ANNEX A1

Assessment of fiscal sustainability challenges criteria used and decision trees

This annex presents the approach followed to assess fiscal sustainability risks over the short, medium and long term. Graph A1.1 provides an overview of the main building blocks. The general approach is similar to that of the 2021 Fiscal Sustainability Report, except that the S1 indicator is now used to assess long-term fiscal sustainability risks, as a complement to the S2 indicator, and no longer to assess medium-term risks. As a result, the assessment of medium-term risks entirely relies on the debt sustainability analysis (DSA).

The remainder of this annex is organised as follows. Sections A1.1, A1.2 and A1.3 describe the approach to assess short-, medium- and long-term fiscal sustainability risks. Section A1.4 provides an overview of the thresholds used for the risk classification throughout the report.

A1.1. THE APPROACH USED TO ASSESS SHORT-TERM RISKS

The analysis of short-term fiscal sustainability risks relies on the composite S0 indicator. This early-detection indicator of fiscal stress follows a signalling approach: it flashes red when certain variables (among a set of 25) exceed critical thresholds beyond which they tended to be associated with episodes of fiscal stress in the past. S0 includes two sub-indices that cover the fiscal side and the financial-competitiveness side. The main benefit of this approach is therefore that it does not only consider purely fiscal factors, but also the risks that may arise from non-fiscal factors, thus recognising the role of structural weaknesses in triggering fiscal stress. Further details on S0 are available in Chapter 1 (in particular in Box 1.1) and Annex A2.

A1.2. THE APPROACH USED TO ASSESS MEDIUM-TERM RISKS

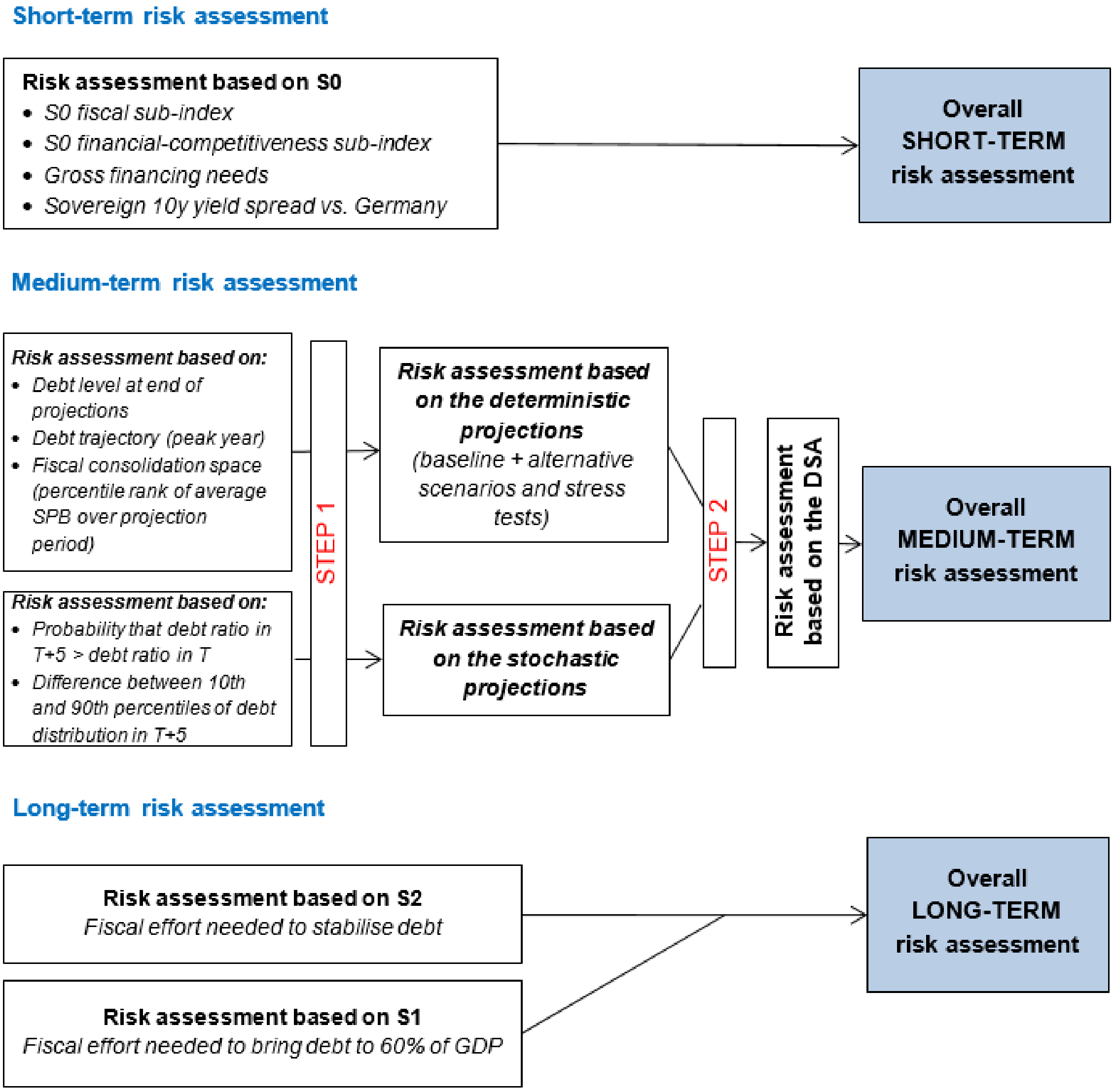
The assessment of medium-term risks is based on the debt sustainability analysis (DSA) risk classification, which is established in two steps. The first step assigns a risk category to the country under consideration for each of the deterministic projections (including the baseline) and for the stochastic projections. The second step combines the risk categories derived from the various deterministic scenarios and from the stochastic projections to conclude on the overall DSA risk classification. Further details on the DSA can be found in Chapter 2.

In the first step, *the risk assessment based on the deterministic scenarios* depends on three criteria. These are (1) the projected debt level in 10 years' time, (2) the projected debt trajectory (as summarised by the year in which debt is projected to peak), and (3) the 'fiscal consolidation space', as measured by the percentile rank of the projected structural primary balance (SPB) in the past distribution of SPBs. The fiscal consolidation space gives an indication of whether the projected SPB is plausible in view of the country's track record, and whether the country has fiscal room for manoeuvre to take corrective measures if necessary.

The decision tree for deterministic projections describes how the three criteria interplay. First, the value of each criterion is associated with a risk category (low, medium or high, according to the thresholds reported in Table A1.1 below), then the risk categories derived from the three criteria are combined along the decision tree presented in Graph A1.2. While the risk classification starts from the risk signal associated with the projected debt level, this signal may be notched up or down by one category depending on the projected debt trajectory and the available 'fiscal consolidation space'. Fiscal consolidation space is measured by the percentile rank of the SPB within the country-specific historical distribution of the SPB. The historical distributions start at the earliest in 1980, depending on data availability. The calculations use 3-year moving averages and exclude major crisis years, namely the Global Financial Crisis (2008-09) and the COVID-19 pandemic (2020-21).

***The risk category based on the stochastic projections* depends on two criteria.** The first one is the probability that the debt level in 5 years' time will not exceed its current level. The second one is the amount of uncertainty, as measured by the difference between the 10th and 90th percentiles of the distribution of debt paths resulting from the stochastic projections (i.e. the difference between the worst and the best possible outcomes, leaving aside tail events). The thresholds associated with these criteria are reported in Table A1.1, and the decision tree

Graph A1.1: **The multi-dimensional approach to assess fiscal sustainability risks**



Source: European Commission.

combining the two criteria is presented in Graph A1.3.

The second step combines the signals from the deterministic and stochastic projections. Each country is first attributed a preliminary risk classification based on the baseline. This preliminary category may then be notched up, but not down. It may be adjusted from low to medium or from medium to high based on the outcome of other scenarios and stochastic projections, as described in Graph A1.4. On the other hand, if a country is considered at high risk under the baseline, the overall DSA risk category is automatically high.

Table A1.1: **DSA: thresholds for the deterministic and stochastic projections**

	Criterion	Threshold	
Deterministic projections	Debt level in 2033	**High:** above 90% of GDP **Medium:** between 60% and 90% of GDP **Low:** below 60% of GDP	
Deterministic projections	Debt trajectory (debt peak year)	**High:** peak year **between** T+7 (2029) and end of projections (2033), or still increasing by end of projections **Medium:** peak year between T+3 (2025) and T+6 (2028) **Low:** peak year within the T+2 forecast horizon (2022-2024)	
Deterministic projections	Fiscal consolidation space (percentile rank of average SPB in 2024-2033)	**High:** up to 25% **Medium:** between 25% and 50% **Low:** above 50%	
Stochastic projections	Probability of debt not stabilising over the next 5 years, i.e. of debt ratio in 2027 exceeding the initial debt ratio	Initial debt ratio ≥ 90%	**High:** if probability > 30% **Medium:** if 0 < probability ≤ 30% **Low:** if probability = 0
Stochastic projections		60 % ≤ initial debt ratio < 90%	**High:** if probability > 60% **Medium:** if 30% < probability ≤ 60% **Low:** if probability ≤ 30%
Stochastic projections		Initial debt ratio < 60%	**Medium:** if probability > 70% **Low:** if probability ≤ 70%
Stochastic projections	Size of macroeconomic uncertainty (diff. btw 10^{th} and 90^{th} percentiles of the distribution of debt paths)	**High:** the third of the countries with highest dispersion **Medium:** the third of the countries with intermediate dispersion **Low:** the third of the countries with lowest dispersion	

Source: European Commission.

Graph A1.2: **DSA, step 1: decision tree for the deterministic projections (including the baseline)**

All deterministic DSA scenarios				
Case	Debt level	Debt path	Consolidation space	Overall
1	HIGH	HIGH/MEDIUM	**ANY**	HIGH
2	HIGH	LOW	HIGH/MEDIUM	HIGH
3	HIGH	LOW	LOW	MEDIUM
4	MEDIUM	HIGH	HIGH/MEDIUM	HIGH
5	MEDIUM	HIGH	LOW	MEDIUM
6	MEDIUM	MEDIUM	ANY	MEDIUM
7	MEDIUM	LOW	HIGH/MEDIUM	MEDIUM
8	MEDIUM	LOW	LOW	LOW
9	LOW	HIGH	HIGH/MEDIUM	MEDIUM
10	LOW	HIGH	LOW	LOW
11	LOW	MEDIUM/LOW	ANY	LOW

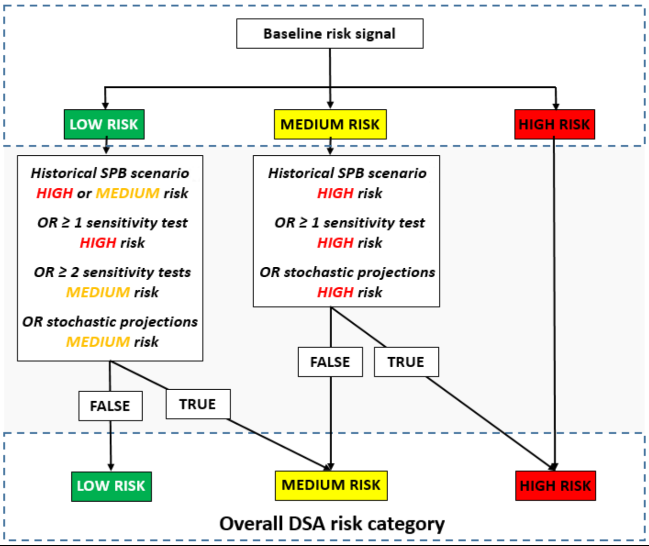
Note: the table is to be read as a decision tree, starting from the debt level then moving on to the debt path and the fiscal consolidation space. The risk category derived from the debt level in T+10 is notched up if the debt path points to high risk and the consolidation space points to medium or high risk (cases 4 and 9). Indeed, in these cases, countries have an increasing debt and limited consolidation space, meaning that there is a chance that there is no feasible adjustment path to curb the debt path. Conversely, the risk is notched down if both the debt path and the consolidation space indicator point to low risk (cases 3 and 8). In these cases, even if the projected debt level is high/medium, the debt path is decreasing, and the country has enough space to take measures in case of adverse shocks.
Source: European Commission.

Graph A1.3: **DSA, step 1: decision tree for the stochastic projections**

Probability of debt not stabilising	Size of uncertainty	Overall
HIGH	ANY	HIGH
MEDIUM	HIGH	MEDIUM
MEDIUM	MEDIUM	MEDIUM
MEDIUM	LOW	LOW
LOW	HIGH	MEDIUM
LOW	MEDIUM	LOW
LOW	LOW	LOW

Note: The table is to be read from left to right as a decision tree, starting from the probability of debt not stabilising then moving on to the size of uncertainty. It gives a strong weight to the probability of debt not stabilising over the next 5 years. Only in cases where the signal associated to this probability is medium and uncertainty is low, is the overall risk category notched down to low risk. Conversely, in cases where this probability is deemed low, but uncertainty is high, the overall risk category is notched up to medium risk.
Source: European Commission.

Graph A1.4: **DSA, step 2: decision tree for the overall DSA risk classification**



Note: It is not possible for a country to be classified at low risk under the baseline and at high risk under the stochastic projections.
Source: European Commission.

A1.3. THE APPROACH USED TO ASSESS LONG-TERM RISKS

The assessment of long-term fiscal sustainability risks is based on the S2 and S1 indicators. The S2 indicator measures the fiscal effort needed to stabilise debt in the long term, regardless of the level, based on the infinite version of the government budget constraint (see Box 3.1). The S1 indicator measures the fiscal effort needed to bring debt to 60% of GDP by 2070. For both indicators, the risk assessment depends on the amount of fiscal consolidation needed: high risk if the required effort exceeds 6 pp. of GDP, medium risk if it lies between 2 pp. and 6 pp. of GDP, and low risk if the effort is negative or below 2 pp. of GDP (see Table A1.3). Finally, the overall long-term risk classification brings together the risk categories derived from S1 and S2. S1 may notch up the risk category derived from S2 when it signals a higher risk than S2. As a result, a country is assessed to be at high risk if (i) the S2 indicator flags high risk, irrespective of the risk category derived from S1, or (ii) S2 signals medium risk but S1 points to high risk (see Table A1.2). Similarly, a country is assessed at medium risk if S2 points to low risk but S1 flags medium or high risk. The aim of these adjustments is to capture risks linked to higher debt levels, as explained in Box 3.1. The long-term risk classification is discussed in Chapter 3, and technical details can be found in Annex A5.

Table A1.2: **Decision tree for the long-term risk classification**

Risk derived from S2	Risk derived from S1	Overall long-term risk category
HIGH	Any	HIGH
MEDIUM	HIGH	HIGH
MEDIUM	MEDIUM	MEDIUM
MEDIUM	LOW	MEDIUM
LOW	HIGH	MEDIUM
LOW	MEDIUM	MEDIUM
LOW	LOW	LOW

Source: European Commission.

A1.4. OVERVIEW OF THE THRESHOLDS USED TO ASSESS FISCAL SUSTAINABILITY RISKS

The thresholds underpinning the various heat maps presented in the report can be found in the following tables:

The thresholds for the DSA risk classification, both for the deterministic and stochastic projections, are reported in Table A1.1.

For the short term, Table A1.3 reports the thresholds used for the S0 indicator, its sub-indices, and each of the variables that they include. The overall S0 index and its sub-indices use only one threshold, beyond which they identify vulnerabilities. For the individual variables, the upper thresholds derived from the signalling approach are complemented by lower thresholds, set at around 80% of the upper thresholds, so that variables may flash red, yellow or not flash at all.

For the S1 and S2 indicators, Table A1.3 reports upper and lower thresholds to distinguish between low, medium and high risk. The percentile ranks of the SPBs required by S1 and S2 are subject to the same thresholds as average SPBs in DSA scenarios (Table A1.1).

Table A1.3: **Overview of thresholds used for the fiscal sustainability risk classification**

	Safety	Upper threshold	Lower threshold
SHORT-TERM RISKS			
S0 overall index	<	0.46	:
S0 fiscal sub-index	<	0.36	:
S0 financial-competitiveness sub-index	<	0.49	:
Fiscal risks from the fiscal context			
Balance (% of GDP)	>	-9.6	-7.7
Primary balance (% of GDP)	>	0.2	0.3
Cyclically-adjusted balance (% of GDP)	>	-2.5	-2.0
Stabilising primary balance (% of GDP)	<	2.3	1.9
Gross debt (% of GDP)	<	68.4	54.8
Change in gross debt (% of GDP)	<	8.1	6.4
Short-term public debt (% of GDP)	<	13.2	10.6
Net debt (% of GDP)	<	59.5	47.6
Gross financing needs (% of GDP)	<	15.9	12.8
Interest-growth rate differential (%)	<	4.8	3.8
Change in governement expenditure (% of GDP)	<	1.9	1.5
Change in governement consumption (% of GDP)	<	0.6	0.5
Fiscal risks from the macro-financial context			
Yield curve (%)	>	0.6	0.7
Real GDP growth (%)	>	-0.7	-0.5
GDP per capita in PPP (% US level)	>	72.7	87.2
Net international investment position (% of GDP)	>	-19.8	-15.8
Net savings households (% of GDP)	>	2.6	3.1
Private debt (% of GDP)	<	164.7	131.8
Private credit flow (% of GDP)	<	11.7	9.4
Short-term debt non-financial corporations (% of GDP)	<	15.4	12.3
Short-term debt households (% of GDP)	<	2.9	2.3
Construction (% of value added)	<	7.5	6.0
Current account balance (% of GDP)	>	-2.5	-2.0
Change in REER (%)	<	9.7	7.7
Change in nominal ULC (%)	<	7.0	5.6
Fiscal risks from financial market developments			
Sovereign yield spreads (bp) - 10 year	<	231.0	184.8
MEDIUM-TERM RISKS			
DSA variables	see Table A1.2		
LONG-TERM RISKS			
S2 indicator	<	6	2
Percentile rank of the SPB implied by S2	>	25%	50%
S1 indicator	<	6	2
Percentile rank of the SPB implied by S1	>	25%	50%
ADDITIONAL VARIABLES			
Structure of public debt			
Share of short-term public debt (% of debt)	<	6.6	5.3
Share of public debt in foreign currency (% of debt)	<	31.6	25.0
Share of public debt held by non-residents (% of debt)	<	49.0	40.0
Contingent liabilites linked to the banking sector			
Bank loans-to-deposits ratio (%)	<	133.4	107.0
Share of non-performing loans (% of loans)	<	2.3	1.8
Change in share of non-performing loans (p.p.)	<	0.3	0.2
NPL coverage ratio (% loans)	>	66.0	33.0
Change in nominal house prix index (%)	<	13.2	11.0

Note: Variables common to the scoreboard used in the macroeconomic imbalances procedure (MIP) have different thresholds here than under the MIP, because the methodologies to calculate them are different.
Source: European Commission.

ANNEX A2
The early-detection indicator of fiscal stress risk (S0)

The analysis of short-term fiscal sustainability risks relies on the composite S0 indicator. This early-detection indicator of fiscal stress follows a signalling approach: it flashes red when certain variables (among a set of 25) exceed critical thresholds beyond which they tended to be associated with episodes of fiscal stress in the past. S0 includes two sub-indices that cover the fiscal side and the financial-competitiveness side.

A2.1. THE METHODOLOGY FOR THE CALCULATION OF THE THRESHOLDS

For each variable used in the composite indicator S0 the optimal threshold is chosen in a way to minimise, based on historical data, the sum of the number of fiscal stress signals sent ahead of no-fiscal-stress episodes (false positive signals – type-I error) and the number of no-fiscal-stress signals sent ahead of fiscal stress episodes (false negative signals – type-II error), with different weights attached to the two components**.** The table below reports the four possible combinations of events.

Table A2.1: **Possible cases based on type of signal sent by the variable at t-1 and state of the world at t**

	Fiscal stress episode	No-fiscal stress episode
Fiscal stress signal	True Positive signal	False Positive signal (Type I error)
No-fiscal stress signal	False Negative signal (Type II error)	True Negative signal

Source: Commission services

Formally, for each variable i the optimal threshold (t_i^*) is chosen to minimise the sum of type I and type II errors for variable i (respectively fiscal stress signals followed by no-fiscal stress episodes - False Positive signals - and no-fiscal-stress signals followed by fiscal stress episodes – False Negative signals) as from the following total misclassification error for variable i (TME_i): (97)

$$t_i^* = \arg\min_{t_i \in T_i} \left(TME_i(t_i) \right) = \qquad (1)$$

$$= \arg\min_{t_i \in T_i} \left(\frac{FN_i(t_i)}{Fs} + \frac{FP_i(t_i)}{Nfs} \right)$$

$i = 1,.., n$

where T_i = set of all values taken by variable i over all countries and years in the panel; $FN_i(t_i)$ = total number of false negative signals sent by variable i (over all countries and years) based on threshold t_i; $FP_i(t_i)$ = total number of false positive signals sent by variable i (over all countries and years) based on threshold t_i; Fs = total number of fiscal stress episodes recorded in the data; Nfs = total number of no-fiscal-stress episodes recorded in the data; (98) n = total number of variables used.

As can be seen from the minimisation problem in (1), `false negative' signals are weighted more than `false positive' signals as:

$$\frac{1}{Fs} > \frac{1}{Nfs}$$

This is due to the fact that the total number of fiscal stress episodes recorded over a (large enough) panel of countries will be typically much smaller than the total number of non-fiscal-stress episodes. This is a positive feature of the model as we might reasonably want to weigh the type II error more than the type I given the more serious consequences deriving from failing to correctly predict a fiscal stress episode relative to predicting a fiscal stress episode when there will be none.

The threshold for variable i (with i = 1,…, n) obtained from (1) is common to all countries in the panel. We define it as a common *absolute* threshold (a critical value for the level of public debt to GDP, or general government balance over GDP, for instance) but it could also be defined as a common *relative* threshold (a common percentage tail of the country-specific distributions). (99) In

(97) Following this methodological approach the optimal threshold will be such as to balance between type I and type II errors. For variables for which values above the threshold would signal fiscal stress, a relatively low threshold would produce relatively more false positive signals and fewer false negative signals, meaning higher type I error and lower type II error; the opposite would be true if a relatively high threshold was chosen.

(98) Here we simplify on the total number of fiscal stress and non-fiscal-stress episodes as in fact also these numbers vary across variables. This is due to the fact that data availability constraints do not allow us to use the whole series of episodes for all variables.

(99) See, for instance, Reinhart, M., Goldstein, G. and Kaminsky, C. (2000), Assessing financial vulnerability in emerging economies: A summary of empirical results, *East Asian Economic Review*, 4(2), 101-147, June. Hemming, R., Kell, M. and Schimmelpfennig, A. (2003), Fiscal

the latter case, while the optimal percentage tail obtained from (1) is the same for all countries, the associated absolute threshold will differ across countries reflecting differences in distributions (country *j*'s absolute threshold for variable i will reflect the country-specific history with regard to that variable). Both the aforementioned methods were applied and a decision was made to focus exclusively on the first, given that the second one tends to produce sensitive country-specific absolute thresholds for variable i only for those countries having a history of medium to high values for the variable concerned (or medium to low, depending on what the fiscal-stress-prone side of the distribution is), while country-specific thresholds would not be meaningful for the rest of the sample.

The TME function in equation (1) is the criterion we used to calculate the thresholds but it is not the only possible criterion used in the literature. The minimisation of the noise-to-signal ratio (*NSR*) is another possible option. ([100]) In this case the optimal threshold for variable *i* (t_i^*) is obtained as:

$$t_i^* = \arg\min_{t_i \in T_i}\left(NSR_i(t_i)\right) = \arg\min_{t_i \in T_i}\left(\frac{FP_i(t_i)/Nfs}{TP_i(t_i)/Fs}\right) \qquad (2)$$

$i = 1,\ldots,n$

where $TP_i(t_i)$ = total number of true positive signals sent by variable i (over all countries and years) based on threshold t_i. The TME minimisation was preferred to this alternative criterion based on the size of the total errors produced.

A2.2. THE CALCULATION OF THE COMPOSITE INDICATOR S0

The early-detection indicator of fiscal stress (S0) is constructed in a similar way to what done in Baldacci et al. (2011) and Reinhart et al. (2000). ([101]) To a certain country *j* and year *t*, a 1 is assigned for every variable i that signals fiscal stress for the following year (a dummy d^i is created for each variable i such that $d_{jt}^i = 1$ if a fiscal stress signal is sent by the variable and $d_{jt}^i = 0$ otherwise, i.e. if a no-fiscal-stress signal is sent or the variable is missing). The value of the composite indicator S0 for country *j* and year *t* ($S0_{jt}$) is then calculated as the weighted number of variables having reached their optimal thresholds with the weights given by the "signalling power" of the individual variables:

$$S0_{jt} = \sum_{i=1}^{n} w_i d_{jt}^i = \sum_{i=1}^{n} \frac{z_i}{\sum_{k=1}^{n} h_{jt}^k \cdot z_k} d_{jt}^i \qquad (3)$$

where *n* = total number of variables; z_i = 1 – (type I error + type II error) = signalling power of variable i; and $h_{jt}^k \in \{0,1\}$ is an indicator variable taking value 1 if variable *k* is observed for country *j* at time *t* and 0 otherwise. ([102]) The variables are therefore assigned higher weight in the composite indicator, the higher their past forecasting accuracy. ([103])

vulnerability and financial crises in emerging market economies, *IMF Occasional Paper*, 218.

([100]) See, for instance, Reinhart, M., Goldstein, G. and Kaminsky, C. (2000), Assessing financial vulnerability in emerging economies: A summary of empirical results, *East Asian Economic Review*, 4(2), 101-147, June. Hemming, R., Kell, M. and Schimmelpfennig, A. (2003), Fiscal vulnerability and financial crises in emerging market economies, *IMF Occasional Paper*, 218.

([101]) See Berti et al. (2012). The difference with Baldacci et al. (2011) is that Berti et al. do not use a system of "double weighting" of each variable incorporated in the composite indicator based on the weight of the subgroup of variables it belongs to (fiscal and financial-competitiveness variables here) and the weight of the individual variable within the group. The difference with Reinhart et al. (2000) is in the way the individual variables' weights are computed (Reinhart et al. use as weights the inverse of the noise-to-signal ratios of the individual variables as they apply the NSR criterion, rather than the TME minimisation).

([102]) This ensures that the sum of the weights is equal to 1 regardless of data availability (which is of course necessary to be able to analyse the evolution of the composite indicator).

([103]) Moreover, as evident from (3), the weight attached to each variable is decreasing in the signalling power attached to the other variables, as well as in the number of variables available for a given country and year.

ANNEX A3

Decomposing debt dynamics, projecting the interest rate on government debt and property incomes

A3.1. DECOMPOSING THE DEBT DYNAMICS

Deterministic government debt projections are based on a general identity characterising the evolution of the stock of debt. In a simplified version, the evolution of the government debt to GDP ratio can be described in the following way:

$$d_t = \alpha^n . d_{t-1} . \frac{(1+i_t)}{(1+g_t)} + \alpha^f . d_{t-1} . \frac{(1+i_t)}{(1+g_t)} . \frac{e_t}{e_{t-1}} - pb_t + f_t \qquad (1)$$

where d_t represents the total government debt to GDP ratio in year t

α^n represents the share of total government debt denominated in national currency

α^f represents the share of total government debt denominated in foreign currency

i_t represents the implicit interest rate on government debt ([104])

g_t represents the *nominal* growth rate of GDP (in national currency)

e_t represents the nominal exchange rate (expressed as national currency per unit of foreign currency)

pb_t represents the primary balance over GDP

f_t represents the stock-flow adjustments over GDP.

In order to obtain the debt dynamics, d_{t-1} is subtracted from both sides of equation (1). This gives the following expression:

$$\Delta d_t = \alpha^n . d_{t-1} . \frac{(i_t - g_t)}{(1+g_t)} + \alpha^f . d_{t-1} . \frac{(i_t - g_t) + \varepsilon_t . (1+i_t)}{(1+g_t)} - pb_t + f_t \qquad (2)$$

where $\varepsilon_t = \frac{e_t}{e_{t-1}} - 1$ represents the rate of depreciation of the national currency.

([104]) By simplicity, it is assumed that this interest rate is the same for government debt denominated in national currency and in foreign currency.

Decomposing further the nominal GDP growth rate, and rearranging the different terms, we obtain:

$$\Delta d_t = d_{t-1} . \frac{i_t}{(1+g_t)} - d_{t-1} . \frac{gr_t}{(1+g_t)} - d_{t-1} . \frac{\pi_t (1+gr_t)}{(1+g_t)} + \alpha^f . d_{t-1} . \varepsilon_t . \frac{(1+i_t)}{(1+g_t)} - pb_t + f_t \qquad (2)'$$

where gr_t represents the *real* growth rate of GDP

π_t represents the inflation rate (in terms of GDP deflator, in national currency)

This expression allows us identifying the key drivers of the debt ratio dynamics, in particular the snow-ball effect, which can be further decomposed into four terms:

- (+) the interest rate effect: $d_{t-1} . \frac{i_t}{(1+g_t)}$

- (-) the real GDP growth effect: $-d_{t-1} . \frac{gr_t}{(1+g_t)}$

- (-) the inflation effect: $-d_{t-1} . \frac{\pi_t (1+gr_t)}{(1+g_t)}$

- (+) the exchange rate effect: $\alpha^f . d_{t-1} . \varepsilon_t . \frac{(1+i_t)}{(1+g_t)}$

As can be easily seen from this expression, both the interest rate and the foreign exchange depreciation rate contribute to the increase of the debt ratio. On the other hand, higher real GDP growth and higher inflation erode the debt to GDP ratio. ([105])

Other key contributors to the debt motion are the primary balance (pb_t) (that is further decomposed in our tables between the structural primary balance before cost of ageing, the cost of ageing, the cyclical component and one-offs and other temporary measures) and stock and flow adjustments (f_t).

([105]) This presentation, based on the government debt ratio identity equation, allows grasping the impact of real GDP growth and inflation on the debt motion coming from direct valuation effects (as government debt is expressed as a share of GDP). However, the primary balance is also influenced by economic activity and inflation. Such behavioural effects are explicitly taken into account in the fiscal reaction function scenario presented in chapter 2 of the report.

As can be seen from the exchange rate effect expression, both valuation effects affecting the *stock* of foreign currency denominated debt and *interest rate* payments (on this share of government debt) contribute to the debt dynamic. ([106]) Looking at historical series, Eurostat includes the exchange rate effect on the *stock* of foreign currency denominated debt in stock and flow adjustments, while the impact due to the cost of servicing debt in foreign currency is included in interest payments. In our tables, we follow this convention.

In practice, the equation used in our model is slightly more complex than equation (1), as we consider three currencies: the national currency, the EUR (foreign currency for non-euro area countries) and the USD (foreign currency for all countries). Hence, equation (1) becomes:

$$d_t = \alpha^n . d_{t-1} . \frac{(1+i_t)}{(1+g_t)} + \alpha^{eur} . d_{t-1} . \frac{(1+i_t)}{(1+g_t)} . \frac{e_t}{e_{t-1}} + \alpha^{usd} . d_{t-1} . \frac{(1+i_t)}{(1+g_t)} . \frac{\tilde{e}_{t-1}}{\tilde{e}_t} . \frac{e_t}{e_{t-1}} - pb_t + f_t \qquad (1)'$$

where

- α^{eur} represents the share of total government debt denominated in euros;
- α^{usd} represents the share of total government debt denominated in USD;
- e_t represents the nominal exchange rate between the national currency and the euro (expressed as national currency per EUR);
- $\tilde{e}_t$ represents the nominal exchange rate between the USD and the euro (expressed as USD per EUR).

Such a specification allows taking into account the effect of exchange rate movements on government debt not only in non-euro area countries, but also in euro area countries (among which government debt issued in USD can be significant).

([106]) An indirect effect, due to the fact that exchange rate movements affect the value of GDP in domestic currency through changes in prices in the tradable sector, could also be shown. However, in practice, in line with other institutions practices (e.g. IMF), these effects are not isolated (data limitation would require to impose further assumptions; effect likely to be of second-order).

A3.2. PROJECTING THE IMPLICIT INTEREST RATE ON GOVERNMENT DEBT

As seen from equation (1), a key driver of the debt motion is the implicit interest rate on government debt. Projecting the implicit interest rate on government debt requires not only assumptions on *market* interest rates (for newly issued debt), but also taking into account explicitly the current and future maturity structure of government debt (between short-term and long-term government debt, and between maturing, rolled-over or not, and non-maturing government debt). This allows a differential treatment in terms of interest rates applied to successive "debt vintages", and interestingly captures different levels of exposure of sovereigns to immediate financial markets' pressures.

Formally, in our model, the implicit interest rate is expressed in the following way:

$$iir_t = \alpha_{t-1} . i_t^{ST} + (1 - \alpha_{t-1}) . iir_t^{LT} \qquad (3)$$

where

- iii_t is the implicit interest rate in year t; ([107])
- i_t^{ST} is the *market* short-term interest rate in year t;
- iir_t^{LT} is the implicit long-term interest rate in year t;
- α_{t-1} is the share of short-term debt in total government debt (and $(1 - \alpha_{t-1})$ is the share of long-term debt in total government debt). ([108])

Our model considers two types of government debt in terms of maturity: short-term debt (debt issued with an *original* maturity of less than one year) and long-term debt (debt issued with an *original* maturity of more than one year). Furthermore, government debt can be decomposed between new debt (debt issued to cover new financing requirements), ([109]) maturing debt (i.e. existing

([107]) This corresponds to i_t in the previous section.
([108]) Hence, as indicated by the t index, these shares may vary through time depending on the debt dynamic.
([109]) This amount also corresponds to the yearly budgetary deficit.

debt that is maturing within the year ([110]) and that needs to be repaid), rolled-over (i.e. whose repayment is covered by newly issued debt) or not, and outstanding debt (i.e. existing debt that has not reached maturity). Combining these different aspects, α_{t-1} (and $(1-\alpha_{t-1})$) used in (3) can be described as follows:

$$\alpha_{t-1} = \frac{D_{t-1}^{STN} + D_{t-1}^{STR}}{D_{t-1}} \quad (4)$$

$$1-\alpha_{t-1} = \frac{D_{t-1}^{O} + D_{t-1}^{LTN} + D_{t-1}^{LTR}}{D_{t-1}} \quad (5)$$

where

- D_{t-1}^{STN} is the new short-term government debt in year $t-1$;
- D_{t-1}^{STR} is the maturing and rolled-over short-term government debt (i.e. the existing short-term debt that has reached maturity, and whose repayment is covered by newly issued short-term debt);
- D_{t-1}^{LTN} is the new long-term government debt;
- D_{t-1}^{LTR} is the maturing and rolled-over long-term government debt (i.e. the existing long-term debt that has reached maturity, and whose repayment is covered by newly issued long-term debt);
- D_{t-1}^{O} is the outstanding (non-maturing) long-term government debt.

Moreover, the implicit long-term interest rate used in (3) can be further decomposed:

$$iir_t^{LT} = \beta_{t-1}.i_t^{LT} + (1-\beta_{t-1}).iir_{t-1}^{LT} \quad (6)$$

where β_{t-1} is the share of newly issued long-term debt (corresponding to both new debt and maturing and rolled-over debt) in total long-term government debt in year $t-1$ (and $(1-\beta_{t-1})$ is the share of outstanding long-term debt in total long-term government debt).

i_t^{LT} is the *market* long-term interest rate in year t.

The share of newly issued long-term debt (respectively outstanding debt) in total long-term government debt, used in expression (6), is described as follows:

$$\beta_{t-1} = \frac{D_{t-1}^{LTN} + D_{t-1}^{LTR}}{D_{t-1}^{O} + D_{t-1}^{LTN} + D_{t-1}^{LTR}} \quad (7)$$

$$(1-\beta_{t-1}) = \frac{D_{t-1}^{O}}{D_{t-1}^{O} + D_{t-1}^{LTN} + D_{t-1}^{LTR}} \quad (8)$$

Hence, replacing iir_t^{LT} in (3) by its expression in (6) gives:

$$iir_t = a_{t-1}.i_t^{ST} + b_{t-1}.i_t^{LT} + (1-a_{t-1}-b_{t-1}).iir_{t-1}^{LT} \quad (3)'$$

From equation (3)', we can see that the implicit interest rate on government debt at year t is a weighted average of market short-term and long-term interest rates and of the implicit interest rate on outstanding (i.e. non-maturing) long-term debt in year $t-1$. Hence, depending on the weight of outstanding debt in total government debt, an increase of market interest rates will transmit more or less quickly to the implicit interest rate on government debt.

In the projections, the following assumptions are made:

- i_t^{LT} and i_t^{ST} are supposed to converge linearly by T+10 to the short term and 10 year long term forward rates.
- After T+10, i_t^{LT} is supposed to converge linearly to 4% in nominal terms ([111]) (2% in real terms) for all countries by the T+30 horizon;
- i_t^{ST} is supposed to converge linearly to i_t^{LT} time a coefficient corresponding to the historical (pre-crisis) EA yield curve (currently 0.5) for all countries by the T+30 horizon;
- new debt (D_{t-1}^{STN} and D_{t-1}^{LTN}) is assumed to be issued in the projections, as a proportion of the variation of government debt, based on the shares given by Estat (of short-term and long-

([110]) Another way to describe it is that this existing debt has a *residual* maturity of less than one year.

([111]) For some non-euro countries, the convergence value is higher: PL, RO: 4.5%, HU: 5%, reflecting higher inflation targets by the national central banks.

term government debt), (112) whenever government debt is projected to increase; (113)

- short-term debt issued in year $t-1$ is assumed to entirely mature within the year, and to be rolled-over (D_{t-1}^{STR}) as a proportion of past government debt, based on the share of short-term government debt given by Estat, whenever government debt is projected to increase; (114)

- a fraction of long-term debt issued in the past is assumed to mature every year, and to be rolled-over (D_{t-1}^{LTR}), whenever government debt is projected to increase. (115) This fraction is estimated based on Estat data on the share of long-term government debt and on ECB data on the share of existing long-term debt maturing within the year. (116)

- Finally, the values of the different variables *over the forecast horizon* (especially i_t^{LT}, i_t^{ST} and iir_{t-1}^{LT}) are set consistently with the available forecast values of the implicit interest rate (iir_t) and information on the maturity structure of debt.

A3.3. TECHNICAL OVERVIEW OF THE T+10 METHODOLOGY

The following model is solved from T+3 up to T+10 (note that as of T+6, for the EU-15 without Germany, the model for the capital and investment module deviates from the general framework below and is governed by the rules described further down in the text):

$$YPOT_{it} = LS_{it}^{\alpha}K_{it}^{(1-\alpha)}TFPS_{it}$$

$$TFP_{it} = \frac{Y_{it}}{H_{it}^{\alpha}K_{it}^{(1-\alpha)}}$$

$$K_{it} = I_{it} + (1-\delta)K_{it-1}$$

$$I_{it} = \frac{I_{it}}{YPOT_{it}}YPOT_{it}$$

$$Y_{it} = YPOT_{it}(1 + YGAP_{it}) * 100$$

1. TFP trend: Kalman-filter extension. T+10 TFP is capped (i.e. a ceiling is imposed) on the basis of US TFP growth.

2. Capital:

a) Investment to potential GDP ratio: ARIMA process to produce extended series (extension to avoid end-point bias for HP filter)

b) Depreciation rate: fixed T+2 rate which is calculated on the basis of the capital law of motion

c) Investment rule: (K_{it} and I_{it} as defined in the equation system above) up to T+5; after T+5: a mix between a capital rule (K_{it} defined as $K_{it-1}\frac{YPOT_{it}}{YPOT_{it-1}}$) and I_{it} defined by capital law of motion) and the investment rule for EU-15 (except DE); investment rule for all other member states. The weight of the capital-rule based investment is gradually decreasing.

3. Trend labour: $LS_{it} = \big(POPW_{it}PARTS_{it}(1 - NAWRU_{it})\big)HPERES_{it}$

a) Working age population: use Eurostat projections on population growth ("proj_np")

b) Participation rate: up to T+5: HP-smoothed ARIMA process to produce extended series (extension beyond T+5 to avoid end-point bias for HP filter); for projection up to T+10 we use Ageing Working Group (AWG's) Cohort Simulation Model with a technical transition rule smoothing the break in T+6.

(112) More precisely, we use the average shares over the last 3 years available.
(113) Otherwise, in the cases where government debt is projected to decrease, for instance, in case of a budgetary surplus, no new debt needs to be issued.
(114) Otherwise, in the cases where government debt is projected to decrease, for instance, in case of a budgetary surplus, only part of this maturing debt needs to be rolled-over (none when government debt is assumed to strongly decrease, for example, when a large budgetary surplus allows repaying past maturing debt).
(115) See previous footnote.
(116) More precisely, the starting point (currently 2022) is calculated based on the 2021 ECB data on the share of long-term debt that is maturing within the year. Beyond this year, it is assumed that the share of maturing long-term debt linearly converges from the value taken in the last available year (2022) to the country-specific historical average by the end of the T+10 projection horizon. Additionally, for post-program countries, IE, CY and PT, the redemption profile of official loans has been taken into account for the calculation of the long-term debt maturing within the year.

c) Average hours worked: ARIMA process to produce extended series up to T+5 (extension to avoid end-point bias for HP filter) and HP smoothed. From t+6 to t+10 we forecast hours using a stabilisation rule: hours(t) = hours(t-1)*1.5 – hours(t-2)*.5. Results are comparable with those from the AWG.

d) NAWRU (T+2 = last year of the ECFIN forecast):

Between T+2 and T+5:

$$NAWRU_{iT+1} = NAWRU_{iT} + \frac{NAWRU_{iT} - NAWRU_{iT-1}}{2}$$

$$NAWRU_{iT+2} = NAWRU_{iT+1}$$

$$NAWRU_{iT+3} = NAWRU_{iT+2}$$

Between T+6 and T+10: convergence rule and prudent rule

T+10 anchor based on panel regression (union density, tax wedge, almp, unemployment benefits replacement rate, demographics/education and a set of macro control variables i.e. TFP, real interest rate, construction)

4. Output gap: closure of the output gap by T+5; each year as of T+3, YGAP decreases by 1/3 of the T+2 YGAP. The gap closure rule states that if the gaps are not closed before the end of the medium term (T+5), they should be mechanically closed by that time.

A3.4. PROPERTY INCOME

The evolution of property income over time has been taken into account in the assessment of the medium and long-term sustainability of public finances since the 2007/08 round of assessments.

In the context of this report, property income received by Member States is considered to be the sum of returns from three categories of general government financial and non-financial assets: i) interest from debt securities – bonds, ii) dividends from equity securities – shares and iii) rents from tangible non-produced non-financial assets such as land and subsoil assets (i.e. natural resources water, mineral and fossil fuels). ([117])

Property income is projected up to 2070, affecting both the medium and long term fiscal sustainability assessment in the form of S1 and S2 indicators. ([118]) Property income projections are separate from and additional to present property income accounted for in the actual balances reported every year by Member States under the SCP scenario, as well as to property income reflected in the two-year forecast horizon.

In calculating the sustainability gaps, property income received by governments is explicitly modelled in a way that is different from government revenues in general. Government revenues in general are a function of the tax bases and the rates chosen by the government. Property income differs from this generalised assumption in that it is determined by market conditions rather than policy settings.

However, since the future stocks of assets and the expected rate of return on these assets that generate income for Member States' governments in the future are not always known, to render projections manageable, a number of simplifying assumptions are made.

In order to model the evolution of property income, the key assumption is that there is no stock-flow adjustment, meaning that government debt is only driven by the general government balance and there is no net sale or purchase of assets in the future. As such, projections for the three categories of property income rely on the general assumption that the stock of financial and non-financial assets generating this income remains constant over time ([119]) at the level of

([117]) This definition is somewhat narrower than the one used in national accounts, where property income (D.4) is as well the income from financial assets and non-produced non-financial assets, but sub-categories considered for these assets are more comprehensive. In national accounts the financial instruments giving rise to interest are, in addition to debt securities, monetary gold / SDRs, deposits, loans and other accounts. The use of produced non-financial assets such as buildings is a fee (P.11 / P.131).

([118]) In the calculation of sustainability indicators (S1 and S2), the projected path of property income is conventionally included in the sub-indicator "initial budgetary position" (IBP).

([119]) Exception are natural resources for Denmark and the Netherlands, see below.

latest available data, i.e. at the values posted in T-1. This assumption implies that there is no future sale or redemption of government assets, that when short-term assets (such as bonds) mature, they are implicitly assumed to be replaced with other bonds of the same nominal value, and that property income flows received by a government from the current stock of assets are used to reimburse debt through its contribution to the general government balance, rather than to purchase other assets.

Consequently, future property income is assumed to be generated only from the upcoming returns on the assets stock and property income projections are modelled by just using further assumptions on the future evolution of the rate of return on assets.

In this sense, returns for equity and non-financial assets (rents) are generally considered to occur in line with GDP projections, whereas returns on bonds are underpinned by the additional assumptions described below.

All data for property income projections comes from Eurostat (general government property income subcategories bonds D41, equity D42 and rents D45).

Bond returns projection

These projections are based on an agreement reached in 2009 by the Economic Policy Committee's Working Group on Ageing Populations and Sustainability (AWG) and later supported in 2012 and 2015, as well as on some ad-hoc assumptions.

Returns on bonds (D.41) have been considered to be as follows:

In the short run (between T and T+30): country-specific yields on 10y government bonds apply as starting point in present year T to gradually converge to a 4% yield applied in T+30.

In the medium to long run (as of T+30): a constant 4% yield applies; this horizon and value are in line with the horizon used for government debt projections.

Due to the current low level of government bond yields, an additional assumption was made that the starting point of convergence to a 4% yield in T+30 should not be the current (T) *level* of the 10-y government bond yield that year, but an *average* of the last 10-y government bond yields.

The assumptions regarding the starting yield value and the duration of convergence to a 4% yield intend to compress the yield gap to be bridged and to stretch the timespan available for convergence, thus limiting distortionary impacts on S1 and S2 for countries with high property income.

Equity returns projection

These projections are based on a method agreed by the AWG in 2007.

Using income from equity - D.42 which reports distributed returns - country-specific shares of paid dividends in GDP are calculated for the last year of available data, T-1; for each country it is considered this share remains constant over the projection horizon, thereby implicitly assuming continuing valuation effects in line with nominal GDP growth.

Rents projection

These projections are based on a method agreed by the AWG in 2007.

The share of rents (D45) to GDP is calculated for the last year of available data for each country, T-1. ([120]) This share is assumed to remain constant over the projection horizon for all countries except Denmark and the Netherlands. For these two countries rich in fossil fuels the stock of subsoil assets is assumed to deplete by 2050, so that the share of rents to GDP in these countries would decline linearly to reach the EU average ([121]) by 2050.

Returns on real estate (rentals on buildings etc.) are not included in property income in the National Accounts since they are produced and often consumed by the general government.

In sum, considering these hypotheses, the projected path of property income ultimately

([120]) This is a simplification. Rents projections should combine the size of reserves, the timing of exploitation and the eur value of the commodity (assumption).
([121]) This average excludes excluding Denmark and the Netherlands.

depends on the stock of bonds held at the start of the projection period (the higher the bonds stock, the steeper the decline in property income over time) given that the return on these bonds is assumed to converge to a 4% yield in the medium-long term.

Since both elements can affect property income projections markedly, mitigating assumptions on the starting point and length of bond returns convergence aim to avoid unrealistic boosts to property income projections (and thereby too large of a required SPB adjustment), in particular in countries with significant property income shares.

ANNEX A4
Stochastic debt projections based on a historical variance-covariance matrix

This annex provides a description of the methodology used for stochastic debt projections based on the historical variance-covariance matrix approach and the data used to implement it. ([122]) The annex is organised as follows: section A7.1 presents the method to obtain annual stochastic shocks to the main macroeconomic variables of the model, section A7.2 shows how shocks are applied around the central scenario (i.e. the baseline 'no-fiscal policy change') and section A7.3 provides further details on the data used.

A4.1. THE METHOD TO OBTAIN (ANNUAL) STOCHASTIC SHOCKS TO MACROECONOMIC VARIABLES

Stochastic shocks are simulated for five macroeconomic variables entering the debt dynamic equation: the government primary balance (pb), nominal short-term interest rate (i^{ST}), nominal long-term interest rate (i^{LT}), nominal GDP growth rate (g), and exchange rate (e) (for non-EA countries). We use quarterly data. ([123]) First, the methodology requires transforming the time series for each macroeconomic variable x into series of historical shocks. ([124]) The historical quarterly shocks are defined as the first difference of the quarterly time series of the five macroeconomic variables. δ_q^x as follows:

$$\delta_q^x = x_q - x_{q-1}$$

with x equal to pb, i^{ST}, i^{LT}, g and e (for non-EA countries).

Second, the variance-covariance matrix for the historical quarterly shocks of the five macroeconomic variables is calculated.

([122]) The approach is based on Berti, K. (2013) Stochastic public debt projections using the historical variance-covariance matrix approach for EU countries, *European Economy. Economic Papers* No. 480 and on Beynet and Paviot (2012) Assessing the sensitivity of Hungarian debt sustainability to macroeconomic shocks under two fiscal policy reactions, *OECD Economics Department Working Paper* No. 946.

([123]) A detailed account of the series used is provided in Table 1 of section A7.3.

([124]) Before the quarterly data series are turned into shocks, some adjustments are made to eliminate extreme outliers.

Third, a Monte Carlo simulation is run by extracting two thousand random vectors of quarterly shocks over the projection period (next five years). ([125]) Shocks are drawn from STATA's pseudo-random number functions assuming a joint normal distribution with zero mean and variance-covariance matrix identical to that of historical quarterly shocks. The quarterly shocks (ε_q) obtained in this way are aggregated into annual shocks to primary balance, nominal short-term interest rate, nominal long-term interest rate, nominal GDP growth, and exchange rate (for non-EA countries), as follows:

The shock to **nominal GDP growth** (g) in year *t* is given by the sum of the quarterly shocks to growth:

$$\varepsilon_t^g = \sum_{q=1}^{4} \varepsilon_q^g$$

This equation expresses the annual shock to nominal GDP growth in year t.

The shock to **the primary balance** (pb) in year t is given by the sum of the quarterly shocks to the primary balance:

$$\varepsilon_t^{pb} = \sum_{q=1}^{4} \varepsilon_q^{pb}$$

The shock to **the nominal exchange rate** (e) in year t is given by the sum of the quarterly shocks to the exchange rate:

$$\varepsilon_t^e = \sum_{q=1}^{4} \varepsilon_q^e$$

The shock to **the nominal short-term interest rate** (i^{ST}) in year t is given by the sum of quarterly shocks to the short-term interest rate:

$$\varepsilon_t^{i^{ST}} = \sum_{q=1}^{4} \varepsilon_q^{i^{ST}}$$

The calculation of the shock to the nominal short-term interest rate in annual terms is justified based

([125]) The total matrix size is 2000x5x20 (5 years of 4 quarters).

on the fact that the short-term interest rate is defined here as the interest rate on government bonds with maturity below the year. With the equation above, we rule out persistence of short-term interest rate shocks over time, exactly as done in standard deterministic projections. In other words, unlike the case of the long-term interest rate (see below), a shock to the short-term interest rate occurring in any of the quarters of year *t* is not carried over beyond year *t*.

- The aggregation of the quarterly shocks to **the nominal long-term interest rate** (i^{LT}) into annual shocks takes account of the persistence of these shocks over time. This is due to the fact that long-term debt issued/rolled over at the moment where the shock takes place will remain in the debt stock, for all years to maturity, at the interest rate conditions holding in the market at the time of issuance. (126) A shock to the long-term interest rate in year t is therefore carried over to the following years in proportion to the share of maturing debt that is progressively rolled over (ECB data on weighted average maturity is used to implement this).

- For countries where *average weighted maturity of debt* T is equal or greater than the number of projection years (5 years), the annual shock to long-term interest rate in year *t* is defined according to the following equations:

t = first projection year

$$\varepsilon_t^{i^{LT}} = \frac{1}{T}\sum_{q=1}^{4} \varepsilon_q^{i^{LT}}$$

t = second projection year

$$\varepsilon_t^{i^{LT}} = \frac{2}{T}\sum_{q=-4}^{4} \varepsilon_q^{i^{LT}}$$

t = third projection year

$$\varepsilon_t^{i^{LT}} = \frac{3}{T}\sum_{q=-8}^{4} \varepsilon_q^{i^{LT}}$$

t = fourth projection year

$$\varepsilon_t^{i^{LT}} = \frac{4}{T}\sum_{q=-12}^{4} \varepsilon_q^{i^{LT}}$$

t = fifth projection year

$$\varepsilon_t^{i^{LT}} = \frac{5}{T}\sum_{q=-16}^{4} \varepsilon_q^{i^{LT}}$$

where q = -4, -8, -12, -16 respectively indicate the first quarter of years t-1, t-2, t-3 and t-4.

The set of equations above clearly allows for shocks to the long-term interest rate in a certain year to carry over to the following years, till when, on average, debt issued at those interest rate conditions will remain part of the stock.

For countries where the average weighted maturity of debt is smaller than the number of projection years, the equations above are adjusted accordingly to reflect a shorter carryover of past shocks. For instance, countries with average weighted maturity T = 3 years will have the annual shock to the long-term interest rate defined as follows:

t = first projection year

$$\varepsilon_t^{i^{LT}} = \frac{1}{3}\sum_{q=1}^{4} \varepsilon_q^{i^{LT}}$$

t = second projection year

$$\varepsilon_t^{i^{LT}} = \frac{2}{3}\sum_{q=-4}^{4} \varepsilon_q^{i^{LT}}$$

t = third, fourth and fifth projection year

$$\varepsilon_t^{i^{LT}} = \sum_{q=-8}^{4} \varepsilon_q^{i^{LT}}$$

(126) The implicit assumption is made here that long-term government bonds are issued at fixed interest rates only.

Finally, the weighted average of annual shocks to short-term and long-term interest rates (with weights given by the shares of short-term debt, α^{ST}, and long-term debt, α^{LT}, over total) gives us the annual shock to the implicit interest rate *i*:

$$\varepsilon_t^i = \alpha^{ST}\varepsilon^{i^{ST}} + \alpha^{LT}\varepsilon^{i^{LT}}$$

Where α^{ST} is the share of short-term debt in total government debt and $\alpha^{LT} = (1 - \alpha^{ST})$. These shares are given by ESTAT. ([127])

A4.2. APPLYING STOCHASTIC SHOCKS TO THE CENTRAL SCENARIO

All results from stochastic projections presented in this report refer to a scenario in which shocks are assumed to be temporary. In this case, annual shocks ε are applied to the baseline value of the variables (primary balance *b,* implicit interest rate *i,* nominal growth rate g and exchange rate *e*) each year as follows:

$b_t = \bar{b}_t + \varepsilon_t^b$ with $\bar{b}_t$ = baseline (from standard deterministic projections) primary balance at year *t*

$g_t = \bar{g}_t + \varepsilon_t^g$ with $\bar{g}_t$ = baseline (from standard deterministic projections) nominal GDP growth at year *t*

$i_t = \bar{\imath}_t + \varepsilon_t^i$ with $\bar{\imath}_t$ = baseline (from standard deterministic projections) implicit interest rate at year *t*

$e_t = \bar{e}_t + \varepsilon_t^e$ with $\bar{e}_t$ = nominal exchange rate as in DG ECFIN forecasts if *t* within forecast horizon; nominal exchange rate identical to last forecasted value if *t* beyond forecast horizon.

In other words, if the shock in year *t* were equal to zero, the value of the variable would be the same as in the standard deterministic baseline projections.

([127]) More precisely, we use the average shares over the last 3 years available.

A4.3. THE DEBT DYNAMIC EQUATION

Through the steps described above we obtain series, over the whole projection period, of shocks to government primary balance, nominal growth rate, implicit interest rate and nominal exchange rate that can be used in the debt dynamic equation to calculate debt ratios over a 5-year horizon, starting from the last historical value.

The debt dynamic equation takes the following form:

$$d_t = \alpha^n d_{t-1}\frac{1+i_t}{1+g_t} + \alpha^f d_{t-1}\frac{1+i_t}{1+g_t}\frac{e_t}{e_{t-1}} - b_t$$
$$+ c_t + f_t$$

where d_t = debt-to-GDP ratio in year t

α^n = share of total debt denominated in national currency ([128])

α^f = share of total debt denominated in foreign currency

b_t = primary balance over GDP in year t

c_t = change in age-related costs over GDP in year t relative to starting year ([129])

f_t = stock-flow adjustment over GDP in year t

All the steps above (extraction of random vectors of quarterly shocks over the projection horizon; aggregation of quarterly shocks into annual shocks; calculation of the corresponding simulated series of primary balance, implicit interest rate, nominal growth rate and exchange rate; calculation of the corresponding path for the debt ratio) are repeated 2000 times. This allows us to obtain yearly distributions of the debt-to-GDP ratio over the five projection years, from which we extract the percentiles to construct the fan charts.

([128]) Shares of public debt denominated in national and foreign currency are kept constant over the projection period at the latest ESTAT data (ECB data are used for those countries, for which ESTAT data were not available).

([129]) Figures on age-related costs from the latest European Commission's Ageing Report are used.

A4.4. DATA USED

For the calculation of the historical variance-covariance matrix, quarterly data on government primary balance are taken from ESTAT; nominal short-term and long-term interest rates are taken from IMF-IFS and OECD; quarterly data on nominal growth rate come from ESTAT and IMF-IFS; quarterly data on nominal exchange rate for non-EA countries come from ESTAT.

Results using the methodology described above were derived for all EU countries by using both short-term and long-term interest rates, whenever possible based on data availability, to keep in line with standard deterministic projections. This was indeed possible for the vast majority of EU countries, the only exceptions being Bulgaria, Croatia and Estonia. ([130])

Shocks to the primary balance were simulated for all countries but two (Croatia and Estonia), based on availability of sufficiently long time series of quarterly primary balances.

In general, data starting from the mid 70s until last available data were used to calculate the historical variance-covariance matrix. This period can be shorter in case of limited data availability. Table 1 provides the definition and sources of the data used.

([130]) For Estonia and Croatia we only used the short-term interest rate as quarterly data on the long-term rate were not available; for Bulgaria we used the long-term interest rate only as data on the short-term rate were not available for most recent years.

Table A4.1: **Overview of variables used to run stochastic debt projections**

Variable	Frequency	Definition	Source
Exchange rate	Quarterly	Nominal exchange rate, average in national currency (=national currency for 1 euro).	Eurostat (AVG-NAC in database ERT-BIL-EUR-Q)
Real GDP growth	Quarterly	Gross domestic product at market prices, percentage change compared to corresponding period of previous year, seasonally adjusted and adjusted data by working days. *Complemented for missing values with* Gross domestic product, real, seasonally adjusted. Calculation to compute real GDP growth values: $\frac{rGDP\,(n)-rGDP\,(n-4)}{rGDP\,(n-4)} \times 100$	Eurostat (B1GQ in unit of measure CLV-PCH-SM in database NAMQ-10-GDP) IMF - International Financial Statistics (NGDP-R-SA-XDC)
GDP deflator	Quarterly	Price index, percentage change compared to corresponding period of previous year, based on 2005=100, in national currency, seasonally adjusted and adjusted data by working days. *Complemented for missing values with* Gross domestic product, deflator, seasonally adjusted. Calculation to compute deflator values: $\frac{GDPdefl\,(n)-GDPdefl\,(n-4)}{GDPdefl\,(n-4)} \times 100$	Eurostat (B1GQ in unit of measure PD-PCH-SM-NAC in database NAMQ 10-GDP) IMF - International Financial Statistics (NGDP-D-SA-IX)
Short-term interest rate	Quarterly	Government debt securities, treasury bills, in percent per annum. (For HR: Interbank rates, money market rate, in percent per annum) *Complemented for missing values with* 3-month interbank rate, in percentage.	IMF- International Financial Statistics (FITB-PA, FIMM-PA) OECD - Key short-term economic indicators
Long-term interest rate	Quarterly	Government debt securities, government bonds, in percent per annum. *Complemented for missing values with* Rate on government bonds maturing in 10 years, in percentage.	IMF - International Financial Statistics (FIGB-PA) OECD - Key short-term economic indicators
Primary balance - Net lending/ borrowing	Quarterly	Net lending/borrowing as percentage of GDP calculated based on (1) net lending (+)/net borrowing (-) and (2) nominal GDP, both in million units of national currency and seasonally adjusted.	Eurostat (B9 in GOV-10Q-GGNFA database and B1GQ in NAMQ-10-GDP database)
- Interest payable	Quarterly	Interest expenditure as a percentage of GDP, unadjusted data.	Eurostat (D41PAY in GOV-10Q-GGNFA)

Source: European Commission

ANNEX A5
The long-term fiscal sustainability indicators (S1, S2)

A5.1. NOTATION

t: time index. Each period is one year

t_F: last year before the long-term projection (i.e. last year forecasted in the European Commission Autumn Forecast 2021, 2023).

t_0: last year before the fiscal adjustment (country-specific).

$t_0 + 1$: first year of the long-term projection period (i.e. year of the fiscal adjustment).

t_1: final year of the long-term projection period (2070), which also correspond to the target year for the debt ratio (relevant for S1).

Notice that $t_0 < t_1$.

D_t: debt-to-GDP ratio (at the end of year t).

PB_t: ratio of structural primary balance to GDP

$\Delta\mathrm{PB}_t \equiv \mathrm{PB}_t - \mathrm{PB}_{t_0}$: change in the structural primary balance relative to the base year t_0. In the absence of fiscal adjustment, it equals the change in age related expenditure (ΔA_t) for $t > t_0$.

$\Delta A_t \equiv A_t - A_{t_0}$: change in age-related costs relative to the base year t_0.

r: differential between the nominal interest rate and the nominal GDP growth rate i.e.

$1 + r \equiv \frac{1+R}{1+G}$: where R and G are, respectively, the nominal interest rate and the nominal growth rate.

If the interest-growth rate differential is time-varying, we define:

$$\alpha_{s;v} \equiv (1 + r_{s+1})(1 + r_{s+2}) \dots (1 + r_v)$$

$$\alpha_{v;v} \equiv 1$$

as the accumulation factor that transforms 1 nominal unit in period s to its period v value.

A5.2. DEBT DYNAMICS

By definition, the debt-to-GDP ratio evolves according to:

$$D_t = (1 + r_t)D_{t-1} - \mathrm{PB}_t. \qquad (1)$$

That is, the debt ratio at the end of year t, D_t, is a sum of three components: the debt ratio at the end of the previous year (D_{t-1}), interest accrued on existing debt during year t (rD_{t-1}), and the negative of the primary balance ($-\mathrm{PB}_t$).

Repeatedly substituting for D_t, the debt ratio at the end of some future year $T > t$ can be expressed similarly, as:

$$D_T = D_{t-1}\alpha_{t-1;T} - \sum_{i=t}^{T}(\mathrm{PB}_i\alpha_{i;T}). \qquad (2)$$

The path of the debt ratio is thus determined by the initial debt ratio, accrued interest (net of growth), and the path of primary balances from t through T.

A5.3. DERIVATION OF THE S1 INDICATOR

The S1 indicator is defined as the immediate and permanent one-off improvement in the ratio of structural primary balance to GDP that is required to bring the debt ratio to 60% of GDP by year t_1 (2070).

In addition to accounting for the need to adjust the initial intertemporal budgetary position and the debt level, it incorporates financing for any additional expenditure until the target date arising from an ageing population.

Under the assumed immediate and permanent one-off consolidation, the change in the primary balance is thus given by

$$\mathrm{PB}_i = \mathrm{SPB}_{t_0} + S_1 - \Delta A_i + \Delta PI_i + CC_i \qquad (3)$$

$$\text{for } i > t_0$$

Using (2), the debt ratio target D_{t_1} can then be written as:

$$D_{t_1} = D_{t_0}\alpha_{t_0;t_1} - \sum_{i=t_0+1}^{t_1}(\mathrm{PB}_i\alpha_{i;t_1}) \qquad (4)$$

Replacing (3) into (4) yields:

$$D_{t_1} = D_{t_0}\alpha_{t_0;t_1} - \sum_{i=t_0+1}^{t_1} (\mathrm{SPB}_{t_0} + S_1)\,\alpha_{i;t_2} + \sum_{i=t_0+1}^{t_1} \left((\Delta A_i - \Delta PI_i - CC_i)\,\alpha_{i;t_1}\right) \quad (5)$$

After some straightforward manipulations, ([131]) we can decompose the S1 into the following main components:

$$S_1 \equiv$$

$$= \underbrace{\frac{D_{t_0}(\alpha_{t_0;t_1}-1)}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})} - \mathrm{SPB}_{t_0} - \frac{\sum_{i=t_0+1}^{t_1}(\Delta PI_i\alpha_{i;t_1})}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})} - \frac{\sum_{i=t_0+1}^{t_1}(CC_i\alpha_{i;t_1})}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})}}_{A} + \underbrace{\frac{D_{t_0}-D_{t_1}}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})}}_{B} + \underbrace{\frac{\sum_{i=t_0+1}^{t_1}(\Delta A_i\alpha_{i;t_1})}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})}}_{C} \quad (6)$$

where (A) is the initial budgetary position (IBP) (i.e. the gap to the debt-stabilising primary balance); (B) the required additional adjustment due to the debt target (DR); and (C) the additional required adjustment due to the costs of ageing (LTC).

A5.4. DERIVATION OF THE S2 INDICATOR

The intertemporal budget constraint and the S2 indicator

According to a generally invoked definition, fiscal policy is sustainable in the long term if the present value of future primary balances is equal to the current level of debt, that is, if the intertemporal government budget constraint (IBC) is met. Let us define the S2 as the immediate and permanent one-off fiscal adjustment that would ensure that the IBC is met. This indicator is appropriate for assessing long-term fiscal sustainability in the face of ageing costs. ([132])

([131]) Add and subtract D_{t_0} on the LHS of (5), divide on both sides by $\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})$ and group the terms as in (6).

([132]) Note that the derivation of S2 does not assume that either the initial sequence of primary balances or the fixed annual increase (S2) are optimal according to some criterion. S2 should be considered as a benchmark and not as a policy recommendation or as a measure of the actual adjustment needed in any particular year.

Since the S2 indicator is defined with reference to the intertemporal government budget constraint (IBC), we first discuss which conditions are required for the IBC to hold in a standard model of debt dynamics. From (2), the debt to GDP ratio at the end of any year $t > t_0$ is given by:

$$D_t = D_{t_0}\alpha_{t_0;t} - \sum_{i=t_0+1}^{t} (\mathrm{PB}_i\alpha_{i;t}). \quad (7)$$

Rearranging the above and discounting both sides to their time t_0 values, we obtain the debt ratio on the initial period:

$$D_{t_0} = \left(\frac{D_t}{\alpha_{t_0;t}}\right) + \sum_{i=t_0+1}^{t} \left(\frac{\mathrm{PB}_i}{\alpha_{t_0;i}}\right). \quad (8i)$$

Assuming an infinite time horizon ($t \to \infty$) we get:

$$\begin{aligned} D_{t_0} &= \lim_{t\to\infty}\left(\frac{D_t}{\alpha_{t_0;t}}\right) + \lim_{t\to\infty}\sum_{i=t_0+1}^{t}\left(\frac{\mathrm{PB}_i}{\alpha_{t_0;i}}\right) \\ &= \lim_{t\to\infty}\left(\frac{D_t}{\alpha_{t_0;t}}\right) + \sum_{i=t_0+1}^{\infty}\left(\frac{\mathrm{PB}_i}{\alpha_{t_0;i}}\right) \end{aligned} \quad (8ii)$$

Either both of the limits on right-hand side of equation (8ii) fail to exist, or if one of them exists, so does the other.

Let us define the *no-Ponzi game condition* (also called the *transversality condition*) for debt sustainability, namely that the discounted present value of debt (in the very long term or in the infinite horizon) will tend to zero:

$$\lim_{t\to\infty}\left(\frac{D_t}{\alpha_{t_0;t}}\right) = 0 \quad (9i)$$

Condition (9i) means that asymptotically, the debt ratio cannot grow at a rate equal or higher than the (growth-adjusted) interest rate, which is what would happen if debt and interest were systematically paid by issuing new debt (i.e. a Ponzi game).

Combining the no-Ponzi game condition (9i) with (8ii), one obtains the intertemporal budget constraint, stating that a fiscal policy is sustainable if the present discounted value of future primary balances is equal to the initial value of the debt ratio.

$$D_{t_0} = \sum_{i=t_0+1}^{\infty} \left(\frac{\mathrm{PB}_i}{\alpha_{t_0;i}}\right) \qquad (9ii)$$

On the other hand, substituting the intertemporal budget constraint (9ii) into (8ii) implies the no-Ponzi game condition. This shows that the no-Ponzi game condition (9i) and the IBC (9ii) are, in fact, equivalent.

Assuming that the intertemporal budget constraint is satisfied through a permanent, one-off fiscal adjustment whose size is given by the S2, from $t_0 + 1$ onwards we can write:

$$\mathrm{PB}_i = \mathrm{SPB}_{t_0} + S_2 - \Delta A_i + \Delta PI_i + CC_i \qquad \text{for} \quad i > t_0. \qquad (10)$$

Then the intertemporal budget constraint (9ii) becomes

$$D_{t_0} = \sum_{i=t_0+1}^{\infty} \left(\frac{\mathrm{PB}_{t_0} + S_2 - \Delta A_i + \Delta PI_i + CC_i}{\alpha_{t_0;i}}\right). \qquad (9iii)$$

Here the ratio of structural primary balance to GDP, PB_t is re-expressed in terms of the required annual additional effort, S2, and the change in age-related costs relative to the base year t_0, combining the equation (10) with equation (9ii).

According to the theory on the convergence of series, necessary conditions for the series in equation (9ii)-(9iii) to converge are for the initial path of primary balances to be bounded and the interest rate differential in the infinite horizon to be positive ([133]). The latter is equivalent to the modified golden rule, stating that the nominal interest rate exceeds the real growth rate (i.e. $\lim_{t\to\infty} r_t > 0$). ([134])

After some rearranging, ([135]) we can decompose the S2 into the following two components:

$$S_2 = \underbrace{\frac{D_{t_0}}{\sum_{i=t_0+1}^{\infty}\left(\frac{1}{\alpha_{t_0;i}}\right)} - \mathrm{SPB}_{t_0} - \frac{\sum_{i=t_0+1}^{\infty}\left(\frac{\Delta PI_i + CC_i}{\alpha_{t_0;i}}\right)}{\sum_{i=t_0+1}^{\infty}\left(\frac{1}{\alpha_{t_0;i}}\right)}}_{A} + \underbrace{\frac{\sum_{i=t_0+1}^{\infty}\left(\frac{\Delta A_i}{\alpha_{t_0;i}}\right)}{\sum_{i=t_0+1}^{\infty}\left(\frac{1}{\alpha_{t_0;i}}\right)}}_{B} \qquad (11)$$

where (A) is the initial budgetary position i.e. the gap to the debt stabilising primary balance ([136]); and (B) the additional required adjustment due to the costs of ageing.

If the interest-growth rate differential r is constant, the accumulation factor simplifies to $\alpha_{s;v} = (1 + r_{s+1})(1 + r_{s+2}) \dots (1 + r_v) = (1 + r)^{v-s}$. Then equation (10) can be simplified further by noting that:

$$\sum_{i=t_0+1}^{\infty} \left(\frac{1}{\alpha_{t_0;i}}\right) = \sum_{i=t_0+1}^{\infty} \left(\frac{1}{(1+r)^{i-t_0}}\right) = \frac{1}{r} \qquad (12)$$

Thus, for a constant discounting factor, (11) can be rewritten as:

$$S_2 = \underbrace{rD_{t_0} - \mathrm{SPB}_{t_0} - r \sum_{i=t_0+1}^{\infty} \left(\frac{\Delta PI_i + CC_i}{\alpha_{t_0;i}}\right)}_{A} + \underbrace{r \sum_{i=t_0+1}^{\infty} \left(\frac{\Delta A_i}{\alpha_{t_0;i}}\right)}_{B} \qquad (13i)$$

If the interest-growth rate differential and the structural primary balance are constant after a certain date (here $t_1 = 2070$), equation (11) can be rewritten as:

$$S_2 = \frac{D_{t_0}}{\sum_{i=t_0+1}^{2069}\left(\frac{1}{\alpha_{t_0;i}}\right) + \frac{1}{r\alpha_{t_0;2069}}} - \mathrm{SPB}_{t_0} - \frac{\sum_{i=t_0+1}^{2069}\left(\frac{\Delta PI_i + CC_i}{\alpha_{t_0;i}}\right) + \frac{\Delta PI_{2070} + CC_{2070}}{r\,\alpha_{t_0;2069}}}{\sum_{i=t_0+1}^{2069}\left(\frac{1}{\alpha_{t_0;i}}\right) + \frac{1}{r\,\alpha_{t_0;2069}}} + \frac{\sum_{i=t_0+1}^{2069}\left(\frac{\Delta A_i}{\alpha_{t_0;i}}\right) + \frac{\Delta A_{2070}}{r\,\alpha_{t_0;2069}}}{\sum_{i=t_0+1}^{2069}\left(\frac{1}{\alpha_{t_0;i}}\right) + \frac{1}{r\,\alpha_{t_0;2069}}} \qquad (13ii)$$

where $r_t = r$ and $\Delta A_t = \Delta A_{2070}$ for $t \geq t_1 = 2070$.

([133]) The latter is an application of the ratio test for convergence.
([134]) See Escolano (2010) for further details on the relationships among the stability of the debt ratio, the IBC and the no-Ponzi game condition.
([135]) In addition, constant multiplicative terms are systematically taken out of summation signs.
([136]) In practical calculations, the present value of property income is also accounted for in the initial budgetary position. Property income enters the equation in an identical manner as age-related costs ΔA_t (i.e. term (B)), but with an opposite sign.

Derivation of the steady state debt level (at the end of the projection period) corresponding to the S2

Assuming that the intertemporal budget constraint is satisfied and that the primary balance and the interest-growth rate differential are constant at their long-run levels after the end of the projection period, then the debt ratio remains constant at the value attained at the end point of the projection period (i.e. at $t_1 = 2070$).

To see this, rewrite (9ii) as:

$$D_{t_0} = \sum_{i=t_0+1}^{\infty}\left(\frac{\mathrm{PB}_i}{\alpha_{t_0;i}}\right) = \sum_{i=t_0+1}^{t_1}\left(\frac{\mathrm{PB}_i}{\alpha_{t_0;i}}\right) + \sum_{i=t_1+1}^{\infty}\left(\frac{\mathrm{PB}_i}{\alpha_{t_0;i}}\right) \qquad (14i)$$

Using (7) and the fact that for $t \geq t_1$ the primary balance and interest-growth rate differential stay constant at $\mathrm{PB}_t = \mathrm{PB}_{t_1}$ we can rearrange (14i) to obtain the debt ratio at t_1:

$$\begin{aligned} D_{t_1} &= D_{t_0}\alpha_{t_0;t_1} - \sum_{i=t_0+1}^{t_1}\left(\mathrm{PB}_i\alpha_{i;t_1}\right) = \sum_{i=t_1+1}^{\infty}\left(\frac{\mathrm{PB}_i}{\alpha_{t_1;i}}\right) \\ &= \sum_{i=1}^{\infty}\left(\frac{\mathrm{PB}_{t_1}}{(1+r_{t_1})^i}\right) = \frac{\mathrm{PB}_{t_1}}{r_{t_1}} \end{aligned} \qquad (14ii)$$

We can generalising the above to each $t \geq t_1$ by using (7) with the initial year changed to t_1 instead of t_0, we see that for each year after t_1, the debt ratio remains unchanged at this value:

$$\begin{aligned} D_t &= D_{t_1}\alpha_{t_1;t} - \sum_{i=t_1+1}^{t}\left(\mathrm{PB}_i\alpha_{i;t}\right) \\ &= \frac{\mathrm{PB}_{t_1}}{r_{t_1}}(1+r_{t_1})^{t-t_1} - \mathrm{PB}_{t_1}\sum_{i=t_1+1}^{t}(1+r_{t_1})^{t-i} \\ &= \underbrace{\left[(1+r_{t_1})^{t-t_1} - r_{t_1}\left(\frac{1-(1+r_{t_1})^{t-t_1}}{1-(1+r_{t_1})}\right)\right]}_{=1}\frac{\mathrm{PB}_{t_1}}{r_{t_1}} \\ &= \frac{\mathrm{PB}_{t_1}}{r_{t_1}} \equiv \bar{\bar{D}} \quad \text{for} \quad t \geq t_1 \end{aligned} \qquad (15)$$

where $\bar{\bar{D}}$ is the constant debt ratio reached after the end of the projection period.

Using (4), the primary balance at the end of the projection period can be calculated as:

$$\mathrm{PB}_{t_1} = \mathrm{SPB}_{t_0} + \Delta PI_{t_3} + CC_{t_1} + S_2 - \Delta A_{t_1} \qquad (16)$$

Replacing (16) into (15), the constant (steady-state) debt ratio ($\bar{\bar{D}}$) is given by:

$$\bar{\bar{D}} = \frac{\mathrm{PB}_{t_1}}{r_{t_1}} = \frac{\mathrm{SPB}_{t_0} + \Delta PI_{t_1} + CC_{t_1} + S_2 - \Delta A_{t_1}}{r_{t_1}} \qquad (17)$$

$$\text{for} \quad t \geq t_1$$

The S2 adjustment implies that the sum of debt and the discounted present value of future changes in aged-related expenditure is (approximately) constant over time

Replacing equations (16) and (13i) into (15), and assuming a constant interest rate differential, the following equation is obtained:

$$\begin{aligned} &D_t + \sum_{i=t+1}^{\infty}\left(\frac{\Delta A_i}{(1+r)^{i-t}}\right) - \sum_{i=t+1}^{\infty}\left(\frac{\Delta PI_i + CC_i}{(1+r)^{i-t}}\right) \\ &= D_{t_0} + \sum_{i=t_0+1}^{\infty}\left(\frac{\Delta A_i}{(1+r)^{i-t_0}}\right) - \sum_{i=t_0+1}^{\infty}\left(\frac{\Delta PI_i + CC_i}{(1+r)^{i-t_0}}\right) \end{aligned} \qquad (18)$$

Equation (18) can be interpreted as follows. Implementing a permanent annual improvement in the primary balance amounting to S2 (equation 5), which is both necessary and sufficient to secure intertemporal solvency, implies that the sum of explicit debt (the first term in both sides) and the variation in age-related expenditure or implicit debt (the second terms in both sides) is (approximately) constant over time. Equation (17) is exact in the steady state (e.g. after 2070), holding only as an approximation during transitory phases (i.e. for time-varying interest rate differentials). ([137])

([137]) Moreover, equations (17) and (18) imply that both the debt and the variation in age-related expenditure are constant over time in the steady state.

ANNEX A6

Estimating the potential impact of simulated bank losses on public finances based on the SYMBOL model

SYMBOL approximates the probability distributions of individual bank's losses using publicly available information from banks' financial statements. In particular, the model estimates an average implied default probability of the individual banks' asset/loan portfolios by inverting the Basel FIRB formula for capital requirements ([138]).

The main data source on banks' financial statements is Orbis Bank Focus, a commercial database of the private company Bureau van Dijk (part of Moody's analytics). For the reference year 2021, unconsolidated data for commercial, saving and cooperatives banks are included. The data as provided by Orbis Bank Focus occasionally lacks information on specific variables for some banks in the sample (e.g. capital, risk weighted assets, provisions, gross non-performing loans). In those cases, capital is imputed via a robust regression by using common equity, while risk weighted assets are approximated using the total regulatory capital ratio (at bank or country level) ([139]). While gross loans are available for all banks, values for provisions and non-performing loans are available only for two thirds of the sample. Missing values for provisions have thus been estimated by country aggregates coming from the EBA dashboard ([140]), while missing values for non-performing loans have been imputed by applying a robust regression using provisions as explanatory variable. Information on the sample is presented in Table A9.1, and Table A9.2 reports statistics at aggregated Member State level for non-performing loans (NPLs) and loans provisions, taken from the EBA dashboard, while recovery rates (country aggregates) are taken from the World Bank (2020). ([141])

Similarly to past exercises, the sample covers roughly 75% of all EU banking assets. ([142]) When the sample, as illustrated in Table A11.1, either includes a small number of banks or covers a low share of total assets, results should be interpreted with caution, since a minor change to any bank's data or the addition of a new bank could have large effects on results.

Table A9.1: **Descriptive statistics of samples used for SYMBOL simulations**

	Sample ratio *(Sample TA/ Population TA)*	Nbr.of banks	Total assets (TA)	Capital	Risk weighted assets (RWA)	RWA/TA	Capital/RWA
	%		EUR bn	EUR bn	EUR bn	%	%
AT	85.3%	398	844.0	77.1	374.9	44.4%	20.6%
BE	95.0%	26	966.1	64.2	326.9	33.8%	19.6%
BG	86.3%	15	60.9	7.1	31.1	51.0%	22.8%
CY	78.8%	21	55.0	3.8	18.2	33.0%	21.1%
CZ	69.6%	17	238.9	21.3	89.5	37.5%	23.9%
DE	71.9%	1123	6278.2	465.1	2625.1	41.8%	17.7%
DK	50.8%	53	580.2	60.9	242.7	41.8%	25.1%
EE	101.2%	3	37.6	3.9	16.2	43.2%	23.8%
ES	88.6%	83	2533.1	192.2	1082.7	42.7%	17.8%
FI	96.1%	101	610.9	45.0	194.5	31.8%	23.2%
FR	76.0%	149	8233.4	445.4	2255.9	27.4%	19.7%
GR	95.9%	7	310.8	23.0	148.2	47.7%	15.5%
HR	92.1%	19	63.8	7.9	31.0	48.6%	25.5%
HU	53.5%	12	87.3	9.7	42.1	48.2%	23.0%
IE	27.6%	21	364.6	37.0	157.6	43.2%	23.5%
IT	75.1%	273	2814.9	219.4	1052.2	37.4%	20.9%
LT	72.4%	4	32.1	2.2	10.0	31.0%	21.9%
LU	37.3%	40	390.1	35.1	166.7	42.7%	21.0%
LV	99.5%	10	20.2	2.2	9.0	44.8%	24.4%
MT	65.0%	9	27.7	2.2	10.1	36.6%	22.0%
NL	72.7%	15	1837.0	131.5	569.3	31.0%	23.1%
PL	68.9%	93	386.9	36.9	203.5	52.6%	18.1%
PT	87.5%	92	358.8	28.8	156.3	43.6%	18.4%
RO	83.8%	15	104.3	10.3	45.6	43.7%	22.6%
SE	56.3%	78	739.4	55.1	196.9	26.6%	28.0%
SI	84.9%	10	42.0	4.3	22.3	53.2%	19.4%
SK	94.9%	9	88.5	7.2	44.2	50.0%	16.3%

(1) 2021 unconsolidated data.
Source: Commission services.

([138]) For more detail on the SYMBOL model see European Commission (2016), Fiscal Sustainability Report, European Economy Institutional Papers, 18 January, Section 5.2.2 and Annex A7.

([139]) The procedure for the imputation of missing values of capital and RWA is described in "SYMBOL database and simulations for 2013, P. Benczur, J. Cariboni, F.E. Di Girolamo, A. Pagano, M. Petracco, JRC European Commission, Technical Report, JRC9298".

([140]) EBA Risk Dashboard - data as of Q4 2021.

([141]) Due to issues in the data, the World Bank paused the 2021 Doing Business report to start a series of audits in the methodology. Thus, we use the recovery rates as of end 2020.

([142]) The sample ratio changes per each MS ranging from 27.5% in Ireland to higher than 100% in EE. This variability calls for caution when reading the results in particular for country with a low coverage ratio and small number of banks.

	Gross loans	NPL Ratio Gross NPL/Gross loans	NPL/TA Gross NPL/TA	NPL/Capital Gross NPL/Capital	Provisions	Recovery rate Baseline Scenario	NPL losses Baseline Scenario
	EUR bn	%	%	%	EUR bn	%	EUR bn
AT	403.2	2.5%	1.4%	16.1%	5.5	79.9%	1.8
BE	472.9	1.2%	0.6%	7.9%	3.8	89.4%	0.1
BG	30.6	8.4%	4.7%	39.5%	1.6	37.7%	0.6
CY	25.9	13.9%	6.9%	83.4%	1.7	73.8%	0.5
CZ	111.8	2.3%	1.3%	14.2%	2.2	67.5%	0.3
DE	2607.1	1.1%	0.6%	7.4%	15.5	79.8%	4.4
DK	165.9	4.3%	1.4%	15.6%	5.2	88.5%	0.0
EE	24.1	1.8%	1.1%	10.0%	0.2	36.1%	0.2
ES	1178.3	3.4%	1.8%	20.7%	29.4	77.5%	2.6
FI	234.0	2.2%	0.9%	11.9%	2.7	88.0%	0.3
FR	2489.7	2.3%	0.7%	13.0%	29.6	74.8%	14.5
GR	75.2	32.9%	17.3%	175.9%	11.3	32.0%	9.0
HR	35.6	7.6%	4.6%	34.9%	2.2	35.2%	0.2
HU	32.2	2.7%	1.1%	9.0%	1.0	44.2%	0.0
IE	117.0	6.5%	2.4%	22.4%	5.0	86.1%	0.0
IT	1606.2	5.1%	3.2%	37.6%	54.9	65.6%	9.4
LT	13.7	2.2%	1.1%	14.0%	0.1	41.4%	0.1
LU	162.7	1.5%	0.6%	6.3%	1.3	43.9%	0.7
LV	9.6	4.4%	2.1%	18.7%	0.2	41.4%	0.2
MT	12.0	5.0%	2.4%	27.9%	0.4	39.2%	0.1
NL	938.5	0.8%	0.4%	5.4%	5.1	90.1%	0.1
PL	222.0	6.3%	3.8%	35.3%	10.5	60.9%	0.2
PT	146.1	4.3%	2.3%	25.0%	6.3	64.8%	0.0
RO	49.4	5.1%	2.7%	24.5%	2.7	34.4%	0.0
SE	308.4	1.6%	0.7%	9.0%	3.1	78.1%	0.1
SI	20.1	3.0%	1.6%	14.9%	0.5	90.0%	0.0
SK	49.0	2.6%	2.0%	22.6%	1.3	46.1%	0.0

Table A9.2: **Descriptive statistics for non-performing loans (NPL)**

	Gross loans	NPL Ratio Gross NPL/Gros	NPL/TA Gross NPL/TA	NPL/ Capital Gross NPL/Capi	Provisions	Recovery rate Baseline Scenario	NPL losses Baseline Scenario
	EUR bn	%	%	%	EUR bn	%	EUR bn
AT	453.4	3.5%	1.9%	20.6%	5.5	79.9%	5.6
BE	498.6	1.0%	0.5%	7.8%	3.3	89.4%	0.2
BG	34.4	6.3%	3.5%	30.3%	1.5	37.7%	0.5
CY	21.0	7.6%	2.9%	41.5%	0.6	73.8%	0.5
CZ	136.4	2.1%	1.2%	13.4%	2.4	67.5%	0.2
DE	3364.8	1.6%	0.9%	11.9%	14.7	79.8%	22.1
DK	175.4	3.5%	1.1%	10.0%	4.9	88.5%	0.0
EE	25.1	1.3%	0.8%	8.2%	0.2	36.1%	0.1
ES	1326.4	3.5%	1.9%	24.5%	31.2	77.5%	3.5
FI	245.1	2.1%	0.8%	11.4%	2.5	88.0%	0.5
FR	2727.8	2.1%	0.7%	12.7%	28.8	74.8%	15.3
GR	149.7	11.6%	5.6%	75.2%	8.0	32.0%	7.0
HR	36.7	6.3%	3.6%	29.3%	2.0	35.2%	0.2
HU	34.7	3.1%	1.2%	11.0%	1.0	44.2%	0.1
IE	113.7	5.6%	1.7%	17.2%	4.0	86.1%	0.3
IT	1692.0	3.8%	2.3%	29.3%	47.1	65.6%	6.1
LT	15.2	1.2%	0.6%	8.5%	0.1	41.4%	0.0
LU	156.8	1.7%	0.7%	7.5%	1.2	43.9%	0.9
LV	10.5	3.6%	1.9%	17.3%	0.1	41.4%	0.2
MT	12.7	5.2%	2.4%	29.5%	0.4	39.2%	0.2
NL	932.6	0.6%	0.3%	4.3%	4.1	90.1%	0.1
PL	229.8	5.0%	3.0%	31.2%	9.0	60.9%	0.4
PT	188.1	3.5%	1.8%	22.8%	6.7	64.8%	0.1
RO	56.3	4.3%	2.3%	23.7%	2.8	34.4%	0.0
SE	319.0	1.1%	0.5%	6.2%	2.7	78.1%	0.1
SI	22.4	2.3%	1.2%	11.9%	0.5	90.0%	0.0
SK	60.7	2.2%	1.5%	18.7%	1.4	46.1%	0.0

(1) 2021 unconsolidated data.
Source: Commission services.

Computation of aggregate banking losses and estimated impact on public finances

Starting from the estimated average probability of default of the asset portfolio of each bank, SYMBOL generates realisations for each individual bank's credit losses via Monte Carlo simulation using the Basel FIRB loss distribution function and assuming a correlation between simulated shocks hitting different banks in the system ([143]). In the short-term scenario, losses from SYMBOL are added on top of losses due to current stocks of non-performing loans, adjusted for moratoria.

Individual bank losses are then transformed into excess losses and recapitalisation needs to be covered and finally aggregated at country and EU27 system level. Based on the bank-level balance sheet data and losses simulation, the model can then implement the loss allocation cascade (e.g. own funds, bail-in of eligible liabilities, Resolution Fund interventions), distinguishing between excess losses and recapitalisation needs. Excess losses are losses in excess of available total capital of a bank (negative equity), while recapitalisation needs are the funds necessary to restore the bank's minimum level of capitalisation given by the regulatory scenario under consideration. ([144])

Throughout the cascade of safety net interventions, it can then be traced how much of each of these two types of financing needs are picked up by the different tools. If after depletion of capital, a bank is failing or left undercapitalised with respect to the minimum level established in the scenarios, the bail-in tool is applied at individual bank level up to 8% of its total liabilities and own funds (TLOF) (or total assets, TA). ([145]) When a Resolution Fund

([143]) The correlation is assumed to be 0.5 for all banks in the current simulation. All EU banks are simulated together.

([144]) European Commission (2016), Fiscal Sustainability Report, European Economy Institutional Papers, 18 January, Annex A7.

([145]) The BRRD does not establish a harmonised level of liabilities eligible for bail-in, but Art. 44 sets out that the RF can kick in only after shareholders and holders of other eligible instruments have made a contribution to loss absorption and recapitalisation of at least 8% of total liabilities and own funds (TLOF). Since bank-level data on bail-inable liabilities is unavailable, the bail-in tool is modelled in both the short- and long-term by imposing that individual banks hold a LAC of at least

(RF) is available, it is then assumed to intervene up to 5% of the total assets of each bank. ([146]) Given that the sample coverage in terms of the number and total assets of banks in the sample is not complete, the RF is assumed to have ex-ante funding equal to the appropriate percentage of covered deposits of the banks in the sample. Any leftover losses or recapitalisation needs not covered after all available tools have intervened are finally assumed to be covered by the government, taking into account the ratio between the total assets (TA) in the sample and the population of all banks.

In the baseline scenario, for the purposes of determining the course of action in case of failure, banks are split into two groups. Those that are not designated as 'significant institutions for SSM purposes', are assumed to be always liquidated (i.e. resolution probability equal to 0%). Those that are designated as 'significant institutions in case of distress' might go into resolution or liquidation. In the category of 'significant institutions', for global systemically important institutions (G-SIIs) and their subsidiaries the probability of going into resolution is set to 100% (i.e. we assume that G-SIIs will be always resolved), while for the other entities we assume an 80% resolution probability ([147]).

The results are used to provide an estimate of the implicit contingent liabilities - banking losses and recapitalisation needs after the safety net– in case of a financial crisis. Notably, in the current exercise, this is done by using a sub-additive measure, the Expected Shortfall, to calculate the expected losses in the tail of the distribution. This methodological development of the estimation technique is illustrated in Bellia *et al*. (forthcoming 2023). In practical terms, we select all the simulations where the factor is above a threshold (fixed for values of the common factor above 3 standard deviations) and we calculate the average value in this selected tail of the distribution. This represents the expected value of the portfolio losses under a stressed economic situation. ([148])

Table A9.3 visualises the role of the various safety-net tools in absorbing unexpected losses.

8% of their TLOF. In practice banks with total capital under this threshold are assumed to meet the 8% minimum threshold via bail-inable liabilities. In the simulation, bail-in stops once the 8% of TA limit has been reached. If a bank holds capital above 8% of TA, there would be no bail-in, but capital might be bearing losses above 8% of TLOF.

([146]) Art. 44 of the BRRD sets out that the contribution of the resolution financing arrangement cannot exceed 5% of the total liabilities. In case of excess demand for SRF funds, funds are rationed in proportion to demand (i.e., proportionally to excess losses and recapitalisation needs after the minimum bail-in, capped at 5% of TA at bank level).

([147]) Please note that (i) in practice, Most of the SRB's banks (82% of the total number of SRB banks accounting for 97% of total exposure at risk) are earmarked for resolution. In contrast, liquidation is foreseen for 18% of the banks, which account for 3% of total exposure at risk, mostly made up of public development banks and smaller banks with a specific business model. (2022-07-13_SRB-Resolvability-Assessment.pdf (europa.eu)). (ii) Up until last year, for DSA exercises, the standard assumptions were either that only significant institutions go into resolution, or that all banks go into resolution. The current set up is thus more favorable to resolution funds, because a share of the significant banks (20%) is now assumed to go into liquidation. However, recent resolutions procedures also involved very small banks, thus it might be that this assumption is not fully aligned with the actual choice of liquidating versus resolving a bank.

([148]) Values of the common factor greater or equal to 3 corresponds to values 3 standard deviations away to the mean, which implies a (one tail) cumulative percentile equal to 99.865. In other words, we focus on the 0.135% of the extreme values of the factor. Replicating the methodology with 2009 data (as in the original SYMBOL implementation), using the expected shortfall approach yields 657 billion of losses, a value 2.6% smaller with respect to the 99.95th percentile under the original calibration (675 EUR billion). We also verify that all runs of simulations used for the original calibration with the percentile approach have a common factor larger or equal than 3. No runs of the simulations where at least one bank defaults have a common factor smaller than 3 (with more than 6 million simulations).

Table A9.3: **Leftover financial needs after each safety net tool (% of GDP 2021), under the short and long term scenarios**

	Initial (2023) short term scenarios			Final (2033) long term scenarios		
	Excess losses plus recap	Excess losses plus recap after bail in	Excess losses plus recap after RFs	Excess losses plus recap	Excess losses plus recap after bail in	Excess losses plus recap after RFs
AT	1.0%	0.6%	0.2%	0.4%	0.3%	0.1%
BE	0.6%	0.4%	0.1%	0.4%	0.3%	0.1%
BG	0.2%	0.2%	0.1%	0.1%	0.1%	0.0%
CY	2.7%	1.4%	0.5%	0.3%	0.2%	0.1%
CZ	0.2%	0.2%	0.1%	0.1%	0.1%	0.1%
DE	0.6%	0.2%	0.1%	0.3%	0.1%	0.0%
DK	0.3%	0.3%	0.3%	0.2%	0.2%	0.1%
EE	0.1%	0.1%	0.0%	0.1%	0.1%	0.0%
ES	1.3%	1.0%	0.6%	0.8%	0.6%	0.2%
FI	0.3%	0.2%	0.1%	0.3%	0.2%	0.0%
FR	1.8%	0.7%	0.4%	0.8%	0.3%	0.1%
GR	2.7%	1.8%	0.6%	0.4%	0.2%	0.0%
HR	0.2%	0.2%	0.1%	0.1%	0.1%	0.0%
HU	0.2%	0.2%	0.1%	0.1%	0.1%	0.1%
IE	0.5%	0.5%	0.2%	0.3%	0.3%	0.1%
IT	1.6%	1.0%	0.5%	0.6%	0.4%	0.1%
LT	0.1%	0.1%	0.0%	0.1%	0.1%	0.0%
LU	6.3%	4.8%	2.0%	3.7%	2.8%	0.7%
LV	0.1%	0.1%	0.0%	0.1%	0.1%	0.0%
MT	0.5%	0.4%	0.2%	0.2%	0.1%	0.0%
NL	0.6%	0.4%	0.2%	0.3%	0.2%	0.1%
PL	0.7%	0.7%	0.5%	0.3%	0.3%	0.2%
PT	1.4%	1.0%	0.4%	0.9%	0.5%	0.1%
RO	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%
SE	0.2%	0.1%	0.1%	0.1%	0.1%	0.0%
SI	0.3%	0.3%	0.1%	0.2%	0.2%	0.0%
SK	1.3%	1.3%	0.7%	0.7%	0.6%	0.2%

Source: Commission services.

Scenarios settings

SYMBOL can be used to illustrate how the regulatory framework set up by the Commission in recent years would, under certain assumptions, limit the impact of a hypothetical future systemic banking crisis on public finances.

Three pieces of legislation are considered: the Capital Requirement Regulation and Directive IV (CRR, CRDIV) ([149]), which improved the definitions of regulatory capital and risk-weighted assets, increased the level of regulatory capital by introducing the capital buffers, including extra capital buffers for European Global Systematically Important Institutions (G-SIIs) and Other Systemically Important Institutions (O-SII) ([150]); the Bank Recovery and Resolution Directive (BRRD) ([151]), which introduced bail-in ([152]) and national resolution funds ([153]), and the Single Resolution Mechanism Regulation (SRMR), ([154]) which established the Single Resolution Board and the Single Resolution Fund (SRF). To reflect the phasing-in ([155]) of the safety-net tools foreseen by this body of legislation, two regulatory scenarios are modelled.

An initial (2023) short-term baseline scenario with safety net in progress, comprising:

- Asset correlation is fixed to 50% (traditional SYMBOL assumption, compatible with default regulatory parameter);
- Bank total capital and initial risk-weighted assets (RWAs) taken directly from the banks' balance sheets. RWA are then updated to reflect the stress condition.

([149]) See European Parliament and Council (2013), Directive 2013/36/EU of the 26 June 2013 on Access to the Activity of Credit Institutions and the Prudential Supervision of Credit Institutions and Investment Firms, Amending Directive 2002/87/EC and Repealing Directives 2006/48/EC and 2006/49/EC, 2013, Official Journal of the European Union, L 176/338

([150]) Very few banks which are OSII are affected by extra buffer (not considered).

([151]) See European Parliament and Council (2014a), Directive 2014/59/EU of the European Parliament and of the Council of 15 May 2014 Establishing a Framework for the Recovery and Resolution of Credit Institutions and Investment Firms and Amending Council Directive 82/891/EEC, and Directives 2001/24/EC, 2002/47/EC, 2004/25/EC, 2005/56/EC, 2007/36/EC, 2011/35/EU, 2012/30/EU and 2013/36/EU, and Regulations (EU) No 1093/2010 and (EU) No 648/2012, of the European Parliament and of the Council" Official Journal of the European Union, L 173/190.

([152]) A legal framework ensuring that part of the distressed banks' losses are absorbed by unsecured creditors. The bail-in tool entered into force on 01/01/2016.

([153]) Funds financed by banks to orderly resolve failing banks, avoiding contagion and other spill-overs.

([154]) See European Parliament and Council (2014b), Regulation (EU) No 806/2014 of the European Parliament and of the Council of 15 July 2014 establishing uniform rules and a uniform procedure for the resolution of credit institutions and certain investment firms in the framework of a Single Resolution Mechanism and a Single Resolution Fund and amending Regulation (EU) No 1093/2010, Official Journal of the European Union, L 225/1.

([155]) CRR/CRDIV increased capital requirements are being phased-in from 2014 to 2019 and banks are progressively introducing the capital conservation buffer; according to BRRD and SRMR, national RFs and the SRF have a target of 1% of covered deposits to be collected over 10 years from 2015 onwards and 8 years from 2016 onwards, respectively.

- Current stocks of non-performing loans contribute to losses in the banking system of each country and their magnitude has been estimated as explained in the main text, including the potential effects of the moratoria.

- Extra capital buffers for G-SIIs prescribed by the Financial Stability Board (FSB) are considered.

- Bail-in: modelled as a scenario whereby a Loss Absorbing Capacity (LAC) is built to represent, together with regulatory capital, 8% of TLOF.

- Resolution Funds - national (NRFs, for Member States not part of the Banking Union) and single (SRF, for Banking Union members) – phased-in in proportion of 8/10 of their target or long-run level and contributing to resolution absorbing losses up to 5% of the TA of the insolvent bank, provided that at least 8% LAC has already been called in ([146]). No backstop (other than public finances) nor ex-post contributions ([156]) are considered.

- No DGS contribution or intervention is modelled.

- Extra losses generated by loans granted by the State are directly transferred to debt or deficit without passing through the safety net cascade.

A final (long-term) 2033 baseline scenario as of when a completely phased-in safety net comprises:

- Asset correlation is fixed to 50% (traditional SYMBOL assumption, compatible with default regulatory parameter).

- Bank total capital taken directly from the banks' balance sheets and reflecting an increased minimum requirement topped-up to 10.5% RWA ([157]). RWA as reported, without Stress Test adjustments.

- Losses on current NPL stocks are not considered, moratoria and guarantees are assumed to be expired ([158]).

- Extra capital buffers for G-SIIs prescribed by the Financial Stability Board (FSB) are considered.

- Bail-in: modelled as a scenario whereby a Loss Absorbing Capacity (LAC) is built to represent, together with regulatory capital, 8% of TA ([159]).

- Resolution Funds ([160]) - national (NRFs, for Member States not part of the Banking Union) and single (SRF, for Banking Union members) – fully phased-in and contributing to resolution absorbing losses up to 5% of the TA of the insolvent bank, provided that at least 8% TA has already been called in ([161]). No backstop (other than public finances) nor ex-post contributions ([162]) are considered.

([156]) Given the aim to portray worst-case fiscal consequences, ex-post contributions to the NRFs/SRF are not modelled, but these can actually go up to 3 times the ex-ante contributions, further reducing the impact on public finances.

([157]) Only mandatory requirements, i.e. the 8% total capital requirement and the 2.5% capital conservation buffer, are included. The discretionary counter-cyclical capital buffer (at the regulator's choice) is not.

([158]) The impact of non-performing loans (NPLs) is considered only in the current situation and the effect is assumed to become negligible in the long-term.

([159]) Same assumptions regarding 8% TA hold under BRRD2 once it will become applicable in December 2020.

([160]) In practice, under the Agreement on the mutualisation and transfer of contributions to the SRF (IGA), in the short-term only a part of current SRF contributions would be mutualised (i.e. available to all banks irrespective of their location), while the rest of the fund is only available to banks from their country of origin. Since a system-wide waterfall under IGA with sequential intervention of national and mutualised SRF is complex to model and since in the short-term only 10% of the SRF would be in place, the model assumes that the entire SRF is already mutualised.

([161]) In case of excess demand for SRF funds, funds are rationed in proportion to demand (i.e., proportionally to excess losses and recapitalisation needs after the minimum bail-in, capped at 5% of TA at bank level).

([162]) Given the aim to portray worst-case fiscal consequences, ex-post contributions to the NRFs/SRF are not modelled, but these can actually go up to 3 times the ex-ante contributions, further reducing the impact on public finances.

- No DGS contribution or intervention is modelled.

- Graph A9.2 illustrates the order of intervention of different tools. The first cushion assumed to absorb simulated losses is capital, the second tool is bail-in, and the last are RFs, as legally foreseen ([163]).

- Moreover, alternative scenario settings are considered, as summarised in Table A9.4 and Graph A9.2.

Calibrating the heat map

The model allows estimating the probability distribution of the amount of public funds needed to cover losses after exhausting the protection provided by the financial safety net. To obtain the input for the heat map on government's implicit contingent liability risks, a minimum size of government's contingent liabilities is fixed, and the theoretical probability of the materialisation of such an event is assessed.

Table A9.5 shows a heat map illustrating the relative riskiness of countries in terms of public finances being hit by a shock of a given minimum share of GDP (3%, 5%, and 10%), conditional on having (a) the banking sector in distress, (2) at least three countries with government's contingent liabilities. The colour coding reflects the relative magnitude of the theoretical probabilities of such an event. ([164])

([163]) Additional tools are available to absorb residual losses and recapitalisation needs, including additional bail-in liabilities, leftover resolution funds and the deposit guarantee scheme. See for a discussion Benczur P., Berti K., Cariboni J., Di Girolamo F. E., Langedijk S., Pagano A., and Petracco Giudici M. (2015), Banking stress scenarios for public debt projections, European Economy Economic Papers 548. In addition, by 2024 at the latest a common backstop to the SRF will be introduced.

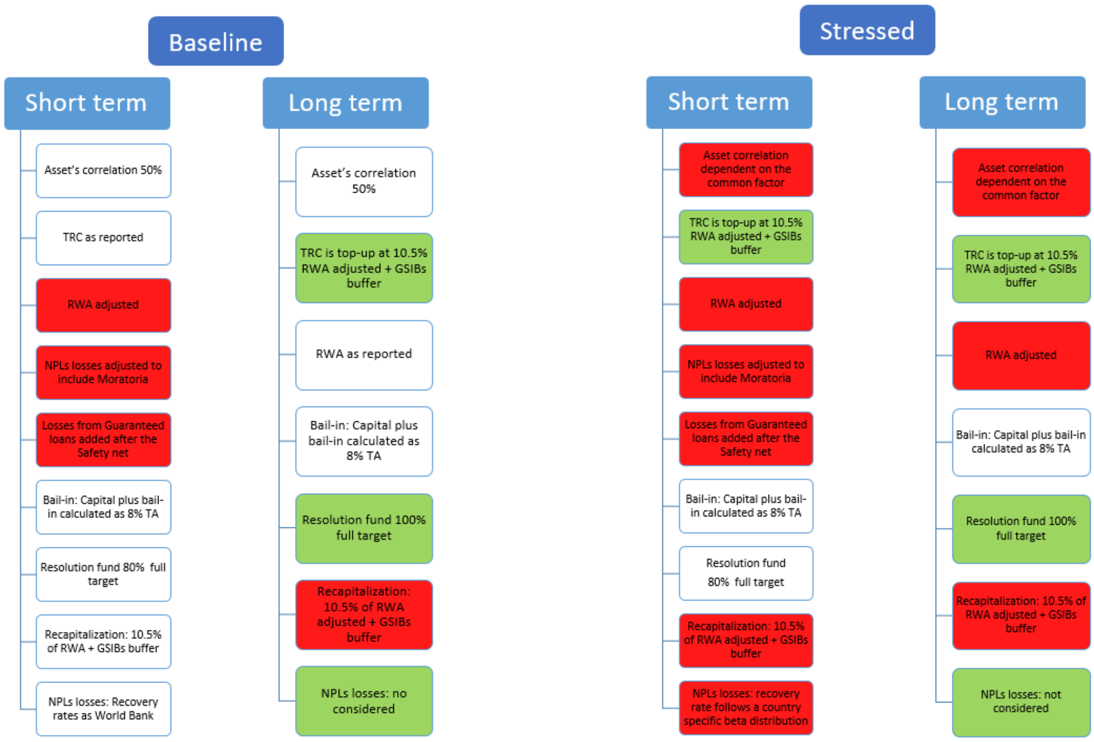
([164]) The absolute levels of the probabilities reported in the heatmap are not to be interpreted as actual probabilities, but rather theoretical probabilities derived from the modelling framework.

Table A9.4: **Theoretical probability of public finances being hit by more than 3%, 5% or 10% of GDP, in the event of a severe crisis (i.e. involving excess loses and recapitalisation needs in at least three different EU countries)**

	Initial (2023) short term scenarios						Final (2033) long term scenarios					
	Baseline (a)			Stress (b)			Baseline (a)			Stress (b)		
	3% GDP	5% GDP	10% GDP	3% GDP	5% GDP	10% GDP	3% GDP	5% GDP	10% GDP	3% GDP	5% GDP	10% GDP
AT	0.01%	0.00%	0.00%	0.43%	0.15%	0.02%	0.00%	0.00%	0.00%	0.22%	0.08%	0.01%
BE	0.02%	0.01%	0.00%	0.38%	0.15%	0.02%	0.02%	0.01%	0.00%	0.29%	0.11%	0.02%
BG	0.01%	0.00%	0.00%	0.09%	0.02%	0.00%	0.00%	0.00%	0.00%	0.04%	0.01%	0.00%
CY	0.15%	0.07%	0.02%	2.50%	1.36%	0.39%	0.02%	0.01%	0.00%	0.30%	0.15%	0.04%
CZ	0.01%	0.00%	0.00%	0.14%	0.05%	0.01%	0.01%	0.00%	0.00%	0.12%	0.04%	0.01%
DE	0.01%	0.00%	0.00%	0.12%	0.04%	0.00%	0.00%	0.00%	0.00%	0.10%	0.03%	0.00%
DK	0.19%	0.09%	0.03%	0.55%	0.28%	0.08%	0.05%	0.03%	0.01%	0.50%	0.26%	0.08%
EE	0.00%	0.00%	0.00%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%
ES	0.15%	0.05%	0.00%	1.28%	0.59%	0.12%	0.06%	0.02%	0.00%	0.67%	0.32%	0.06%
FI	0.03%	0.02%	0.00%	0.29%	0.14%	0.03%	0.02%	0.01%	0.00%	0.23%	0.11%	0.03%
FR	0.06%	0.02%	0.00%	0.65%	0.32%	0.08%	0.02%	0.01%	0.00%	0.45%	0.21%	0.06%
GR	0.11%	0.02%	0.00%	1.50%	0.50%	0.04%	0.01%	0.00%	0.00%	0.23%	0.08%	0.01%
HR	0.00%	0.00%	0.00%	0.09%	0.02%	0.00%	0.00%	0.00%	0.00%	0.07%	0.02%	0.00%
HU	0.02%	0.00%	0.00%	0.12%	0.03%	0.00%	0.01%	0.00%	0.00%	0.11%	0.03%	0.00%
IE	0.06%	0.03%	0.00%	0.65%	0.33%	0.10%	0.03%	0.02%	0.00%	0.32%	0.18%	0.06%
IT	0.06%	0.02%	0.00%	0.79%	0.30%	0.05%	0.02%	0.01%	0.00%	0.29%	0.12%	0.02%
LT	0.01%	0.00%	0.00%	0.03%	0.01%	0.00%	0.00%	0.00%	0.00%	0.02%	0.01%	0.00%
LU	1.45%	0.72%	0.15%	5.62%	3.55%	1.46%	0.36%	0.19%	0.05%	3.26%	2.15%	0.97%
LV	0.00%	0.00%	0.00%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.02%	0.00%	0.00%
MT	0.04%	0.02%	0.01%	0.46%	0.23%	0.05%	0.01%	0.01%	0.00%	0.22%	0.11%	0.03%
NL	0.08%	0.03%	0.01%	0.59%	0.29%	0.07%	0.02%	0.01%	0.00%	0.43%	0.23%	0.06%
PL	0.02%	0.00%	0.00%	0.80%	0.17%	0.01%	0.01%	0.00%	0.00%	0.22%	0.06%	0.00%
PT	0.07%	0.02%	0.00%	1.18%	0.50%	0.08%	0.03%	0.01%	0.00%	0.43%	0.19%	0.03%
RO	0.00%	0.00%	0.00%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.00%	0.00%
SE	0.03%	0.01%	0.00%	0.07%	0.03%	0.01%	0.01%	0.01%	0.00%	0.06%	0.03%	0.01%
SI	0.00%	0.00%	0.00%	0.21%	0.06%	0.00%	0.00%	0.00%	0.00%	0.15%	0.03%	0.00%
SK	0.04%	0.00%	0.00%	0.71%	0.15%	0.00%	0.01%	0.00%	0.00%	0.23%	0.06%	0.00%

(1) Green: low risk (probability lower than 0.50%); Yellow: medium risk (probability between 0.50% and 1%); Red: high risk (probability higher than 1%).
Source: Commission services.

Graph A9.2: **Schematic representation of the scenarios**



Source: Commission services

Table A9.5: **Detailed scenarios description**

Components: / *Scenario:*	Asset correlation	TRC	RWAs	Bail-in	National/ Single RF	Recapitalization	Extra losses due to NPLs	Deposit Guarantee Scheme	Banks in resolution
Initial Baseline (2023) Short term	50%	K	RWA Adjusted	Yes Capital plus bail-in 8% TA	Yes, 5% TA cap, after LAC of 8% has been called in 8/10 of full target No ex-post contributions	10.5% RWA Adjusted + Buffers	- Yes to all banks - NPL including loans under moratoria - RR as reported by World Bank	No	Random significant banks
Initial Stressed (2023) Short term	Depending on common factor	K	RWA Adjusted	Yes Capital plus bail-in 8% TA	Yes, 5% TA cap, after LAC of 8% has been called in 8/10 of full target No ex-post contributions	10.5% RWA Adjusted + Buffers	- Yes to all banks - NPL including loans under moratoria - RR follows a country specific beta distribution depending on the size of the shock	No	Random significant banks
Final Baseline (2033) Long term	50%	K	RWA	Yes Capital plus bail-in 8% TA	Yes, 5% TA cap, after LAC of 8% has been called in No ex-post contributions	10.5% RWA + Buffers	No	No	Random significant banks
Final Stressed (2033) Long term	Depending on common factor	K	RWA	Yes Capital plus bail-in 8% TA	Yes, 5% TA cap, after LAC of 8% has been called in No ex-post contributions	10.5% RWA + Buffers	No	No	Random significant banks

(1) The size of the Single Resolution Fund was on Q2 2021 €52 billion (https://www.srb.europa.eu/en/content/single-resolution-fund#build-up) which is around 65% of its target size (i.e. 1% of deposits, around €80 billion)
Source: Commission services.