonlinear solution is associated with a much smaller fiscal spending multiplier than the linearized solution in long-lived liquidity traps. More precisely, when the ZLB is expected to bind for 3 years, the nonlinear solution implies a multiplier of about  $2/3$  while the linearized solution of the same model implies a multiplier slightly above 2. As the multiplier in our benchmark model with real rigidities equals  $1/3$  in normal times when the ZLB does not bind, the nonlinear solution implies roughly a doubling of the multiplier in a long-lived trap, whereas the linearized solution implies that the multiplier is elevated seven times.

What accounts for the large difference between the nonlinear and linearized solutions in a prolonged liquidity trap? We document that the difference can almost entirely be accounted for by the nonlinearities in the price setting block of the model – the Phillips curve. Key here is the nonlinearity implied by the Kimball aggregator. The Kimball aggregator implies that the demand elasticity for intermediate goods is state-dependent, i.e. the firms' demand elasticity is an increasing function of its relative price. In short, the demand curve is quasi-kinked. While the fully nonlinear model takes this state-dependency explicitly into account, a linear approximation replaces that nonlinearity by a linear function. Put differently, linearization replaces the quasi-kinked demand

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<sup>1</sup> We rule out the well-known problems associated with steady state multiplicity emphasized by Benhabib, Schmitt-Grohe and Uribe (2001) by restricting our attention to the steady state with a positive inflation rate.

<sup>2</sup> There is also a recent literature which studies models where the effects of government spending is state dependent (i.e. differs in booms and recessions even absent zero lower bound considerations due to labor market frictions), see e.g. Michailat (2014), Rendahl (2016) and Roulleau-Pasdeloup (2016). This paper does not address this literature.



curve with a linear function.<sup>3</sup> Intuitively, in a deep recession that triggers the ZLB to bind for a long time, the Kimball aggregator carries the implication that firms do not find it attractive to cut their prices much since that reduces the demand elasticity and thereby does not crowd in more demand. With more fiscal spending in such a situation, firms also find it less attractive to increase their prices. Thus – with policy rates stuck at zero – aggregate inflation increases only little and therefore the real interest rate falls by little: the multiplier does not increase to the same extent with the duration of the ZLB. When the model is linearized, the response of aggregate inflation is notably stronger due to the nature of a linear approximation of a quasi-kinked demand curve at the steady state with no dispersion. Hence, the drop in the real interest rate is elevated following a spending hike and the multiplier is magnified. The bottom line: the linearized version of the model exaggerates the rise in expected and actual inflation due to a sizable approximation error and thereby elevates the magnitude of the fiscal multiplier in long-lived liquidity traps.<sup>4</sup>

Our results have potentially important implications for the scope of fiscal stimulus to be self-financing, and the extent to which fiscal consolidations can be self-defeating. In the nonlinear model, fiscal stimulus is never a “free lunch” as in Erceg and Lindé (2014); and conversely, fiscal consolidations are never self-defeating. The linearized solution arrives at the opposite conclusions: fiscal stimulus can be self-financing in a sufficiently long-lived liquidity trap and fiscal consolidations can be self-defeating. However, although these findings cast doubt on the findings of the existing literature on the fiscal implications of stimulus, it should be recalled that we are considering a model environment where the fiscal output multiplier is small in normal times ( $1/3$  as mentioned earlier). Had we instead considered a model in which the multiplier were in the mid-range of the empirical evidence (i.e. a multiplier slightly less than unity) when monetary policy is unconstrained, it is possible that the multiplier could be magnified sufficiently in a long-lived liquidity trap to obtain a “fiscal free lunch” for a transient hike in spending.<sup>5</sup> We elaborate further on this issue in the conclusions.

Our paper is related to Boneva, Braun and Waki (2016), Christiano, Eichenbaum and Johannsen (2016), Fernandez-Villaverde et al. (2015), Nakata (2015) and Eggertsson and Singh (2016). Importantly, none of the above papers considers the case of a Kimball (1995) aggregator. Boneva,

<sup>3</sup> It is well known that in a linearized model, the Kimball (1995) and Dixit-Stiglitz (1977) aggregator – the latter featuring a constant demand elasticity – are observationally equivalent up to a factor of proportionality.

<sup>4</sup> The small rise in inflation expectations is consistent with Dupor and Li (2015), who argue that expected inflation reacted little to spending shocks in the United States during the Great Recession.

<sup>5</sup> A large empirical literature has examined the effects of government spending shocks, mainly focusing on the post-WWII pre-financial crisis period when monetary policy had latitude to adjust interest rates. The bulk of this research suggests a government spending multiplier in the range of 0.5 to somewhat above unity (1.5). See e.g. Hall (2009), Ramey (2011), Blanchard, Erceg and Lindé (2016) and the references therein.

Braun and Waki (2016) report that the multiplier is smaller in a fully nonlinear model. Their model features a Dixit-Stiglitz aggregator. Eggertsson and Singh (2016) report that the multipliers of the nonlinear and linearized model differ only very little. Their model features a Dixit-Stiglitz aggregator and assumes firms-specific labor markets, implying that price dispersion is irrelevant for the nonlinear model dynamics. By contrast, our analysis shows how important these assumptions are: moving to the frequently used Kimball aggregator and allowing for price dispersion alters the conclusions about the multiplier substantially. Nakata (2015) and Fernández-Villaverde et al. (2015) show that shock uncertainty may have potentially important implications for equilibrium dynamics. Even so, robustness analysis shows that allowing for shock uncertainty have very limited impact on our results when the degree of price adjustment (and thus the extent to which how quickly expected inflation adjusts) is calibrated to fit the macroevidence on the slope of Phillips curve. Christiano, Eichenbaum and Johannsen (2016) analyze multiplicity of equilibria in a nonlinear New Keynesian model. They document that there is a unique stable-under-learning rational expectations equilibrium in their model and that all other equilibria are not stable under learning.

The remaining of the paper is organized as follows. Section 2 presents the small scale New Keynesian model and Section 3 the results. Section 4 examines the robustness of the results to various perturbations of the model. Section 5 concludes.

## **2. A Stylized New Keynesian Model**

The simple model we study is very similar to the one developed Erceg and Linde (2014), which in turn builds on the baseline Eggertsson and Woodford (2003) model with the exception that it allows for real effects of price distortions by dropping the assumption that labor cannot be reallocated between different firms (or industries). We deviate from Erceg and Linde (2014) in two ways; first by allowing for a Kimball (1995) aggregator (with the standard Dixit and Stiglitz (1977) specification as a special case), and second, by including a discount factor (or savings) shock. Below, we outline the model and its key nonlinear equations. In the Appendix A, we describe the linearized version.

### **2.1. Model**

#### **2.1.1. Households**

The utility functional for the representative household is

$$\max_{\{C_t, N_t, B_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \varsigma_t \left\{ \log(C_t - C\nu_t) - \frac{N_t^{1+\chi}}{1+\chi} \right\} \quad (1)$$

where the discount factor  $\beta$  satisfies  $0 < \beta < 1$  and is subject to an exogenous shock  $\varsigma_t$ . As in Erceg and Lindé (2014), the utility function depends on the household's current consumption  $C_t$  as deviation from a "reference level"  $C\nu_{t+j}$ . The exogenous consumption taste shock  $\nu_t$  raises the reference level and marginal utility of consumption. The utility function also depends negatively on hours worked  $N_t$ .

The household's budget constraint in period  $t$  states that its expenditure on goods and net purchases of (zero-coupon) government bonds  $B_{G,t}$  must equal its disposable income:

$$P_t C_t + B_{G,t} = (1 - \tau_N) W_t N_t + (1 + i_{t-1}) B_{G,t-1} - T_t + \Gamma_t \quad (2)$$

Thus, the household purchases the final consumption good at price  $P_t$ . The household is subject to a constant distortionary labor income tax  $\tau_N$  and earns after-tax labor income  $(1 - \tau_N) W_t N_t$ . The household pays lump-sum taxes net of transfers  $T_t$  and receives a proportional share of the profits  $\Gamma_t$  of all intermediate firms.

Utility maximization yields the standard consumption Euler equation

$$1 = \beta \delta_t E_t \left\{ \frac{(1 + i_t)}{1 + \pi_{t+1}} \frac{C_t - C\nu_t}{C_{t+1} - C\nu_{t+1}} \right\}, \quad (3)$$

where we have defined

$$\delta_t \equiv \frac{E_t \varsigma_{t+1}}{\varsigma_t}, \quad (4)$$

and introduced the notation  $1 + \pi_{t+1} = P_{t+1}/P_t$ . We also have the following labor supply schedule:

$$N_t^\chi = \frac{1 - \tau_N}{C_t - C\nu_t} \frac{W_t}{P_t}. \quad (5)$$

Equations (3) and (5) are the key equations for the household side of the model.

### 2.1.2. Firms and Price Setting

*Final Goods Production* The single final output good  $Y_t$  is produced using a continuum of differentiated intermediate goods  $Y_t(f)$ . Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G \left( \frac{Y_t(f)}{Y_t} \right) df = 1. \quad (6)$$

As in Dotsey and King (2005) and Levin, Lopez-Salido and Yun (2007), we assume that  $G(\cdot)$  is given by the following strictly concave and increasing function:

$$G\left(\frac{Y_t(f)}{Y_t}\right) = \frac{\omega}{1+\psi} \left[ (1+\psi) \frac{Y_t(f)}{Y_t} - \psi \right]^{\frac{1}{\omega}} - \left[ \frac{\omega}{1+\psi} - 1 \right], \quad (7)$$

where  $\psi = \frac{(1-\phi_p)\epsilon_p}{\phi_p}$  and  $\omega = \frac{\phi_p - (\phi_p - 1)\epsilon_p}{1 - (\phi_p - 1)\epsilon_p}$ . Here  $\phi_p \geq 1$  denotes the gross markup of the intermediate firms. The parameter  $\epsilon_p$  governs the degree of curvature of the intermediate firm's demand curve, and in Figure 1 we show how relative demand is affected by the relative price under alternative assumptions about  $\epsilon_p$  (and thus  $\psi$ ) for given  $\phi_p$ .<sup>6</sup> When  $\epsilon_p = 0$ , the demand curve exhibits constant elasticity as under the standard Dixit-Stiglitz aggregator, implying a linear relationship between relative demand and relative prices ( $\psi = 0$  in Figure 1). When  $\epsilon_p$  is positive – as in Smets and Wouters (2007) – the firm's instead face a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a large fall in demand. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost, especially when  $\epsilon_p$  is high ( $\psi = -8$  in Figure 1). Finally, we notice that  $G(1) = 1$ , implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index  $Y_t$ , taking as given the price  $P_t(f)$  of each intermediate good  $Y_t(f)$ . Moreover, final goods producers sell units of the final output good at a price  $P_t$ , and hence solve the following problem:

$$\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f) Y_t(f) df \quad (8)$$

subject to the constraint (6). The first order conditions can be written as

$$\begin{aligned} \frac{Y_t(f)}{Y_t} &= \frac{1}{1+\psi} \left( \left[ \frac{P_t(f)}{P_t} \frac{1}{\Lambda_t^p} \right]^{-\frac{\phi_p - (\phi_p - 1)\epsilon_p}{\phi_p - 1}} + \psi \right), \\ P_t \Lambda_t^p &= \left[ \int P_t(f)^{-\frac{1 - (\phi_p - 1)\epsilon_p}{\phi_p - 1}} df \right]^{-\frac{\phi_p - 1}{1 - (\phi_p - 1)\epsilon_p}}, \\ \Lambda_t^p &= 1 + \psi - \psi \int \frac{P_t(f)}{P_t} df, \end{aligned} \quad (9)$$

<sup>6</sup> The figure is taken from Levin, Lopez-Salido and Yun (2007), and the mapping between  $\psi$  and  $\epsilon_p$  is given by  $\epsilon_p = \frac{-\phi_p \psi}{(\phi_p - 1)}$ . A value of  $\psi = -8$  thus implies that  $\epsilon_p$  equals 88 when the gross markup  $\phi_p$  equals 1.1.

where  $\Lambda_t^p$  denotes the Lagrange multiplier on the aggregator constraint (7). Note that for  $\epsilon_p = 0$  (and thus  $\psi = 0$ ),  $\Lambda_t^p = 1 \forall t$  and the first-order conditions in (9) simplifies to the usual Dixit and Stiglitz (1977) expressions

$$\frac{Y_t(f)}{Y_t} = \left[ \frac{P_t(f)}{P_t} \right]^{-\frac{\phi_p}{\phi_p-1}}, \quad P_t = \left[ \int P_t(f)^{\frac{1}{1-\phi_p}} df \right]^{1-\phi_p}$$

*Intermediate Goods Production* A continuum of intermediate goods  $Y_t(f)$  for  $f \in [0, 1]$  is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand schedule from the final goods firms through the solution to the problem in eq. (8) that varies inversely with its output price  $P_t(f)$  and directly with aggregate demand  $Y_t$ .

Aggregate capital ( $K$ ) is assumed to be fixed, so that aggregate production of the intermediate good firm is given by

$$Y_t(f) = K(f)^\alpha N_t(f)^{1-\alpha}. \quad (10)$$

Despite the fixed aggregate stock  $K \equiv \int K(f) df$ , shares of it can be freely allocated across the  $f$  firms, implying that real marginal cost,  $MC_t(f)/P_t$  is identical across firms and equal to

$$\frac{MC_t}{P_t} \equiv \frac{W_t/P_t}{MPL_t} = \frac{W_t/P_t}{(1-\alpha) K^\alpha N_t^{-\alpha}}, \quad (11)$$

where  $N_t = \int N_t(f) df$ .

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. In each period, each firm  $f$  faces a constant probability,  $1 - \xi_p$ , of being able to re-optimize its price  $P_t(f)$ . The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price according to the following formula

$$\tilde{P}_t = (1 + \pi) P_{t-1}, \quad (12)$$

where  $\pi$  is the steady-state (net) inflation rate and  $\tilde{P}_t$  is the updated price.

Given Calvo-style pricing frictions, firm  $f$  that is allowed to re-optimize its price,  $P_t^{opt}(f)$ , solves the following problem

$$\max_{P_t^{opt}(f)} E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \varsigma_{t+j} \Lambda_{t,t+j} \left[ (1 + \pi)^j P_t^{opt}(f) - MC_{t+j} \right] Y_{t+j}(f)$$

where  $\Lambda_{t,t+j}$  is the stochastic discount factor (the conditional value of future profits in utility units, recalling that the household is the owner of the firms), and demand  $Y_{t+j}(f)$  from the final goods firms is given by the equations in (9).

### 2.1.3. Monetary and Fiscal Policies

The evolution of nominal government debt is determined by the following equation

$$B_{G,t} = (1 + i_{t-1}) B_{G,t-1} + P_t G_t - \tau_N W_t N_t - T_t \quad (13)$$

where  $G_t$  denotes real government expenditures on the final good  $Y_t$ . Following the convention in the literature on fiscal multipliers, we assume that lump-sum taxes stabilize government debt as share of nominal trend GDP,  $b_{G,t} \equiv \frac{B_{G,t}}{P_t Y}$ . Specifically, we follow Erceg and Linde (2014) and assume that net lump-sum taxes as share of nominal trend GDP,  $\tau_t \equiv \frac{T_t}{P_t Y}$ , follow the simple rule:

$$\tau_t - \tau = \varphi_b (b_{G,t-1} - b_G), \quad (14)$$

where variables without time subscript denote steady state. Finally, government spending,  $g_{y,t} \equiv \frac{G_t}{Y}$  is exogenous.

Turning to the central bank, it is assumed to adhere to a Taylor-type policy rule that is subject to the zero lower bound:

$$1 + i_t = \max \left( 1, (1 + i) \left[ \frac{1 + \pi_t}{1 + \pi} \right]^{\gamma_\pi} \left[ \frac{Y_t}{Y_t^{pot}} \right]^{\gamma_x} \right) \quad (15)$$

where  $Y_t^{pot}$  denotes the level of output that would prevail if prices were flexible, and  $i$  the steady-state (net) nominal interest rate, which is given by  $r + \pi$  where  $r \equiv 1/\beta - 1$ . In the linearized model, (15) is written

$$i_t = \max(0, i + \gamma_\pi (\pi_t - \pi) + \gamma_x x_t) \quad (16)$$

where  $x_t \equiv \ln \left( Y_t / Y_t^{pot} \right)$  is the model-consistent output gap.

### 2.1.4. Aggregate Resource Constraint

We now turn to discuss the derivation of the aggregate resource constraint. Let  $Y_t^{sum}$  denote the unweighted average (sum) of output for each firm  $f$ , i.e.

$$Y_t^{sum} = \int_0^1 Y_t(f) df.$$

which from (10) and the observation that all firms have the same capital-labor ratio can be rewritten as

$$\begin{aligned} Y_t^{sum} &= \int \left( \frac{K(f)}{N_t(f)} \right)^\alpha N_t(f) df \\ &= \left( \frac{K}{N_t} \right)^\alpha \int N_t(f) df \\ &= K^\alpha N_t^{1-\alpha} \end{aligned} \quad (17)$$

Recalling that  $Y_{t+j}(f)$  is given from (9), it follows that

$$Y_t^{sum} = Y_t \int_0^1 \frac{1}{1+\psi} \left( \left[ \frac{P_t(f)}{P_t} \frac{1}{\Lambda_t^p} \right]^{-\frac{\phi_p - (\phi_p - 1)\epsilon_p}{\phi_p - 1}} + \psi \right) df,$$

or equivalently, using (17):

$$Y_t = (p_t^*)^{-1} K^\alpha N_t^{1-\alpha}, \quad (18)$$

where

$$p_t^* \equiv \int_0^1 \frac{\phi_p}{\phi_p - (\phi_p - 1)\epsilon_p} \left( \left[ \frac{P_t(f)}{P_t} \frac{1}{\Lambda_t^p} \right]^{-\frac{\phi_p - (\phi_p - 1)\epsilon_p}{\phi_p - 1}} + \psi \right) df.$$

In a technical appendix, available upon request, we show how to develop a recursive formulation of the sticky price distortion term  $p_t^*$ .

Now, because actual output  $Y_t$  is what is available for private consumption and government spending purposes, it follows that:

$$\underbrace{C_t + G_t}_{\equiv Y_t} \leq (p_t^*)^{-1} \underbrace{K^\alpha N_t^{1-\alpha}}_{\equiv Y_t^{sum}}. \quad (19)$$

The sticky price distortion introduces a wedge between input use and the output available for private and government consumption.<sup>7</sup> Even so, this term vanishes in the log-linearized version of the model.

## 2.2. Parameterization

Our benchmark calibration – essentially adopted from Erceg and Linde (2014) – is fairly standard at a quarterly frequency. We set the discount factor  $\beta = 0.995$ , and the steady state net inflation rate  $\pi = .005$ ; this implies a steady state interest rate of  $i = .01$  (i.e., four percent at an annualized rate). We set the intertemporal substitution elasticity  $\sigma = 1$  (log utility), the capital share parameter  $\alpha = 0.3$ , the Frisch elasticity of labor supply  $\frac{1}{\chi} = 0.4$ , and the steady state value for the consumption taste shock  $\nu = 0.01$ .<sup>8</sup> Three parameters determine the direct sensitivity of prices to marginal costs: the gross markup  $\phi_p$ , the stickiness parameter  $\xi_p$  and the Kimball parameter  $\epsilon_p$ . We have direct evidence on two of these –  $\phi_p$  and  $\xi_p$ . When it comes the latter, a large body of *microeconomic* evidence, see e.g. Klenow and Malin (2010) and Nakamura and Steinsson (2012) and the references therein, suggest that firms change their prices rather frequently, on average

<sup>7</sup> As the economy is assumed to be endowed with a fixed aggregate capital stock  $K$  which does not depreciate, no resources is devoted to investment. An alternative formulation would have embodied a constant capital depreciation rate in which case output would have been used for  $C_t$ ,  $I$  and  $G_t$ .

<sup>8</sup> By setting the steady value of the consumption taste shock to a small value, we ensure that the dynamics for the other shocks are roughly invariant to the presence of  $-C\nu_t$  in the period consumption utility function.

somewhat more often than once a year. Based on this micro evidence, we set  $\xi_p = 0.667$ , implying an average price contract duration of 3 quarters ( $\frac{1}{1-0.667}$ ). For the gross markup, we set  $\phi_p = 1.2$  as a compromise between the low estimate of  $\phi_p$  in Altig et al. (2011) and the higher estimated value by Smets and Wouters (2007). In addition, this is the estimated value by Christiano, Eichenbaum and Evans (2005) and Christiano, Trabandt and Walentin (2010), and used by e.g. Levin, Lopez-Salido and Yun (2007) and Eggertsson and Singh (2016). To pin down  $\epsilon_p$ , our starting point is the loglinearized New Keynesian Phillips Curve

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_{mc} \widehat{mc}_t, \quad (20)$$

which obtains in our model where  $\widehat{mc}_t$  denotes marginal cost as log-deviation from its steady state value. The parameter  $\kappa_{mc}$ , i.e. the slope of the Phillips curve, is given by

$$\kappa_{mc} \equiv \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} \frac{1}{1+(\phi_p-1)\epsilon_p}. \quad (21)$$

The *macroeconomic* evidence suggest that the sensitivity of aggregate inflation to variations in marginal cost is very low, see e.g. Altig et al. (2011). To capture this, we adopt a value for  $\epsilon_p$  so that the slope of the Phillips curve ( $\kappa_{mc}$ ) – given our adopted values for  $\beta$ ,  $\xi_p$  and  $\phi_p$  – equals 0.012.<sup>9</sup> This calibration allows us to match both the micro- and macroevidence on price setting behavior and is aimed at capturing the resilience of core inflation, and measures of expected inflation, during the recent global recession.

We assume a government debt to annualized output ratio of 0.6 (consistent with U.S. pre-crisis federal debt level). We set government consumption as a share of output  $g_y = 0.2$ . Further, we set net lump-sum taxes  $\tau = 0$  in steady state. The above assumptions imply a steady state labor income tax  $\tau_N = 0.33$ . The parameter  $\varphi$  in the tax rule (14) is set equal to 0.01, which implies that the contribution of lump-sum taxes to the response of government debt is negligible in the first couple of years following a shock (so that almost all variation in tax revenues reflect fluctuations in labor tax revenues). For monetary policy, we use the standard Taylor (1993) rule parameters  $\gamma_\pi = 1.5$  and  $\gamma_x = .125$ .

In order to facilitate comparison between the nonlinear and linear model, we specify processes for the exogenous shocks such that there is no loss in precision due to an approximation. In particular, the preference, discount and government spending shocks are assumed to follow AR(1)

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<sup>9</sup> The median estimates of the Phillips Curve slope in recent empirical studies by e.g. Adolfson et al (2005), Altig et al. (2011), Galí and Gertler (1999), Galí, Gertler and López-Salido (2001), Lindé (2005), and Smets and Wouters (2003, 2007) are in the range of 0.009 – .014.



processes:

$$\begin{aligned}
g_{y,t} - g_y &= \rho_g (g_{y,t-1} - g_y) + \varepsilon_{g,t}, \\
\nu_t - \nu &= \rho_\nu (\nu_{t-1} - \nu) + \sigma_{\nu,t}, \\
\delta_t - \delta &= \rho_\delta (\delta_{t-1} - \delta) + \sigma_\delta \varepsilon_{\delta,t},
\end{aligned} \tag{22}$$

where  $\delta = 1$ . Our baseline parameterization of these processes adopts a persistence coefficient of 0.95, so that  $\rho_\nu = \rho_g = \rho_\delta = 0.95$  in (22). But following some prominent papers in the literature on fiscal multipliers, we also investigate the sensitivity of our results when the processes are assumed to be moving average (MA) processes. Those results are reported in Appendix A.

### 2.3. Solving the Model

We compute the linearized and nonlinear solutions using the Fair and Taylor (1983) method. This method imposes certainty equivalence on the nonlinear model, just as the linearized solution does by definition. In other words, the Fair and Taylor solution algorithm traces out the implications of not linearizing the equilibrium equations for the resulting multiplier without shock uncertainty. An alternative approach would have been to compute solutions where uncertainty about future shock realizations matters for the dynamics of the economy following for instance Adam and Billi (2006, 2007) within a linearized framework and Fernández-Villaverde et al. (2015) and Gust, Herbst, López-Salido and Smith (2016) within a nonlinear framework. These authors have shown that allowing for future shock uncertainty can potentially have important implications for equilibrium dynamics, especially when inflation expectations are less well anchored because the conduct of monetary policy is far from non-optimal and prices are quick to adjust. We nevertheless confine ourself to study perfect foresight simulations for the following three reasons. First, because much of the existing literature have in fact used a perfect foresight approach, retaining this approach allows us to parse out the effects of going from a linearized to a nonlinear framework. Second, the high degree of real rigidities we introduce in order to fit the micro- and macroeconomic evidence implies that expected inflation adjusts slowly, which in turn means that the impact of future shock uncertainty is modest. Even so, allowing for shock uncertainty only strengthens our finding of a pronounced difference between the linearized and nonlinear solutions in a long-lived trap as the responses in the linearized equilibrium is amplified relatively more than the nonlinear solution by future shock uncertainty.<sup>10</sup> Third, the perfect foresight assumption allows us to readily study the

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<sup>10</sup> We have verified this by comparing the decision rules under perfect foresight with the decision rules obtained when allowing for shock uncertainty (calibrating the variance of the shocks in the model so that the probability is

robustness in a larger scale model with many state variables. So far, the solution algorithms used to solve models with shock uncertainty have typically not been applied to models with more than 4-5 state variables.<sup>11</sup>

To solve the model, we feed the relevant equations in the nonlinear and log-linearized versions of the model to Dynare. Dynare is a pre-processor and a collection of MATLAB routines which can solve nonlinear models with forward looking variables, and the details about the implementation of the algorithm used can be found in Juillard (1996). We use the perfect foresight simulation algorithm implemented in Dynare using the ‘simul’ command.<sup>12</sup> The algorithm can easily handle the ZLB constraint: one just writes the Taylor rule including the max operator in the model equations, and the solution algorithm reliably calculates the model solution in fractions of a second. Thus, apart from gaining intuition about the mechanisms embedded into the models, there is no need anymore to linearize models in order to solve and simulate them.

For the linearized model, we used the algorithm outlined in Hebden, Linde and Svensson (2012) to check for uniqueness of the local equilibrium associated with a positive steady state inflation rate. As noted earlier, we rule the well-known problems associated with steady state multiplicity emphasized by Benhabib, Schmitt-Grohe and Uribe (2001) by restricting our attention to the steady state with a positive inflation rate. However, for the nonlinear version of the model we cannot rule out the possibility that there exists other solutions in addition to the one found by Dynare, but note that the work by Christiano, Eichenbaum and Johannsen (2016) suggest that alternative solutions may not be relevant (i.e. not stable under learning).

### 3. Results

In this section, we report the benchmark results. As mentioned earlier, our aim is to compare spending multipliers in linearized and nonlinear versions of the model economy. Specifically, we seek to characterize how the difference between the multiplier in the linear and nonlinear frameworks varies with the expected duration of the liquidity trap. We start out by reporting how we construct the baseline scenarios and then report the marginal fiscal multipliers.

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hitting the ZLB is 0.2 percent each quarter of the stochastic simulations with the model, in line with Post WWII U.S. evidence).

<sup>11</sup> A recent paper by Judd, Maliar and Maliar (2011) provides a promising avenue to compute the stochastic solution of larger scale models efficiently.

<sup>12</sup> The solution algorithm implemented in Dynare’s simul command is the method developed in Fair and Taylor (1983).

### 3.1. Construction of Baseline Scenarios

To construct a baseline where the interest rate is bounded at zero for  $ZLB_{DUR} = 1, 2, 3, \dots, T$  periods, we follow the previous fiscal multiplier literature (e.g. Christiano, Eichenbaum and Rebelo, 2011) and assume that the economy is hit by a large adverse shock that triggers a deep recession and drives interest rates to zero. The longer the expected liquidity trap duration (i.e. the larger value of  $ZLB_{DUR}$ ) we want to consider, the larger the adverse shock has to be. The particular shock we consider is a negative consumption taste shock  $\nu_t$  (see the equations 1 and 22) following Erceg and Linde (2014).<sup>13</sup>

To provide clarity on how we pick the shock sizes, Figure 2 reports the linear and nonlinear solutions for the same negative taste shock (depicted in the bottom right panel). The economy is in the deterministic steady state in period 0, and then the shock hits the economy in period 1. As is evident from Figure 2, the same-sized shock has a rather different impact on the economy depending on whether the model is linearized or solved in its original nonlinear form. For instance, we see from panel 3 that while the nominal interest rate is bounded by zero from periods 1 to 8 in the linearized model, the same-sized consumption demand shock (panel 9) only generates a two quarter trap in the nonlinear model. Hence, we need to subject the nonlinear model to a more negative consumption demand shock – as shown by the red dotted line in panel 9 in Figure 3 – to generate  $ZLB_{DUR} = 8$  for the interest rate (panel 3).<sup>14</sup>

Important insights about the differences between the linearized and nonlinear solutions can be gained from Figures 2 and 3. Starting with Figure 2, we see from the fifth panel that the drop in the potential real rate is about the same in both models. Still, the linearized model generates a much longer liquidity trap because inflation and expected inflation falls much more (panel 2), which in turn causes the actual real interest rate (panel 4) to rise much more initially. The larger initial rise in the actual real interest rate, and thus the gap between the actual and potential real rates, triggers a larger fall in the output gap (panel 1) and consequently real GDP falls more in the linearized model as well (because the impact on potential GDP is about the same, as implied by the similarity of the potential real interest rate response).

Turning to Figure 3, we first note from the third panel that the paths for the policy rate are

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<sup>13</sup> In Appendix A, we present results when the recession is instead assumed to be triggered by the discount factor shock  $\delta_t$  that was used in the seminal papers by Eggertsson and Woodford (2003), and Christiano, Eichenbaum and Rebelo (2011). For the linearized solution, the results are invariant w.r.t. the choice of the baseline shock (see Erceg and Lindé, 2014, for proof). And for the nonlinear solution, Figure A.1 shows that the multiplier schedules are nearly identical.

<sup>14</sup> Figure 3 also depicts a third line (“Nonlinear model with linear NKPC and Res. Con.”), which we will discuss further in Section 3.2.

bounded at zero for 8 quarters and display a very similar path upon exit from the liquidity trap. Moreover, panel 9 shows that it takes a much larger adverse consumption demand shock in the nonlinear model to trigger a liquidity trap of the same expected duration as in the linearized model. This implies that the drop in the potential real rate and real GDP (panels 5 and 7) is much more severe in the nonlinear model. Even so, and perhaps most important, we see that inflation – panel 2 – falls substantially less in the nonlinear model. This suggests that the difference between the linearized and nonlinear solutions to a large extent is driven by the pricing block of the model.

### 3.2. Marginal Fiscal Multipliers

As previously noted, we are seeking to compare fiscal multipliers in liquidity traps of same expected duration in the linearized and nonlinear frameworks. Accordingly, we allow for differently sized shocks in the linearized and nonlinear models so that each model variant generates a liquidity trap with the same expected duration  $ZLB_{DUR} = 1, 2, 3, \dots, T$ . Let  $\left\{B_t^{linear} \left(\sigma_{\nu,i}^{linear}\right)\right\}_{t=1}^T$  and  $\left\{B_t^{nonlin} \left(\sigma_{\nu,i}^{nonlin}\right)\right\}_{t=1}^T$  denote vectors with simulated variables in the linear and nonlinear models in periods  $t = 1, 2, \dots, T$ , respectively. This notation reflects that the innovations,  $\sigma_\nu$ , to the consumption demand shock  $\nu_t$ , in eq. (22) are set so that

$$\sigma_{\nu,i}^{linear} \Rightarrow ZLB_{DUR} = i,$$

and

$$\sigma_{\nu,i}^{nonlin} \Rightarrow ZLB_{DUR} = i,$$

where we consider  $i = 1, 2, \dots, T$ . In the specific case of  $i = 8$ , panel 9 in Figure 3 shows that  $\sigma_{linear,8}^\nu = -.18$  and  $\sigma_{nonlin,8}^\nu = -.42$ .

To these different baseline paths, we add the fiscal response in the first period the ZLB binds, which happens in the same period as the adverse shock hits ( $t = 1$ ). By letting  $\left\{S_t^{linear} \left(\sigma_{\nu,i}^{linear}, \sigma_G\right)\right\}_{t=1}^T$  and  $\left\{S_t^{nonlin} \left(\sigma_{\nu,i}^{nonlin}, \sigma_G\right)\right\}_{t=1}^T$  denote vectors with simulated variables in the linear and nonlinear solutions when both the negative baseline shock  $\sigma_\nu$  and the positive government spending shock  $\sigma_G$  hits the economy, we can compute the partial impact of the fiscal spending shock as

$$I_t^j(ZLB_{DUR}) = S_t^j \left(\sigma_{\nu,i}^j, \sigma_G\right) - B_t^j \left(\sigma_{\nu,i}^j\right)$$

for  $j = \{linear, nonlin\}$  and where we write  $I_t^j(ZLB_{DUR})$  to highlight its dependence on the liquidity trap duration. Notice that the fiscal spending shock is the same for all  $i$  and is scaled so

that  $ZLB_{DUR}$  remains the same as under the baseline shock only. By setting the fiscal impulse so that the liquidity trap duration remains unaffected, we retrieve “marginal” spending multipliers in the sense that they show the impact of a “tiny” change in the fiscal instrument.<sup>15</sup>

In Figure 4 we report the results of our exercise. The upper panels report results for the benchmark calibration with the Kimball aggregator. The lower panels report results under the Dixit-Stiglitz aggregator, in which case  $\epsilon_p = 0$ . This parametrization implies a substantially higher slope of the linearized Phillips curve (see eq. 21) and thus a much stronger sensitivity of expected inflation to current and expected future marginal costs (and output gaps). We will first discuss the results under the Kimball parameterization, and then turn to the Dixit-Stiglitz results.

The left panels report the output-spending multiplier on impact, i.e. simply

$$m_i = \frac{\Delta Y_{t,i}}{\Delta G_{t,i}} = \frac{\Delta Y_{t,i}/Y}{\Delta g_{y,t}}$$

where the  $\Delta$ -operator represents the difference between the scenario with the spending change and the baseline without the spending change and  $Y$  denotes the steady state level of output. We compute  $m_i$  for  $ZLB_{DUR} = 1, \dots, 12$ , but also include results for the case when the economy is at the steady state, so that  $ZLB_{DUR} = 0$ .

As the linear approximation is more accurate the closer the economy is to the steady state, it is not surprising that the difference between the “linear” and “nonlinear” multiplier increases with the duration of the liquidity trap. In a three-year liquidity trap, the multiplier in the nonlinear solution is about twice as high (0.65) compared to normal times (when it is about 0.30), whereas it is about 7 times higher (2.1) in the linearized solution. So for a three year liquidity trap, the multiplier in the linearized solution is over three times (2.1/0.65) as large as in the nonlinear solution. For shorter-lived liquidity traps, the differences are notably more modest, and in the special case when the economy is in the steady state ( $ZLB_{DUR} = 0$  in the figure) we note that the multipliers are identical in both economies. The difference in government debt (as share of actual annualized GDP) response after 1 year, shown in the upper right panel, largely follows the pattern for  $m_i$  and increases with  $ZLB_{DUR}$ .<sup>16</sup>

The substantial differences in the output and debt responses between the linearized and nonlinear solutions begs the question of which factors account for them. The middle upper panel, which

<sup>15</sup> Had we considered a larger fiscal intervention that altered the duration of the liquidity trap, there would have been an important distinction between the *average* (i.e. the total response) and *marginal* (i.e. the impact of a small change in  $g_t$  which leaves  $ZLB_{DUR}$  unchanged) multiplier as discussed in further detail in Erceg and Linde (2014).

<sup>16</sup> For ease of interpretability, we have normalized the response of debt and inflation so that they correspond to a initial change in government spending (as share of steady state output) by one percent.

shows the response of the one-period ahead expected annualized inflation rate (i.e.,  $4E_t\pi_{t+1}$ ), sheds some light on this. As can be seen from the panel, expected inflation responds much more in a long-lived trap in the linearized model than in the nonlinear model. The sharp increase in expected inflation triggers a larger reduction in the actual real rate relative to the potential real rate (not shown) in the linearized model, and thereby induces a more favorable response of private consumption which helps to boost output relative to the nonlinear model.

Turning to the Dixit-Stiglitz case shown in the lower panels, we see that the differences between the linear and nonlinear solutions are even more pronounced in this case, with the multiplier in an 8-quarter trap being over 100 ( $27.5/0.25$ ) times larger than in normal times in the linearized solution, but only roughly 10 times higher in the nonlinear solution.<sup>17</sup> The larger discrepancy in the Dixit-Stiglitz case is to a large extent driven by the fact that we are in effect allowing a substantially higher slope (i.e.  $\kappa_{mc}$ ) of the New Keynesian Phillips curve in eq. (20). Taken together, the results in Figure 4 suggest that the findings of the papers in the previous literature which relied on linearized models were more distorted to the extent that they relied on a calibration with a higher slope of the Phillips curve and thus a larger sensitivity of expected inflation.

The key question is then why expected inflation responds so much more in the linearized economy, and particularly so in the Dixit-Stiglitz case? To shed light on this, we simulate two additional variants of the nonlinear model. In the first, we linearize the pricing equations of the model, e.g. replace all pricing equations in the nonlinear model with the standard linearized Phillips curve. In the second, we linearize all the pricing equations and remove the price distortion term from the aggregate resource constraint (19). Following the approach with the linear and fully nonlinear models, we construct baseline scenarios for the two additional variants of the model as described in Section 3.1 for  $ZLB_{DUR} = 1, \dots, 12$ . The blue dash-dotted line in Figure 3 depicts the eight quarter liquidity trap baseline in the variant with linearized pricing equations and resource constraint (second additional variant described above). Clearly, the simulated paths of the variables in this variant of the model are very similar to those in the fully linearized solution. Therefore, given that the consumption demand  $\nu_t$  generating the baseline and the added government spending shock both work through the demand side of the economy, is it not surprising that the results in Figure 5 for this model (blue dashed-dotted line, referred to as “Linearized Resource Constraint and NKPC”) are very similar to those obtained with the linearized model, both under Kimball and Dixit-Stiglitz aggregators. Hence, we can draw the conclusion that it is the linearization of the resource constraint

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<sup>17</sup> We only show results up to 8 quarters with the Dixit-Stiglitz aggregator to be able to show the differences more clearly in the graph.

and the Phillips curve (20), and not the aggregate demand part of the model, which account for the bulk of the differences between in the linear and nonlinear models under Kimball in a long-lived liquidity trap. In fact, as shown by the green dash-dotted line in the upper panels of Figure 5, it is almost sufficient to just linearize the NKPC to account for most of the discrepancy between the linearized and nonlinear solution under the Kimball aggregator. Together, these findings show that the lower inflation response in Figure 4 in the nonlinear solution relative to the linear solution is driven by an effectively lower response to marginal costs when the price dispersion term is elevated. The price dispersion term per se does not matter much, but the fact that the price dispersion is elevated following an adverse shock implies that many firms perceive that their demand elasticity is high and are therefore reluctant to change prices much at all in response to changes in marginal costs (in terms of Figure 1, they are located in the upper left quadrant).

The lower panels in Figure 5, however, show that log-linearization of the New Keynesian Phillips curve only is not sufficient to explain the large discrepancies between the linear and nonlinear solutions under the Dixit-Stiglitz aggregator. In this case, accounting for the price distortion in the aggregate resource constraint is necessary, i.e. log-linearizing the resource constraint so that movements in the price distortion term  $p_t^*$  in eq. (18) becomes irrelevant for equilibrium dynamics. The reason for this difference between the Kimball and the Dixit-Stiglitz aggregators is that the price distortion variable moves much more for the latter specification as re-optimizing firms will adjust their prices much more under Dixit-Stiglitz compared to Kimball for given  $\xi_p$  (recalling the insights from Figure 1). So in a Dixit-Stiglitz world where firms adjust prices a lot when they re-optimize, the bulk of the difference between the linearized and nonlinear solutions is driven by movements in the price distortion, whereas in the Kimball world where firms adjust prices by less, the bulk of the differences is driven by the pricing equations directly.

Boneva, Braun and Waki (2016) argue that the key is to account for the price distortion, and our results suggest their claim is valid given that they are considering a model framework in line with the Dixit-Stiglitz aggregator. In terms of multiplier sizes, it is important to note that we report lower multipliers than Boneva, Braun and Waki in Figure 4 (for comparable degree of price adjustment) because our spending process is assumed to be a fairly persistent AR(1) process. If we assume that spending follows a uniform MA process and is only increased as long as policy rates are constrained by the ZLB, we obtain a marginal multiplier of unity in both the linear and nonlinear solutions already in a one-quarter liquidity trap (as we should, see Woodford, 2011, for

proof).<sup>18</sup> Even so, there is an important difference between the linearized and nonlinear solution for longer-lived liquidity traps, where multiplier rises to 5 in a three-year trap in the former case but only to 1.03 in the nonlinear solution. This is in line with Boneva, Braun and Waki, who reports a maximum multiplier of 1.05 in their model.

## 4. Robustness analysis

In this section, we examine the robustness of the results w.r.t. the aggregator specification (Kimball vs. Dixit-Stiglitz) as well as to the indexation schedule for firms which are not able to re-optimize their prices.

### 4.1. Dixit-Stiglitz vs. Kimball

To further tease out the difference between the Kimball vs. Dixit-Stiglitz aggregator, Panel A in Figure 6 compares outcomes when the sticky price parameter  $\xi_p$  is adjusted in the Dixit-Stiglitz version so that the slope of the linearized Phillips curve (20) is the same as in our benchmark Kimball calibration. Both the Kimball and Dixit-Stiglitz versions hence now feature a linearized Phillips curve with an identical slope coefficient ( $\kappa_{mc} = 0.012$ , see 21), but the Dixit-Stiglitz version of the model achieves this with a substantially higher value of  $\xi_p$  (0.90). However, since only the value of  $\kappa_{mc}$  matters in the linearized solution, the multiplier schedules are invariant w.r.t. the mix of  $\xi_p$  and  $\epsilon_p$  that achieves a given  $\kappa_{mc}$  in the linearized models. Consequently, the linearized solution for the Dixit-Stiglitz aggregator is thus identical to the Kimball solution depicted by the solid black line in the upper panel in Figure 4.

Even so, the nonlinear solutions shown in Panel A in Figure 6 differ. In particular, we see that

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<sup>18</sup> These results are discussed and presented in Appendix A, see Figure A.2.



the Dixit-Stiglitz aggregator implies that expected inflation and output multiplier responds more when the duration of the liquidity trap increases. Thus, when the Kimball parameter  $\epsilon_p$  is reduced, the more will expected inflation and output multiplier respond when  $ZLB_{DUR}$  increases; conversely, increasing  $\epsilon_p$  and lowering  $\xi_p$  flattens the output multiplier schedule even more. The explanation behind this finding is that a higher value of  $\epsilon_p$  induces the elasticity of demand to vary more with the relative price differential among the intermediate good firms as shown in Figure 1, and this price differential increases when the economy is far from the steady state. Thus, intermediate firms which only infrequently are able to re-optimize their price will optimally choose to respond less to a given fiscal impetus far from the steady state when price differentials are larger as they perceive that they may have a much larger impact on their demand for a given change in their relative price. As a result, aggregate current and expected inflation are less affected far from the steady state in the Kimball case relative to the Dixit-Stiglitz case for which the elasticity of demand is independent of relative price differentials. This demonstrates that the modeling of price frictions matters importantly within a nonlinear framework, especially so when nominal wages are flexible.

#### 4.2. Indexation of non-optimizing firms

So far, we have followed the convention in the literature and assumed that non-optimizing firms index their prices w.r.t. the steady state inflation rate, see eq. (12). This is a convenient benchmark modelling assumption as it simplifies the analysis by removing steady state price distortions. However, this assumption has been criticized for being inconsistent with the microevidence on price setting. According to micro evidence on price setting, many firms' prices remain unchanged for several subsequent quarters, whereas they always change under our benchmark indexation scheme. Thus – there is an important issue to what extent this matters for aggregate dynamics, especially in the nonlinear solution. To examine this, we re-specify the model assuming no indexation among the non-optimizing firms following e.g. Ascari and Ropele (2007), i.e.

$$\tilde{P}_t = P_{t-1}. \tag{23}$$

In Panel B in Figure 6, we report the results when comparing the nonlinear baseline model (black solid line, which features indexation) with the nonlinear variant without indexation for the non-optimizing firms (red dotted line). From the panels, we see that abandoning the conventional assumption of full indexation results in a somewhat steeper multiplier schedule, mostly explained by the somewhat higher sensitivity of expected inflation in the “no-indexation” model. As the

marginal multipliers in the linearized solution is roughly unchanged (verified, but not shown in the figure), the message in Panel B is that the results by and large hold up well for this perturbation of the model as well.

## **5. Conclusions**

We have calculated the magnitude of the fiscal spending multiplier in linearized and nonlinear solutions of a New Keynesian model at the zero lower bound. Importantly, we use a model that is amended with real rigidities to simultaneously account for the macroeconomic evidence of a low Phillips curve slope and the microeconomic evidence of frequent price re-optimization. We have shown that the nonlinear solution is associated with a much smaller multiplier than the linearized solution in long-lived liquidity traps, and pinned down the key features in the model which account for the difference. Our results caution against the common practice of using linearized models to calculate fiscal multipliers in long-lived liquidity traps.

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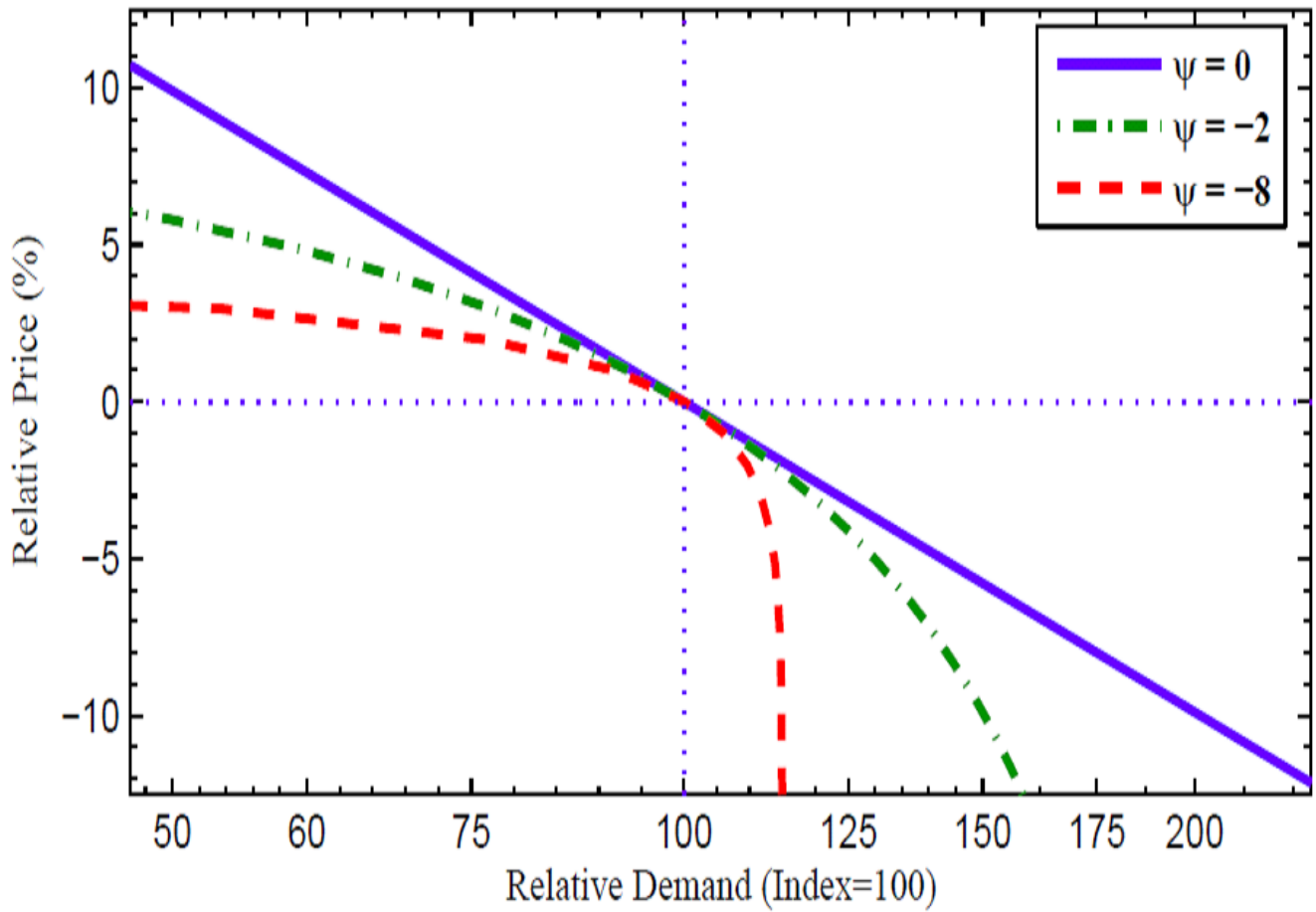
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Figure 1: Quasi-Kinked Demand



Notes: This figure is taken from Levin, Lopez-Salido and Yun, 2007, "Strategic Complementarities and Optimal Monetary Policy", CEPR Discussion Paper No. 6423

Figure 2: Baselines in Linear and Nonlinear Models for an Equally-Sized Consumption Demand Shock

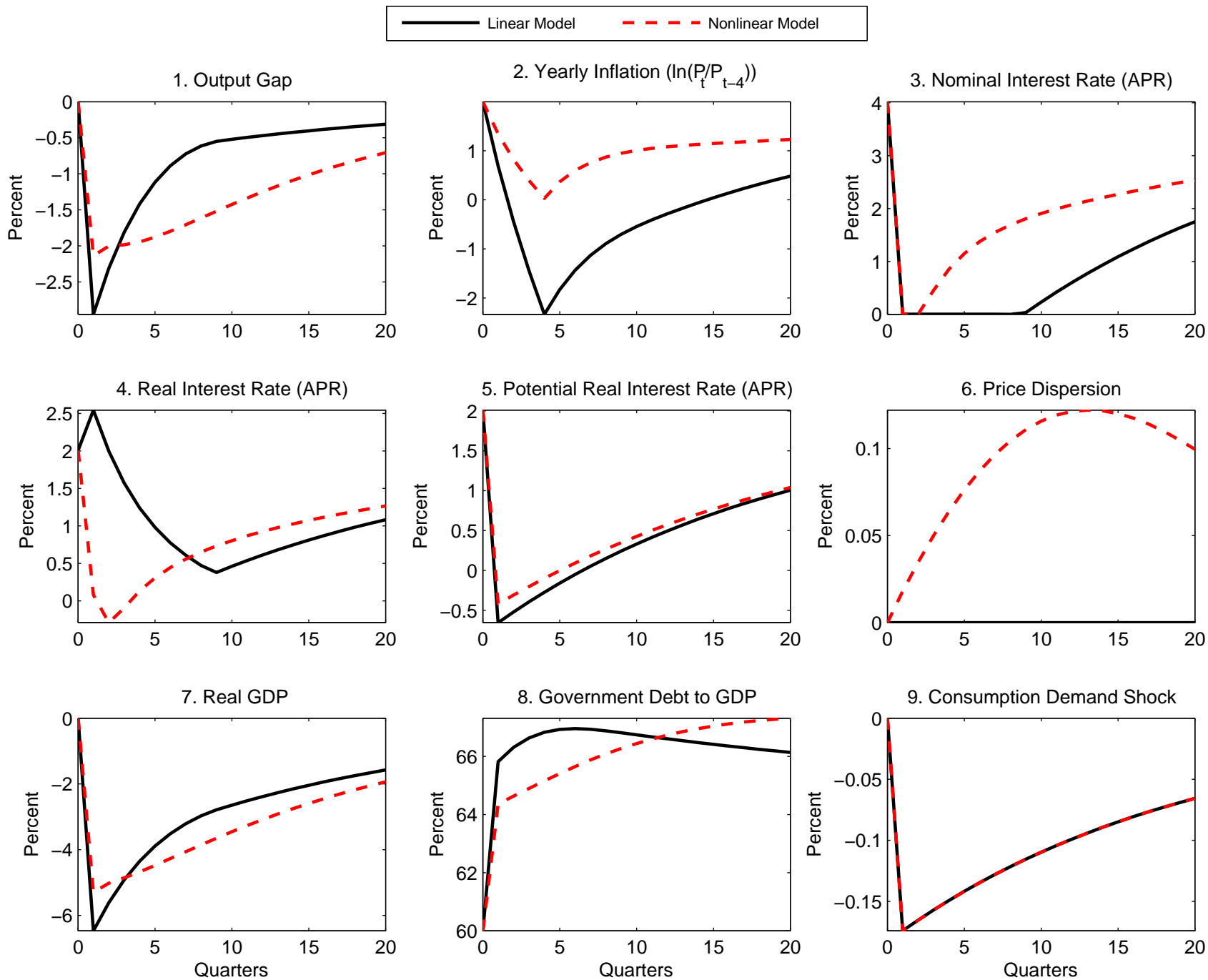




Figure 3: Baselines for 8-Quarter Liquidity Trap

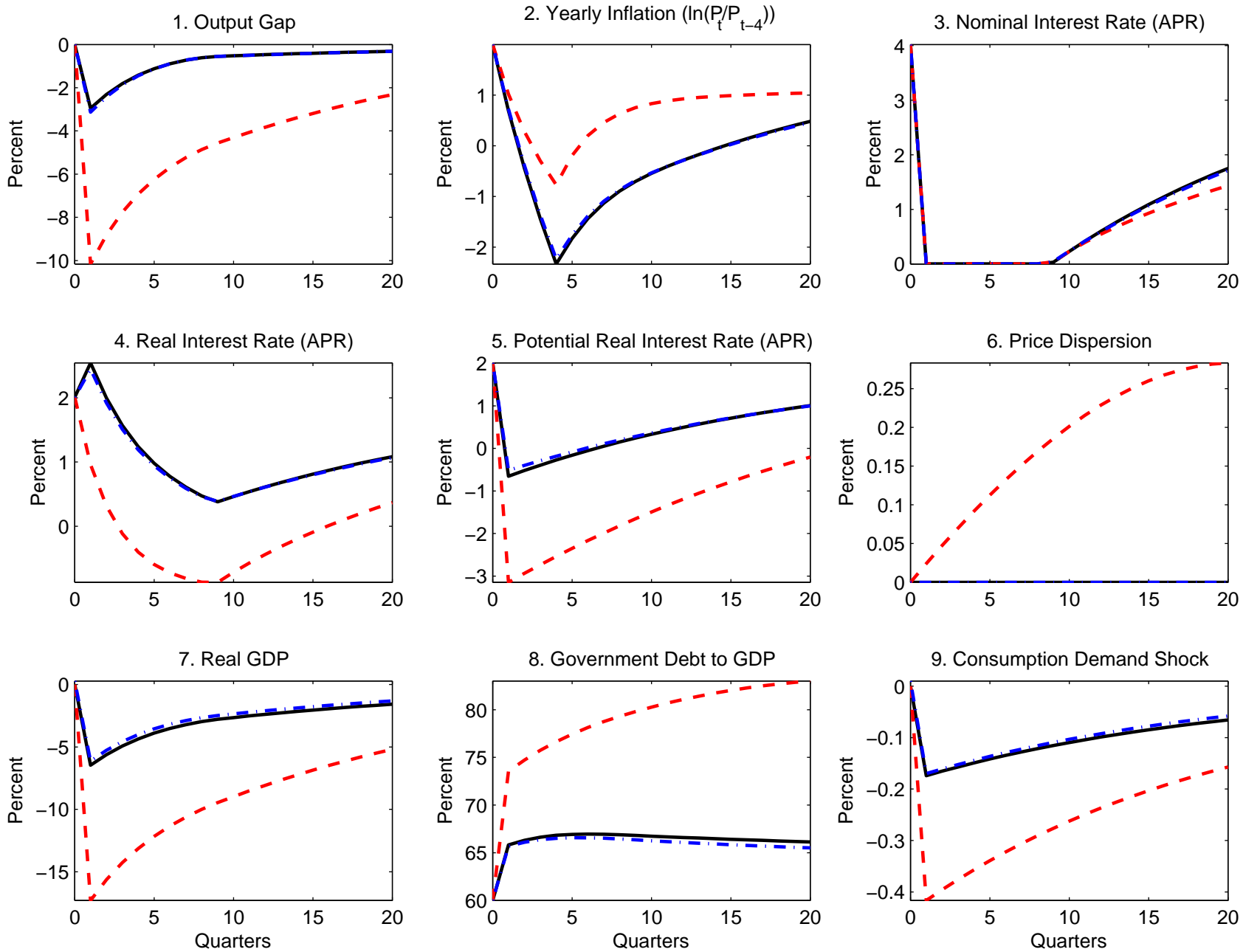


Figure 4: Marginal Multipliers

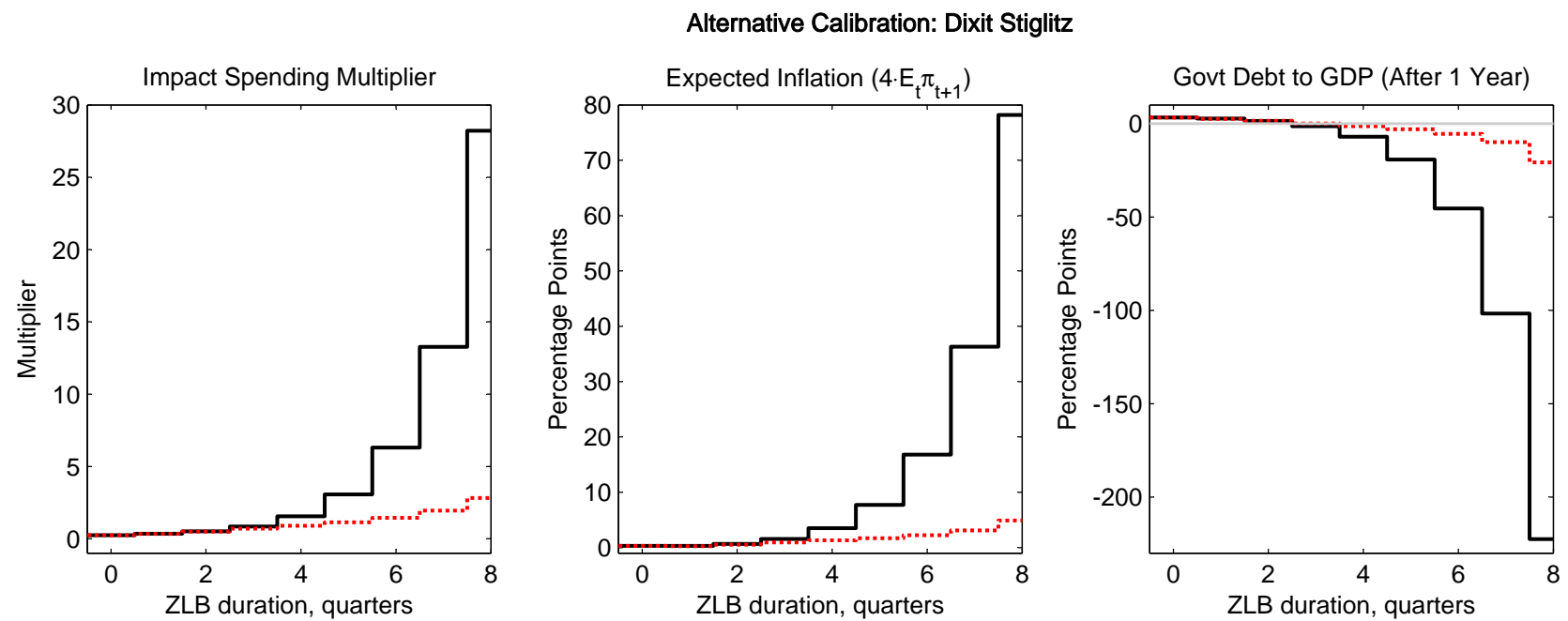
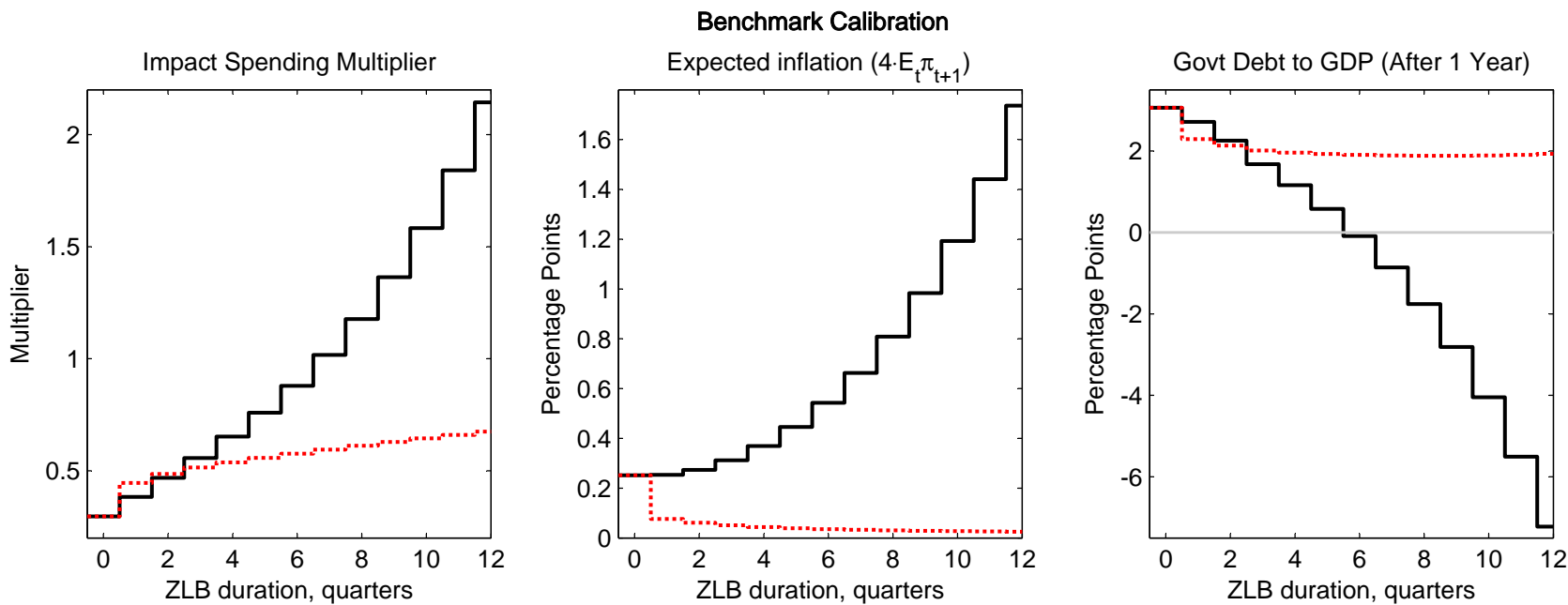


Figure 5: Marginal Multipliers

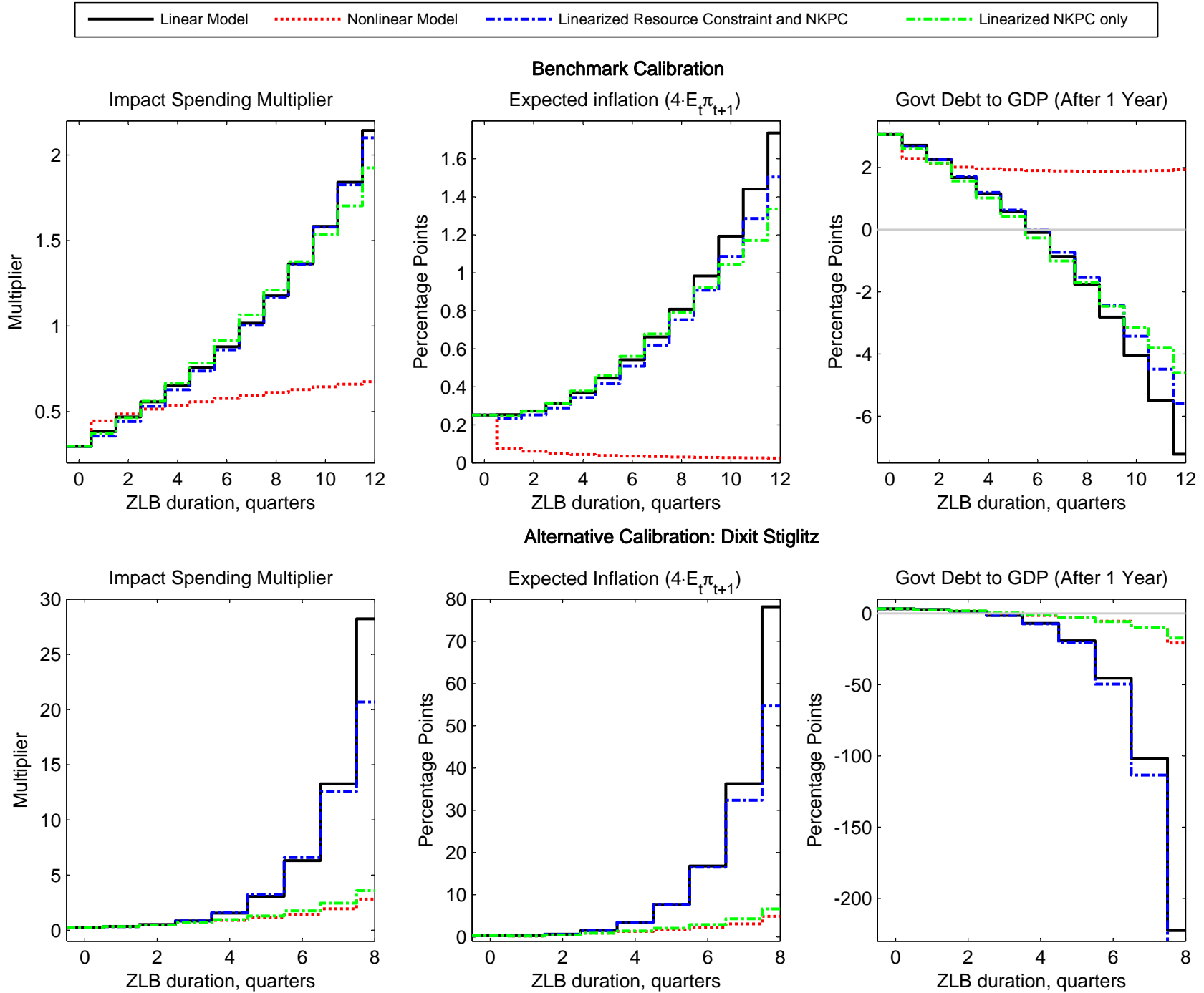
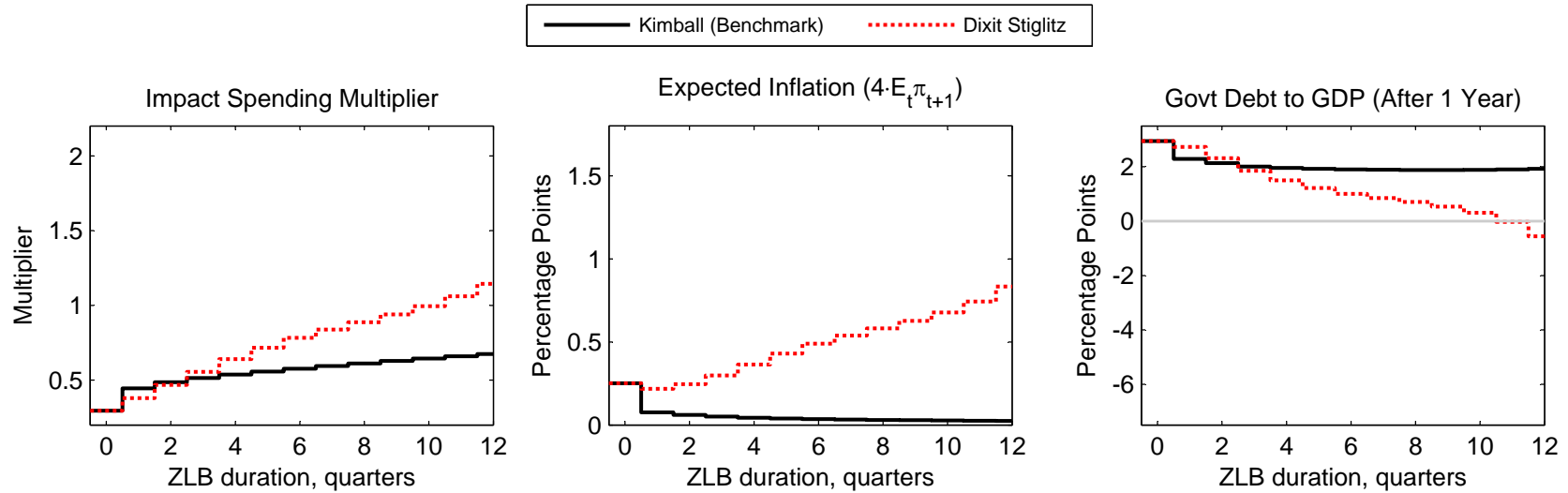
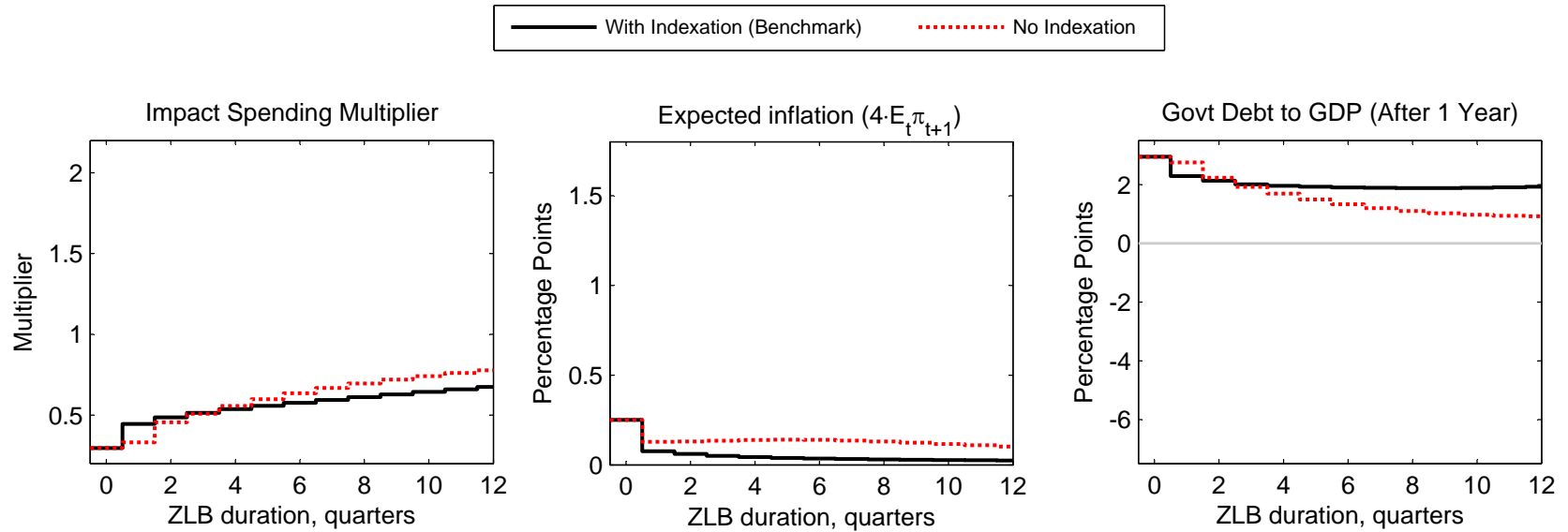


Figure 6: Marginal Impact of Changes in Spending: Sensitivity Analysis in Nonlinear Model

Panel A: Kimball ( $\xi_p=0.667; \varepsilon_p>0$ ) vs. Dixit-Stiglitz ( $\xi_p=0.9; \varepsilon_p=0$ )



Panel B: Impact of Indexation Assumption for Non-Optimizing Firms



## Appendix A. Additional Results

In this appendix, we state the log-linearized equations of the model and present some additional results.

### A.1. The Log-linearized Stylized Model

As shown in the technical appendix (available upon request from the authors), the equations of the log-linearized model can be written as follows:

$$x_t = x_{t+1|t} - \hat{\sigma}(i_t - \pi_{t+1|t} - r_t^{pot}), \quad (\text{A.1})$$

$$\pi_t = \beta\pi_{t+1|t} + \kappa_p x_t, \quad (\text{A.2})$$

$$y_t^{pot} = \frac{1}{\phi_{mc}\hat{\sigma}}[g_{y,t} + (1 - g_y)\nu\nu_t], \quad (\text{A.3})$$

$$r_t^{pot} = -\delta_t + \frac{1}{\hat{\sigma}} \left(1 - \frac{1}{\phi_{mc}\hat{\sigma}}\right) [(g_{y,t} - g_{y,t+1|t}) + (1 - g_y)\nu(\nu_t - \nu_{t+1|t})], \quad (\text{A.4})$$

$$b_{G,t} = (1 + r)b_{G,t-1} + b_G(i_{t-1} - \pi_t) + g_{y,t} - \tau_N s_N(y_t + \phi_{mc}x_t) - \tau_t, \quad (\text{A.5})$$

$$y_t = x_t + y_t^{pot} \quad (\text{A.6})$$

where  $\hat{\sigma}$ ,  $\kappa_p$ ,  $\phi_{mc}$  and  $s_N$  are composite parameters defined as:

$$\hat{\sigma} = \sigma(1 - g_y)(1 - \nu_c), \quad (\text{A.7})$$

$$\kappa_p = \frac{(1 - \xi_p)(1 - \beta\xi_p)}{\xi_p} \frac{1}{1 + (\phi_p - 1)\epsilon_p} \phi_{mc}, \quad (\text{A.8})$$

$$\phi_{mc} = \frac{\chi}{1 - \alpha} + \frac{1}{\hat{\sigma}} + \frac{\alpha}{1 - \alpha}, \quad (\text{A.9})$$

$$s_N = \frac{1 - \alpha}{\phi_p}. \quad (\text{A.10})$$

In slight abuse of previous notation, all variables above are measured as percent or percentage point deviations from their steady state level.<sup>A.1</sup>

<sup>A.1</sup> We use the notation  $y_{t+j|t}$  to denote the conditional expectation of a variable  $y$  at period  $t + j$  based on information available at  $t$ , i.e.,  $y_{t+j|t} = E_t y_{t+j}$ . The superscript ‘pot’ denotes the level of a variable that would prevail under completely flexible prices, e.g.,  $y_t^{pot}$  is potential output.

Equation (A.1) expresses the “New Keynesian” IS curve in terms of the output and real interest rate gaps. Thus, the output gap  $x_t$  depends inversely on the deviation of the real interest rate ( $i_t - \pi_{t+1|t}$ ) from its potential rate  $r_t^{pot}$ , as well as on the expected output gap in the following period. The parameter  $\hat{\sigma}$  determines the sensitivity of the output gap to the real interest rate; as indicated by (A.7), it depends on the household’s intertemporal elasticity of substitution in consumption  $\sigma$ , the steady state government spending share of output  $g_y$ , and a (small) adjustment factor  $\nu_c$  which scales the consumption taste shock  $\nu_t$ . The price-setting equation (A.2) specifies current inflation  $\pi_t$  to depend on expected inflation and the output gap, where the sensitivity to the latter is determined by the composite parameter  $\kappa_p$ . Given the Calvo-Yun contract structure, equation (A.8) implies that  $\kappa_p$  varies directly with the sensitivity of marginal cost to the output gap  $\phi_{mc}$ , and inversely with the mean contract duration ( $\frac{1}{1-\xi_p}$ ). The marginal cost sensitivity equals the sum of the absolute value of the slopes of the labor supply and labor demand schedules that would prevail under flexible prices: accordingly, as seen in (A.9),  $\phi_{mc}$  varies inversely with the Frisch elasticity of labor supply  $\frac{1}{\chi}$ , the interest-sensitivity of aggregate demand  $\hat{\sigma}$ , and the labor share in production  $(1 - \alpha)$ . The equations (A.3) and (A.4) determinate potential output and the potential (or natural) real rate. The evolution of government debt is determined by equation (A.5), and depends on variations in the service cost of debt, government spending as well as labor income and lump-sum tax revenues. Equation (A.6) is a simple definitional equation for actual output  $y_t$  (in logs). Finally, the policy rate  $i_t$  follows a Taylor rule subject to the zero lower bound (equation 16 in the main text) and the exogenous shocks follows the processes in eqs. (22).

## A.2. Sensitivity Analysis

Figure A.1 report results when the discount factor shock  $\delta_t$  (defined as the expected change in  $\varsigma_t$  shock in eq. 1, see eq. 4) is driving the baseline in Figure 3. For ease of comparison, the benchmark results with the consumption taste shock  $\nu_t$  driving the baseline scenarios are also included. The upper panels in the figure confirm the results in by Erceg and Lindé (2014) by showing that the fiscal spending multiplier is independent of the shock driving the baseline when the model is linearized as long as the different baseline shocks generate an equally long-lived ZLB episode. So our choice to work with the consumption demand shock in  $\nu_{t+j}$  instead of the conventional discount factor shock in  $\varsigma_t$  in (1) to generate the baseline path underlying Figures 4 to 6 has no consequence for our results with the linearized model. However, the results for the non-linear variant may differ. However, the lower panels in Figure A.1 show that the results are very similar even in the nonlinear

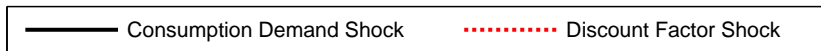
solution, so our choice of baseline appears immaterial.

Another aspect we want to understand is how our results differ from Boneva, Braun, and Waki (2016) due to our AR(1) assumption for government spending instead of the MA-process they work with. Figure A.2 assess this issue by comparing the results under our benchmark AR(1) process with persistence .95 for government spending against the MA process for which  $G_t$  is increased in an uniform fashion as long as the policy rate is bounded at zero for  $ZLB_{DUR} = 1, 2, 3, \dots, T$  and set at its steady state value otherwise. Apart from the fact that our solution procedure does not account for future shock uncertainty, this way of modeling government spending is identical to Boneva et al. who in turns follow Eggertsson (2010).

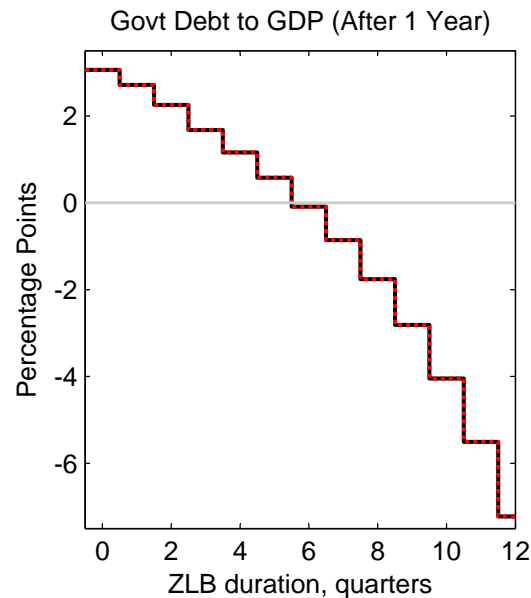
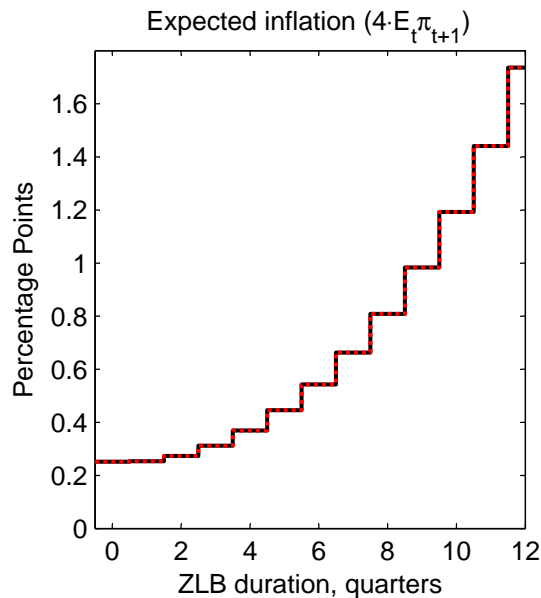
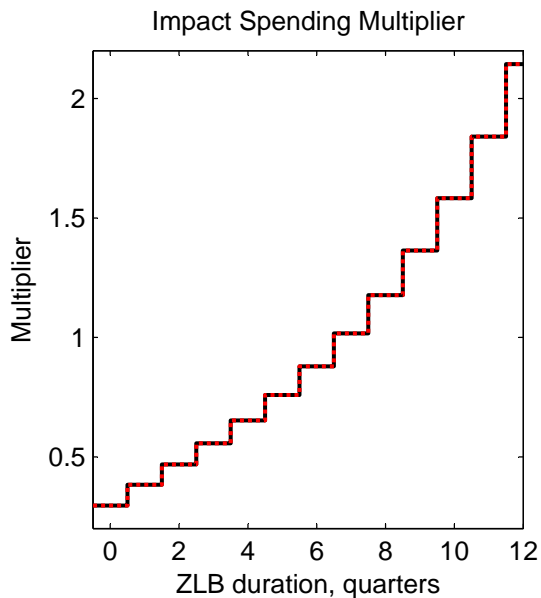
As can be seen from the upper panels of Figure A.2, the MA-process increases the marginal spending multiplier substantially relative to the AR(1) process for the ZLB durations we consider. This happens as increases in government spending has very benign effects on the potential real interest rate when the duration of the spending hike equals the expected duration of the liquidity trap (see e.g. Erceg and Lindé, 2014). For a one quarter liquidity trap the multiplier equals unity, as shown analytically by Woodford (2011). Our fairly persistent AR(1) process tends to dampen the multiplier schedule as a relatively large fraction of the spending comes on line when the ZLB is no longer binding. This feature explains why the AR(1) multiplier is substantially lower in a short lived liquidity trap. However, the AR(1) process is also associated with a substantially lower multiplier even in a fairly long-lived trap compared to the MA process because its has less benign effects on the potential real rate.

All this is well-known from the literature on linearized models. However, the results for the non-linear model, shown in the lower panels, are less explored. We have already discussed the AR(1) case at length in the text. What we see is that the results for the MA process are quite different for longer ZLB durations, because the MA schedule for the nonlinear model stays essentially flat at unity, in line with the findings of Braun et al. (2013); for a 12-quarter trap the multiplier only increases to 1.03 from a multiplier of unity in a one-period liquidity trap. This is sharp contrast to the multiplier schedule for the linearized model where the multiplier is as high as 5 in a liquidity trap lasting 3 years. All told, the results in this appendix shows that our benchmark results holds up well for an MA-process for government spending. If anything, an MA process magnifies the difference between the linearized and nonlinear solutions. Moreover, the results for the linear and nonlinear models in Figure A.2 are in line with the results in the existing literature.

Figure A.1: Marginal Multipliers: Sensitivity With Respect to Baseline Shock



**Linearized Model**



**Nonlinear Model**

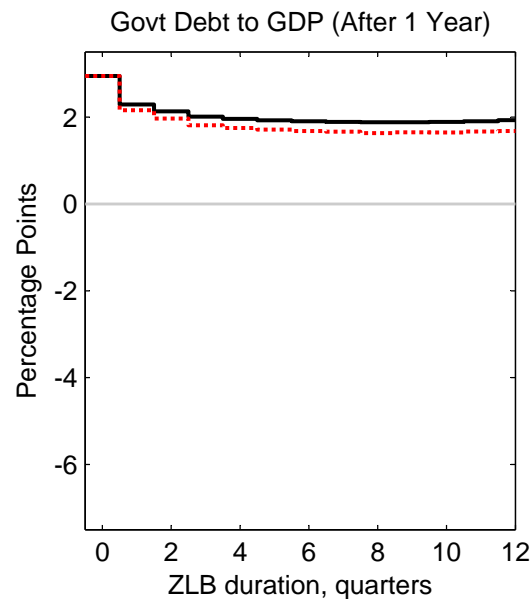
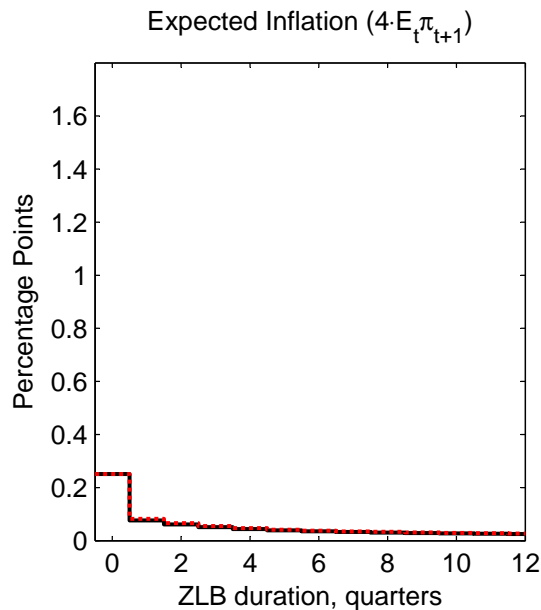
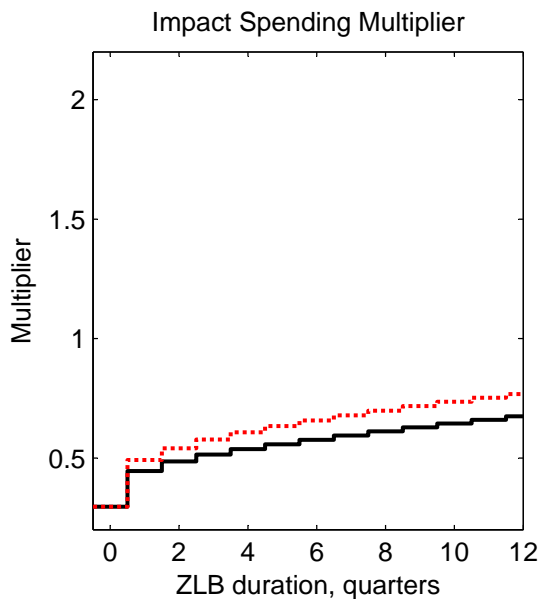
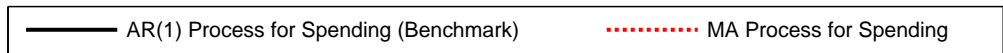
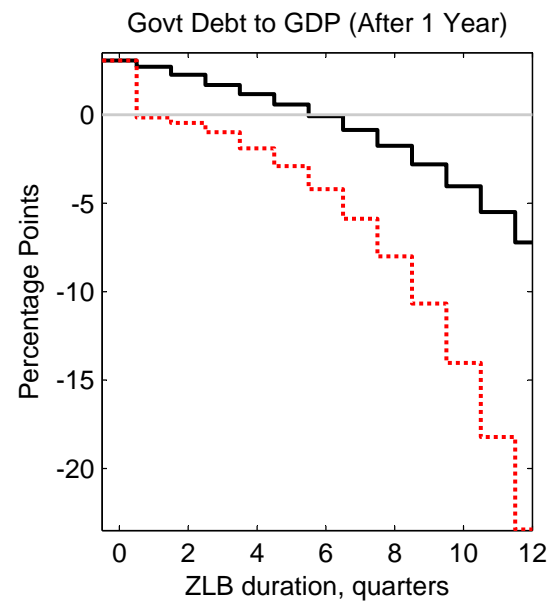
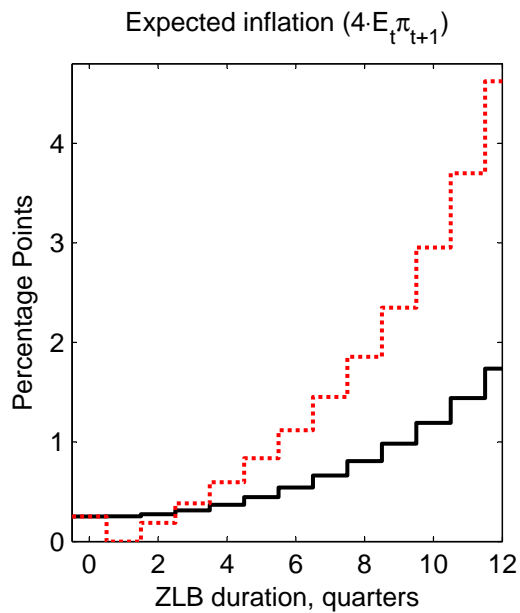
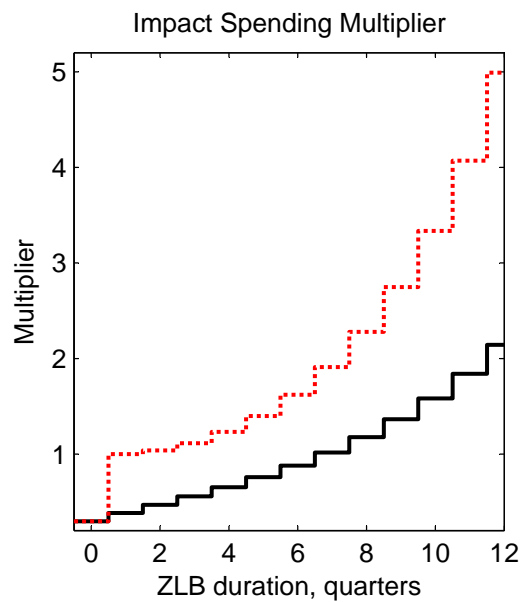




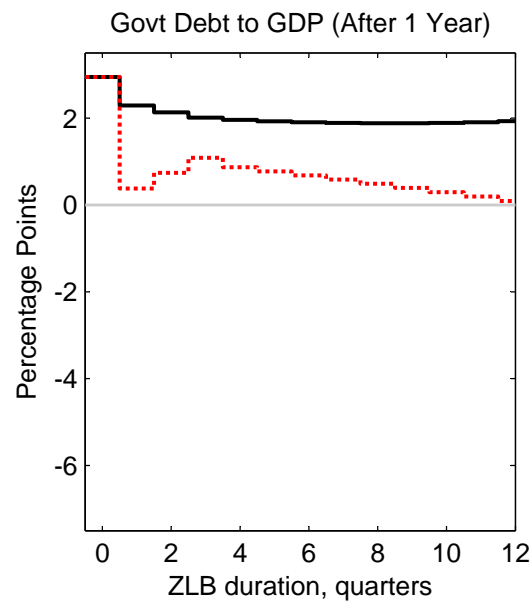
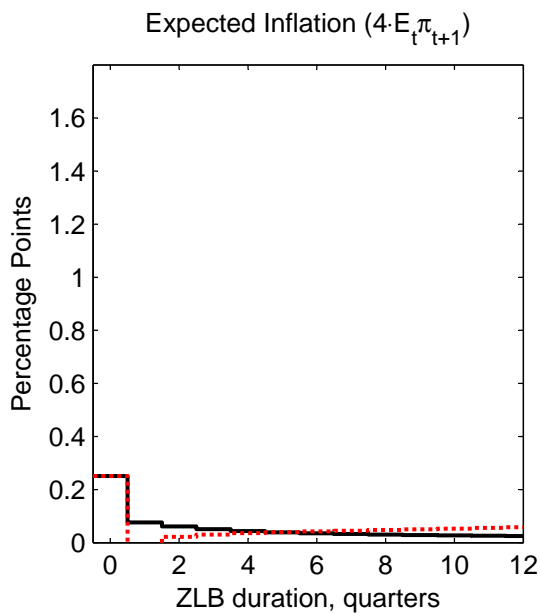
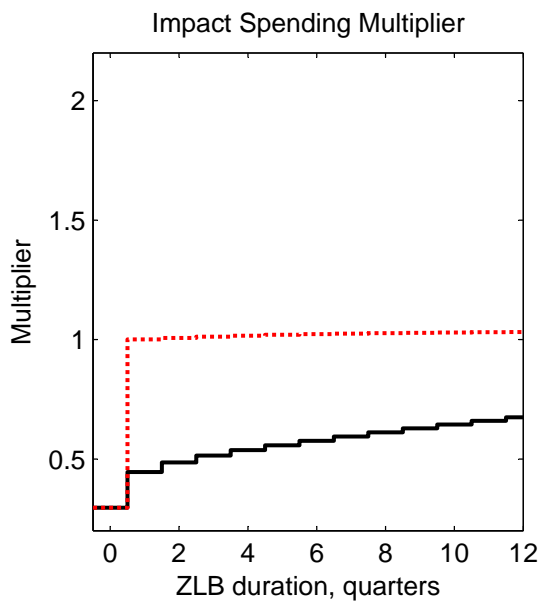
Figure A.2: Marginal Multipliers: Sensitivity With Respect to Specification of Spending Process



**Linearized Model**



**Nonlinear Model**





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