Demographics and Real Interest Rates Across Countries and Over Time*

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Abstract

We develop a tractable multicountry general equilibrium model with imperfect capital mobility to explore the implications of demographic trends for the evolution of real interest rates across countries and over time. We calibrate the model to study how low-frequency movements in a country's real interest rate depend on the interaction between the degree of international financial integration and both domestic and foreign demographic developments. The key insight from the model is that the real interest rate in a more financially integrated country is relatively more sensitive to global developments than to domestic factors. We estimate panel error-correction models relating real interest rates to many of its possible drivers—demographics included—imposing some structure motivated by the lessons from the theory. The empirical results show that global factors and domestic demographic variables are robust determinants of real interest rates. Alternative specifications showcase the importance of accounting for time-varying financial integration and of controlling for a range of possible real rate drivers. Both the model and the empirical analysis suggest demographic trends can account for a non-negligible portion of the global decline in real interest rates. Given available projections, demographic factors should continue to exert downward pressure on real interest rates going forward.

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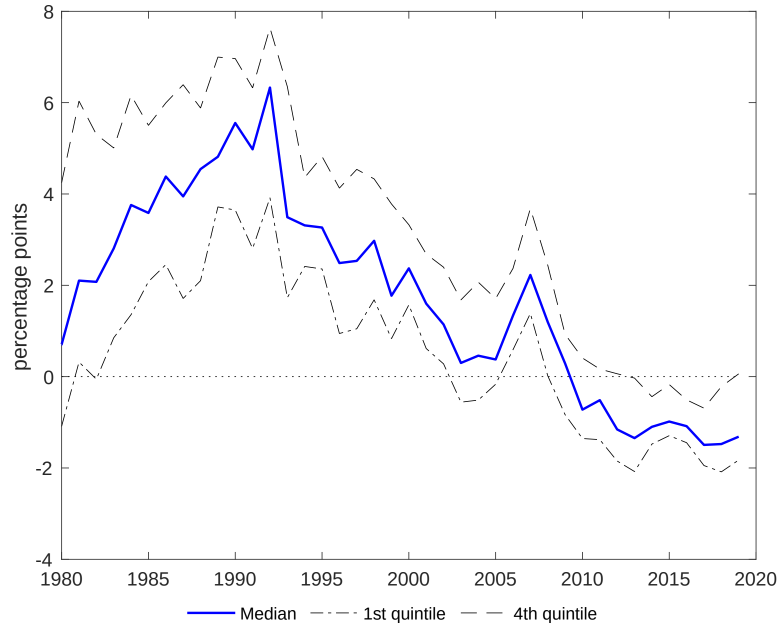
1 Introduction

Figure 1: Ex-ante real short-term interest rates.



Note: Median (solid blue line) and interquintile range (dashed black line) of ex-ante real short-term interest rates for 19 OECD countries between 1979 and 2019. Section 4 describes the data and the calculations in details.

Between 1990 and the onset of the Covid-19 pandemic, real interest rates in advanced economies exhibited a pronounced and persistent decline (Figure 1). Since 2022, however, short-term real rates have increased meaningfully, as central banks worldwide had to tighten monetary policy to fight high inflation. More intriguingly, long-term real rates have also increased markedly, raising the specter of persistently higher real interest rates in the future. Whether or not advanced economies will return to an environment of low real interest rates once central banks manage to tame inflation and normalize monetary policy is a key macroeconomic question, with important implications for both fiscal and monetary policy.

A comprehensive answer to this question ultimately depends on the underlying forces behind the movements of real interest rates. As Blanchard (2023) and Obstfeld (2023) observe, the pattern of the decline prior to the pandemic may indicate a return to a low-rate environment is in fact a plausible conjecture. The basis for this view is the low-frequency nature of the factors that are likely to be important drivers of equilibrium real rates (Rachel and Smith, 2017).[1]

In this paper, we revisit the role of one of the candidate low-frequency drivers of real interest rates: demographic trends. More specifically, we study the role of demographic variables in driving real rates

[1] The views in the literature are far from being one-sided. At the other end of the spectrum, Hamilton et al. (2016) find little evidence that permanent factors account for the decline of global real rates. In their analysis, temporary factors, such as deleveraging, tighter financial regulation, and inflation trends, are the culprit. Borio et al. (2022) argue that over the past one and a half century none of the usual drivers that affect savings and investment appear to matter for the evolution of real rates. Using an even longer sample, with data since the XIV century for a set of developed countries, Rogoff et al. (2024) fail to find a relationship between long-term real rates and output or population growth.

across countries, taking into account the existence of frictions that limit international capital mobility.

The existing literature, including our earlier work (Carvalho et al., 2016), shows past demographic developments and available projections can explain a significant portion of the real interest rate decline observed after 1990. We raise the bar for that conclusion with the observation that if demographics are indeed an important driver of low-frequency movements in real rates, their pattern in cross-sectional and time-series data should align well with demographic developments across countries and over time.

At least two issues, however, make that kind of inference challenging. First, in a world with international capital mobility, a country's real rate should depend not only on its own, but also on global demographic developments. Second, other variables may affect real rates, so that uncovering the role of any given driver requires carefully controlling for other potential explanations. To handle these two issues, we resort to both a novel model and econometric analysis.

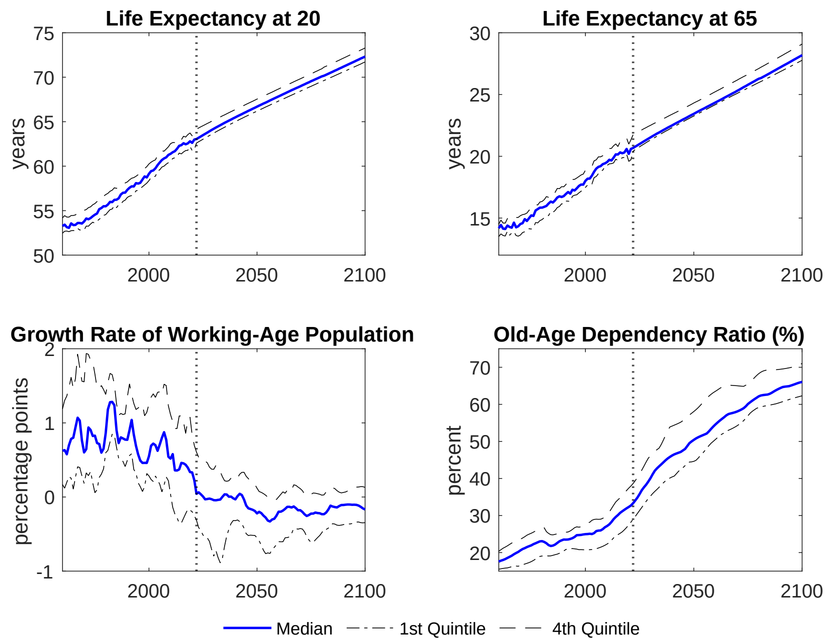
First, we develop a multicountry life-cycle model with imperfect capital mobility to study how domestic and global demographic variables influence a country's real interest rate. Our main finding is demographic trends account for about one third of the fall in real interest rates between 1990 and 2019, and, combined with increased financial integration, about one half of the decline in their dispersion across countries over the same period.[2] Given the available projections, our simulations predict demographic trends will continue to exert downward pressure on real interest rates globally, in line with the aforementioned conjectures in Blanchard (2023) and Obstfeld (2023).

Drawing on the results from the model, we then turn to an empirical analysis of the relationship between demographics and real interest rates. In our preferred empirical specification, the global rate and demographic variables account for between one and a half and two percentage points of the change in real rates averaged across countries between 1979 and 2019. When we focus on the post-1990 sub-sample, the global rate becomes more important, reflecting the increased degree of financial integration. Yet, domestic demographic variables continue to play a meaningful role. Government debt and, especially, pension spending show up as the two factors that have prevented an even larger decline in real interest rates.

The data clearly show advanced economies are aging at a fast pace (Figure 2). Between 1960 and 2020, median life expectancy at 20 has increased by about 9 years, from 53.4 to 62.6 (top-left panel). Over the same period, older generations have also experienced significant longevity gains, with median life expectancy at 65 increasing from 14.2 years to 20.2 years (top-right panel). Meanwhile, fertility rates have fallen sharply, implying a decline in the median growth rate of the working-age population from approximately 1% in 1960 to 0.26% in 2020 (bottom-left panel). The combination of lower fertility and higher longevity has roughly doubled the old-age dependency ratio (the ratio between people 65 years old and above to people 15 to 64 years old) from 15.6 in 1960 to 29.8 in 2020 (bottom-right panel). Going forward, available demographic projections suggest working-age population in advanced economies will

[2] In our model, which features a single good, international real interest rate differentials arise because of differences in their drivers across countries combined with imperfect capital mobility. Models with multiple goods that allow for real exchange rate fluctuations may imply a different mapping between real interest rate differentials and the degree of capital mobility (Kekre and Lenel, 2024).

Figure 2: Demographic variables.



Note: Median (solid blue line) and interquintile range (dashed black line) of four demographic variables. Top-left panel: Years of life expectancy at 20. Top-right panel: Years of life expectancy at 65. Bottom-left panel: Growth rate of working-age population. Bottom-right panel: Number of people 65 and older relative to the number of people 15 to 64 years old. Sample: 19 OECD countries between 1960 and 2100 (projections after 2020). Source: United Nations World Population Prospects (2021 Revision).

contract at a rate of approximately 0.2% per year, while life expectancy will continue to increase. As a result, by the end of this century the dependency ratio will be near 60%.

Our goal is to study how such demographic developments shape real interest rates across countries in a world with imperfect capital mobility. We begin by developing a multicountry life-cycle model in which households can invest in both domestic and foreign assets. Investment in foreign assets is subject to portfolio-holding costs, which proxy for various factors that hinder capital flows in practice. As a result, international capital mobility is imperfect in the model.[3] In our framework, demographic trends affect the equilibrium real interest rate through changes in the growth rate of the labor force and life expectancy. Population composition effects arise endogenously from these two fundamental forces. Crucially, because households can trade assets internationally, demographic developments in one country can affect the others as well.

A calibrated three-country version of the model captures the salient features of the demographic transition in developed economies. We use this framework to study the low-frequency relationship between demographics and real interest rates, and how the degree of financial integration shapes this relationship across countries and over time. In particular, we focus on how a country's real rate depends on its own and on global demographic developments as financial integration changes over time. The main lesson from the

[3]The limiting cases of the two-country version of our model with zero and infinite portfolio-holding costs correspond to Ferrero (2010) and Carvalho et al. (2016), respectively.

model is increased financial integration shifts the sensitivity of a country's real interest rate away from its own demographic developments and towards global determinants. Hence, integration tends to narrow the range of real rates across different countries, in line with the decreased dispersion over time illustrated in Figure 1. As an implication, going forward, developments in the process of international capital markets integration will shape further convergence in real interest rates across countries, or lack thereof (IMF, 2023).

The lessons from the model inform our empirical strategy. Consistently with our theoretical analysis, we take into account that the (time-varying) degree of financial integration should modulate the relative importance of domestic and foreign factors over time. Our empirical findings corroborate the model predictions by highlighting the roles of demographic variables and financial integration in determining real interest rates. Since we focus on low-frequency developments, we estimate panel error-correction models that highlight the long-run relationships between demographics and real interest rates. To address the challenge that factors other than demographics may also affect real interest rates, we augment the regressions with a set of potential drivers motivated by the literature. For the sake of parsimony, we summarize global factors through a measure of the global real interest rate faced by each country, interacted with a measure of its (time-varying) degree of financial integration.[4]

In most specifications we entertain, the coefficients on domestic demographic variables are significant and have the expected sign. Importantly, this result does not hold if we only include demographic variables. This finding showcases the importance of accounting for multiple determinants of real rates, even if the focus is on a specific driver, as in our case. The estimates also show a positive and statistically significant relationship between a country's real rate and the global rate. When we omit financial integration, however, some results change meaningfully. In particular, and perhaps surprisingly, the coefficient on the global rate often turns negative and/or becomes statistically insignificant. This result highlights the importance of accounting for time-varying financial integration in our empirical analysis.

Our paper belongs to a recent wave of research that investigates, both theoretically and empirically, the determinants of real interest rates. A number of existing contributions focus on demographics, calibrating overlapping generation (OLG) models to individual economies, such as the U.S. (Gagnon et al., 2021), the euro area (Kara and von Thadden, 2016; Bielecki et al., 2020), and Japan (Ikeda and Saito, 2014).[5] Our focus on the open economy dimension is closer in spirit to Lisack et al. (2021), Bárány et al. (2023), and, especially, Krueger and Ludwig (2007), who also discuss the interaction between demographics and financial integration. While those contributions consider only the two extreme cases of closed economies

[4] Domestic variables, in turn, are interacted with one minus the degree of financial openness. Federle et al. (2024) adopt a similar specification to estimate the economic effects of war in one country on other regions.

[5] Using a sufficient-statistic approach, Auclert et al. (2021) predict that changes in the age distribution (the compositional effect of aging) will increase wealth-to-GDP ratios, lower asset returns, and widen global imbalances for the rest of this century. Cesa-Bianchi et al. (2023) show that increased longevity, together with the slowdown of productivity growth, explains the secular decline of the global equilibrium real interest rate. Demographic variables feature prominently also among the factors that can explain the secular stagnation hypothesis, both on the demand side (Eggertsson et al., 2019) and on the supply side (Aksoy et al., 2019). Goodhart and Pradhan (2017) express a contrarian view, arguing that demographic trends will contribute to reverting recent observed macroeconomic trends, including for real interest rates. As noted in Carvalho et al. (2016), Blanchard (2023) and Obstfeld (2023), this view neglects the role of increased life expectancy on workers' savings decisions during their employment spell to finance a longer retirement period.

or fully-integrated capital markets, we allow for imperfect capital mobility.[6]

Empirically, Lunsford and West (2019) conclude demographic variables can explain some of the variability in U.S. real interest rates over more than one hundred years, while Fiorentini et al. (2018) highlight the importance of the share of young workers in accounting for the rise and fall of real rates between 1960 and 2016. Our empirical analysis expands on this second paper. We employ an econometric specification that, informed by our structural model, accounts for time-varying financial integration. We also consider a number of additional real rate drivers, such as productivity growth (Holston et al., 2017), fiscal variables (Rachel and Summers, 2019), convenience yields (Del Negro et al., 2017; Del Negro et al., 2019), and inequality (Eggertsson et al., 2019; Mian et al., 2021). The specifications with demographic variables only and the regressions that do not account for time-varying financial openness demonstrate the importance of those two features of our empirical strategy. This approach clearly differentiates our paper from other empirical contributions in the existing literature.

In our model, the real rate is the return on both government bonds, physical capital, and private claims. In practice, these returns differ. As Gomme et al. (2015) have documented for the U.S., while the return on safe assets (primarily government bonds in advanced economies) has declined, the return on risky assets (in particular equity) has remained roughly constant. Reis (2022) finds that this result is robust across countries and to different measures of capital and income. By abstracting from aggregate uncertainty and imperfect competition, our model fails to capture the rise of macroeconomic risk and markups that Farhi and Gourio (2018) and Eggertsson et al. (2021) argue are key drivers of the wedge between the return on equity and the return on government bonds.[7] Therefore, we focus on the comparison between the real interest rate in the model and the real yield on government bonds in the data. This choice is consistent the findings in Davis et al. (2024). In an empirical no-arbitrage macro-finance model, those authors find equilibrium real interest rates and trend inflation rates, rather than bond risk premia, account for the bulk of the variation in bond yields. Moreover, their estimates of equilibrium real interest rates are correlated with demographic variables.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 discusses the calibration and the quantitative experiments that illustrate how the relationship between demographics and real rates varies with the degree of financial integration across countries. Section 4 presents our empirical analysis. Section 5 concludes.

[6]Two other papers that focus on demographics allowing for imperfect capital mobility are Guest (2006), who investigates the effects of declining fertility on savings, and Bonfatti et al. (2022), who characterize capital flows between high-income regions, middle-income regions, and Japan.

[7]Farhi and Gourio (2018) also find a role for the rising importance of intangibles in production. In a calibrated heterogeneous agent life-cycle model with aggregate risk, Kopecky and Taylor (2020) show aging can produce both a downward trend in the real risk-free rate and an increase in the equity risk premium. Marx et al. (2021) also account for both trends in a calibrated OLG model, but argue the divergence between safe and risky rates is due to developments in productivity growth and risk aversion.

2 The Model

This section presents an open-economy life-cycle model with imperfect capital mobility. We build on Gertler (1999) and allow for differential time-varying demographic trends across countries. Portfolio-holding costs, as in Chang et al. (2015), hamper the free flow of capital across countries. The resulting framework nests the closed-economy model of Carvalho et al. (2016) and the open-economy model of Ferrero (2010) as special cases.

2.1 Demographics

The economy consists of $\mathcal{M}$ regions. In each region $m = 1, \dots, \mathcal{M}$, a mass of $(1 - \omega_{mt} + n_{mt})N^w_{mt-1}$ new workers (w) are born in every period t, where N^w_{mt-1} is the number of workers at time $t-1$ and ω_{mt} is the probability a worker remains in the labor force between periods $t-1$ and t. Therefore, the number of workers evolves according to

$$N^w_{mt} = (1 - \omega_{mt} + n_{mt})N^w_{mt-1} + \omega_{mt}N^w_{mt-1} = (1 + n_{mt})N^w_{mt-1},$$

so that n_{mt} is the net growth rate of the labor force.

A worker who exits the labor force becomes a retiree (r). The probability of a retiree surviving between periods $t-1$ and t is γ_{mt}. Therefore, the number of retirees evolves according to

$$N^r_{mt} = (1 - \omega_{mt})N^w_{mt-1} + \gamma_{mt}N^r_{mt-1}.$$

The (old) dependency ratio, $\psi_{mt} \equiv N^r_{mt}/N^w_{mt}$, measures the number of retirees per worker, and evolves according to

$$(1 + n_{mt})\psi_{mt} = (1 - \omega_{mt}) + \gamma_{mt}\psi_{mt-1}. \tag{1}$$

The growth rate of the labor force and the probability of surviving as a retiree are the fundamental demographic factors in the model. In our quantitative exercises, we measure the growth rate of the labor force directly from the data. Conditional on a given retirement age, we then back out the survival probability from the evolution of the dependency ratio using equation (1).

2.2 Retirees

The problem of a retiree r from country m, born in period j, and retired in period k, is

$$V^{rjk}_{mt} = \max_{C^{rjk}_{mt}, \left\{A^{rjk}_{m\ell t}\right\}_{\ell=1}^{\mathcal{M}}} \left[\left(C^{rjk}_{mt}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m\gamma_{mt+1}\left(V^{rjk}_{mt+1}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where V^{rjk}_{mt} is the retiree's value function, C^{rjk}_{mt} denotes the retiree's consumption, $\sigma > 0$ is the elasticity of intertemporal substitution, and $\beta_m > 0$ is the individual discount factor.[8]

The retiree's budget constraint is

$$C^{rjk}_{mt} + \left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta^{rjk}_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] \sum_{\ell=1}^{\mathcal{M}} A^{rjk}_{m\ell t} = \frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^{rjk}_{m\ell t-1} + E^{jk}_{mt}, \tag{3}$$

where $A^{rjk}_{m\ell t}$ are the assets the retiree holds against country ℓ, which pay a gross return $R_{\ell t}$, and $\eta^{rjk}_{m\ell t} \equiv A^{rjk}_{m\ell t}/(\sum_{p=1}^{\mathcal{M}} A^{rjk}_{mpt})$ are portfolio shares. At the beginning of each period, retirees turn their wealth to a perfectly competitive mutual fund that pools the risk of death and pays an extra return equal to the inverse of the survival probability, as in Yaari (1965) and Blanchard (1985). In addition, a retiree receives lump-sum pension benefits E^{jk}_{mt} from the government.

In forming their portfolios, retirees incur a cost (or fee) that depends on the difference between the actual share invested in foreign assets of each country ($\eta^{rjk}_{m\ell t}$) and an exogenous target level ($\bar{\eta}_{m\ell}$), assumed to be independent of type, which pins down the steady-state bilateral net foreign asset positions.[9] Following Chang et al. (2015), we assume that portfolio-holding costs are quadratic and their level depends on a (possibly time-varying) parameter $\Lambda_{m\ell t} \geq 0$. These costs stand in for all the factors that prevent frictionless capital flows and equalization of real interest rates across countries, even after controlling for risk premia.[10]

Appendix A.1 shows that the share a retiree invests in country-p assets (with $p \neq m$) is independent of age and time since retirement ($\eta^{rjk}_{mpt} = \eta^{r}_{mpt}$, $\forall j$ and k), and satisfies

$$\left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta^{r}_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] (R_{pt} - R_{mt}) = \Lambda_{mpt} \left(\eta^{r}_{mpt} - \bar{\eta}_{mp}\right) R_{mt}. \tag{4}$$

In addition, Appendix A.2 shows that the same condition holds for workers. Therefore, retirees and workers invest the same share of their total financial wealth in country-p assets ($\eta^{r}_{mpt} = \eta^{w}_{mpt} = \eta_{mpt}$).

The solution of the optimization problem for a retiree yields a consumption function linear in the sum of total financial wealth and the present discounted value of pension benefits (S^{rjk}_{mt})

$$C^{rjk}_{mt} = \xi^{r}_{mt} \left(\frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^{rjk}_{m\ell t-1} + S^{rjk}_{mt}\right), \tag{5}$$

[8] In our model, all households consume one homogenous good. Sposi (2022) studies demographics and global imbalances in a world with multiple goods, emphasizing the importance of trade frictions. Kekre and Lenel (2024) find that persistent shocks to relative demand, which could include differences in demographics, translate into persistent real interest rate differentials across countries and account for about 75% of the variance of the USD/G10 real exchange rate.

[9] The assumption that the cost applies to total asset holdings keeps the model quite tractable, as will become clear.

[10] When $\Lambda_{m\ell t}$ tends to infinity, the countries in the model become closed, as in Carvalho et al. (2016). Conversely, when frictions are absent, the model corresponds to a multicountry version of Ferrero (2010). One interpretation of our formulation for portfolio costs is that households delegate the management of their portfolios to asset managers who need to pay fees to invest in foreign (and domestic) assets.

where

$$S_{mt}^{jk} = E_{mt}^{jk} + \frac{\gamma_{mt+1} S_{mt+1}^{jk}}{\widetilde{R}_{mt}},$$

and the adjusted gross return $\widetilde{R}_{mt}$ is defined as

$$\widetilde{R}_{mt} \equiv \frac{\sum_{\ell=1}^{\mathcal{M}} \eta_{m\ell t} R_{\ell t}}{1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} (\eta_{m\ell t} - \bar{\eta}_{m\ell})^2}. \tag{6}$$

Finally, the marginal propensity to consume out of total wealth for retirees obeys the Euler equation

$$\frac{1}{\xi_{mt}^{r}} = 1 + \gamma_{mt+1} \beta_m^{\sigma} \widetilde{R}_{mt}^{\sigma-1} \frac{1}{\xi_{mt+1}^{r}}. \tag{7}$$

Like the portfolio shares, the marginal propensity to consume is independent of individual characteristics (birth and retirement age), which allows for easy aggregation across retirees.

2.3 Workers

In maximizing intertemporal utility, workers take into account the probability of retiring. Thus, the continuation value for a worker born in period j is V_{mt+1}^{wj} with probability ω_{mt+1} and V_{mt+1}^{rjt+1} with the complementary probability. The full optimization problem is

$$V_{mt}^{wj} = \max_{C_{mt}^{wj}, \{A_{m\ell t}^{wj}\}_{\ell=1}^{\mathcal{M}}} \left\{ \left(C_{mt}^{wj}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m \left[\omega_{mt+1} V_{mt+1}^{wj} + (1-\omega_{mt+1}) V_{mt+1}^{rjt+1}\right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

subject to

$$C_{mt}^{wj} + \left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{wj} - \bar{\eta}_{m\ell}\right)^2\right] \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^{wj} = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{wj} + W_{mt} - T_{mt}^{j}, \tag{9}$$

where W_t is the real wage and T_{mt}^{j} are lump-sum taxes, which only workers pay.[11]

As already anticipated, all workers optimally choose the same portfolio shares, which also equal the choice of retirees. Workers' consumption is linear in the sum of total financial wealth, the present discounted value of wage income net of taxes ("human wealth"), and the present discounted value of pension benefits

$$C_{mt}^{wj} = \xi_{mt}^{w} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{wj} + H_{mt}^{j} + Z_{mt}^{j} \right), \tag{10}$$

[11] As in Ferrero (2010) and Carvalho et al. (2016), we assume that workers inelastically supply one unit of labor and retirees do not work. Gertler (1999) shows how to relax both these assumptions. With endogenous labor, the optimal response to a declining growth rate of the labor force and an increase in life expectancy would be to supply more hours. Such a behavior of hours worked would be inconsistent with the data for most OECD economies (OECD, 2018).

where human wealth is given by

$$H_{mt}^{j} = W_{mt} - T_{mt}^{j} + \frac{\omega_{mt+1} H_{mt+1}^{j}}{\Omega_{mt+1} \widetilde{R}_{mt}},$$

and the present discounted value of pension benefits for workers is

$$Z_{mt}^{j} = \frac{1}{\Omega_{mt+1} \widetilde{R}_{mt}} \left[(1 - \omega_{mt+1}) \left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}} \right)^{\frac{1}{1-\sigma}} S_{mt+1}^{jt+1} + \omega_{mt+1} Z_{mt+1}^{j} \right].$$

Workers discount both variables by the adjusted gross return $\widetilde{R}_{mt}$, defined in equation (6), times an additional factor that takes into account the probability of retiring and the heterogeneity in the marginal propensity to consume between the two groups

$$\Omega_{mt} \equiv \omega_{mt} + (1 - \omega_{mt}) \left(\frac{\xi_{mt}^{r}}{\xi_{mt}^{w}} \right)^{\frac{1}{1-\sigma}}. \tag{11}$$

Because retirees and workers discount the future at different rates, Ricardian equivalence does not hold in this model, even though taxes are lump-sum.

Finally, as for retirees, the marginal propensity to consume for workers is independent of individual characteristics and evolves according to

$$\frac{1}{\xi_{mt}^{w}} = 1 + \beta_{m}^{\sigma} \left(\Omega_{mt+1} \widetilde{R}_{mt} \right)^{\sigma-1} \frac{1}{\xi_{mt+1}^{w}}, \tag{12}$$

allowing for easy aggregation also across workers.

2.4 Aggregation

Since marginal propensities to consume are independent of individual characteristics and consumption functions are linear, we can aggregate across workers and across retirees by simply adding over individuals in each group.[12]

Aggregate retirees' consumption is given by

$$C_{mt}^{r} = \xi_{mt}^{r} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{r} + S_{mt} \right).$$

Similarly, aggregate workers' consumption is given by

$$C_{mt}^{w} = \xi_{mt}^{w} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{w} + H_{mt} + Z_{mt} \right).$$

[12] In dropping reference to the birth and retirement periods, we use the notation $\int_{r} C_{mt}^{rjk} = N_{mt}^{r} C_{mt}^{rjk} \equiv C_{mt}^{r}$ and $\int_{w} C_{mt}^{wj} = N_{mt}^{w} C_{mt}^{wj} \equiv C_{mt}^{w}$. The same notation applies to asset holdings, human wealth, and the present discounted value of pensions for retirees and workers.

Aggregate consumption in country m is simply the sum of retirees' and workers' consumption

$$C_{mt} \equiv C^w_{mt} + C^r_{mt}.$$

Finally, because of the heterogeneity between workers and retirees over the life cycle, we need to keep track of the distribution of wealth between these two groups. The result that retirees and workers from a given country choose the same portfolio shares is particularly useful in this respect. First, given the total amount of country-p assets held by country-m agents, we define the share held by retirees as

$$\lambda_{mpt} \equiv \frac{A^r_{mpt}}{A^r_{mpt} + A^w_{mpt}}. \tag{13}$$

Second, from the definition of portfolio shares, $A^z_{mpt} = \eta_{mpt} A^z_{mt}$ for $z = \{r, w\}$, where $A^z_{mt} = \sum_{\ell=1}^{\mathcal{M}} A^z_{m\ell t}$ are total asset holdings by either retirees or workers. Using this result in equation (13), we obtain

$$\lambda_{mpt} = \frac{A^r_{mt}}{A^r_{mt} + A^w_{mt}} = \lambda_{mt},$$

that is, the retirees' share of country-p assets held by country-m agents corresponds to the share of wealth accruing to retirees in country m.

In Appendix A.3 we show that we can derive the evolution of the distribution of wealth by combining the budget constraints of retirees and workers to obtain

$$\left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] [\lambda_{mt} - (1 - \omega_{mt+1})] A_{mt}$$
$$= \omega_{mt+1} \left[(1 - \xi^r_{mt}) \lambda_{mt-1} A_{mt-1} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} \eta_{m\ell t-1} + E_{mt} - \xi^r_{mt} S_{mt}\right], \tag{14}$$

where $A_{mt} \equiv A^r_{mt} + A^w_{mt}$ represents the total amount of assets held by country-m residents.

2.5 Firms

A continuum of measure one of perfectly competitive firms operate in each country. Firms hire workers and accumulate capital K_{mt} to produce the single good according to a labor-augmenting Cobb-Douglas technology

$$Y_{mt} = \left(X_{mt} N^w_{mt}\right)^{\alpha} K^{1-\alpha}_{mt-1},$$

where $\alpha \in (0, 1)$ is the labor share and Y_{mt} represents output. Productivity grows exogenously at rate x_{mt}

$$X_{mt} = (1 + x_{mt}) X_{mt-1}.$$

Since we focus on long-run dynamics, we assume no adjustment costs in capital and investment. Firms' investment I_{mt} adds to the capital stock

$$K_{mt} = (1-\delta)K_{mt-1} + I_{mt},$$

where $\delta \in (0,1)$ is the depreciation rate.

The first-order conditions for firms' profit-maximization problem are standard. The one for labor equates the marginal product of labor to the real wage

$$W_{mt} = \alpha \frac{Y_{mt}}{N^w_{mt}}.$$

Since the relative price of capital equals one, the associated first-order condition implies the marginal product of capital net of depreciation equals the real interest rate

$$R_{mt} = (1-\alpha)\frac{Y_{mt+1}}{K_{mt}} + (1-\delta).$$

As we discuss extensively in the next section, demographic variables affect the real interest rate and, therefore, the return on capital.

2.6 Government

In each period, the government issues one-period bonds (B_{mt}) and levies lump-sum taxes on workers (T_{mt}) to fund pension benefits (E_{mt}), wasteful spending (G_{mt}), and to repay maturing debt inclusive of interest to bondholders ($R_{mt-1}B_{mt-1}$). In addition, the government collects two portions of the portfolio fees that households pay. The first corresponds to the fees that foreign investors pay to take a position in country-m assets. The second corresponds to the fees paid by domestic investors that are not associated with foreign asset positions and hence do not accrue to foreign governments.[13]

The resulting government budget constraint is

$$B_{mt} = R_{mt-1}B_{mt-1} + G_{mt} + E_{mt} - T_{mt} - \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2}(\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell mt} - \left[\sum_{\ell \neq m} \sum_{k \neq \ell} \frac{\Lambda_{m\ell t}}{2}(\eta_{m\ell t} - \bar{\eta}_{m\ell})^2 A_{mkt} \right], \tag{15}$$

where the second-to-last term on the right-hand side are the fees that foreigners pay to take a position in country-m assets and the last term corresponds to the fees domestic households pay to take asset positions abroad but are not collected by foreign governments.

We close the model by assuming debt, spending and pensions are exogenous fractions of output

$$G_{mt} = g_{mt}Y_{mt}, \qquad B_{mt} = b_{mt}Y_{mt}, \qquad E_{mt} = e_{mt}Y_{mt}, \tag{16}$$

[13] These additional terms are due to our formulation of portfolio-holding costs, according to which the fees apply to total asset holdings.

so the government budget constraint determines taxes residually. Due to the life-cycle structure, Ricardian Equivalence does not hold in the model. Hence, our choice of fiscal "rules" in (16) is not innocuous. Given our focus on demographics, we consider simple rules that can match the data. We discuss the sensitivity of the real interest rate to government spending, debt and pension spending at the end of the next section.

2.7 Balance of Payments and Equilibrium

Country-m assets held by its residents corresponds to capital and bonds not owned by foreigners

$$A_{mmt} = K_{mt} + B_{mt} - \sum_{\ell \neq m} A_{\ell m t}.$$

Net foreign assets for country m equal the difference between the claims on foreign assets owned by its residents and the claims on country-m assets owned by foreigners

$$F_{mt} \equiv \sum_{\ell \neq m} (A_{m\ell t} - A_{\ell m t}).$$

As customary (see Appendix A.4 for details), the law of motion of net foreign assets follows from consolidating the budget constraints of households and the government

$$F_{mt} = F_{mt-1} + \sum_{\ell \neq m} (R_{\ell t-1} - 1) A_{m\ell t-1} - (R_{mt-1} - 1) \sum_{\ell \neq m} A_{\ell m t-1} - \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 A_{m\ell t} + \sum_{\ell \neq m} \frac{\Lambda_{\ell m t}}{2} (\eta_{\ell m t} - \bar{\eta}_{\ell m})^2 A_{\ell m t} + NX_{mt}, \quad (17)$$

where the trade balance is the difference between production and domestic absorption

$$NX_{mt} \equiv Y_{mt} - C_{mt} - G_{mt} - I_{mt}.$$

Finally, global asset market clearing requires that net foreign assets add up to zero at the world level

$$\sum_{\ell=1}^{\mathcal{M}} F_{\ell t} = 0.$$

Because the labor force and technology grow over time, we focus on a solution for detrended variables.[14] Given exogenous processes for the growth rate of the labor force, life expectancy, the growth rate of technology, and fiscal variables, a competitive equilibrium for the world economy requires that, in each country, retirees and workers maximize utility subject to their budget constraints, firms maximize profits given their technological constraints, the government satisfies its budget constraint, and labor, goods and asset markets clear.

[14]The detrended counterpart of a generic variable D_t is $d_t \equiv D_t/(X_t N_t^w)$. The model admits a well-defined steady state in terms of detrended variables.

Table 1: Demographic variables in the initial steady state (in %).

Parameter	**Young Economy** ($\mathcal{Y}$)	**Old Economy** ($\mathcal{O}$)	**Rest of the World** ($\mathcal{W}$)
Relative size ($N^w_{m0}/N^w_{\mathcal{W}0}$)	0.007	0.007	1
Growth rate of the labor force (n_{m0})	1.13	0.55	0.76
Dependency ratio (ψ_{m0})	20.48	25.08	22.34

We solve the model with an "extended-path" approach, according to which agents form expectations in each period assuming the exogenous processes will remain constant at their current values into the indefinite future. The extended-path solution concatenates the point-wise equilibrium values obtained in each period by solving for the perfect-foresight path from each period onward under those "constant beliefs." Like perfect foresight, the extended-path approach allows us to obtain a fully non-linear solution for the transition between steady states focusing on low-frequency dynamics (that is, abstracting from fluctuations that are the focus of business cycle models with aggregate shocks). Due to the constant-beliefs assumption, however, the extended-path approach avoids the excessive "front-loading" of responses that characterizes the standard perfect-foresight solution.

3 Quantitative Analysis

Our main quantitative experiment analyzes the macroeconomic transition of the world economy between two steady states characterized by differential demographic developments across countries and time-varying degrees of financial integration. For simplicity, we assume that technology growth and fiscal variables are equal across countries (as a share of GDP) and remain constant over time.

3.1 Calibration

Each period corresponds to one year. We calibrate our multicountry model to a world with three regions: a small "young" economy ($\mathcal{Y}$), a small "old" economy ($\mathcal{O}$), and the rest of the world ($\mathcal{W}$). The young economy has a relatively high growth rate of the labor force and a relatively low dependency ratio, while the old economy has the opposite characteristics.

The source for the demographic data is the United Nation (U.N.) World Population Database 2022, as further discussed in Section 4. We assume the initial size of the rest of the world is equal to the sum of the working-age population across the 19 OECD economies in our sample in 1990 (3.78 billion), and set the initial relative sizes of both the young and old economies to $\approx 0.7\%$ of the world economy, which corresponds to the first quintile of the cross-sectional distribution of population sizes in that same year.

Table 1 reports the values for the demographic variables in the initial steady state.[15] The young

[15] The paths for demographic variables, including their initial steady state values, come from an HP filter. See Section 3.2.

Table 2: Common parameters across countries.

Parameter value			Description
ω	=	0.978	Average employment duration
σ	=	0.500	Elasticity of intertemporal substitution
α	=	0.667	Labor share
δ	=	0.100	Depreciation rate
x	=	0.005	Growth rate of productivity
$\bar{\eta}_{m\ell}$	=	0	Target net foreign asset positions
b	=	0.600	Debt/GDP
g	=	0.250	Government spending/GDP
e	=	0.075	Pensions/GDP

economy has a relatively high growth rate of the labor force and a relatively low dependency ratio.[16] We match the initial growth rate of the working-age population and the dependency ratio in this region to the 80^{th} and 20^{th} percentiles of their empirical counterparts, respectively, which correspond to $n_{\mathcal{Y}0} = 1.13\%$ and $\psi_{\mathcal{Y}0} = 20.48\%$. Conversely, for the old economy we target the 20^{th} percentile for the growth rate of the working-age population and the 80^{th} percentile for the dependency ratio ($n_{\mathcal{O}0} = 0.55\%$ and $\psi_{\mathcal{O}0} = 25.08\%$, respectively). Finally, for the rest of the world, we target the population-weighted average of the countries in our sample for both the growth rate of the working-age population and the dependency ratio ($n_{\mathcal{W}0} = 0.76\%$ and $\psi_{\mathcal{W}0} = 22.34\%$).

We follow Gertler (1999) and Carvalho et al. (2016) in setting most remaining parameters that are common to all countries (Table 2). Agents are born workers at age 20. We fix the probability of remaining employed to match an average employment duration of 45 years, so that, on average, individuals retire at age 65. We set the elasticity of intertemporal substitution σ to 0.5, consistent with the evidence in Hall (1988) and Yogo (2004). We further set the labor share α equal to 0.667 and the depreciation rate δ equal to 0.1, which are standard values in the literature. We assume the growth rate of technology to be $x = 0.5\%$, roughly in line with the average growth rate for the countries in our dataset since 1990. For simplicity, we assume zero target shares for foreign asset holdings (*i.e.*, $\bar{\eta}_{m\ell} = 0 \ \forall m, \ell$).[17] Finally, we calibrate fiscal variables (debt, government spending and pensions) as a fraction of GDP to match the average values for OECD countries since 1990, which implies $b = 60\%$, $g = 25\%$ and $e = 7.5\%$, respectively.

The remaining parameters to calibrate are the initial values of the portfolio-holding costs and agents' time-discount factors. We jointly choose these parameters to target real interest rates and external positions in the initial steady state. The real interest rate measure we use is a three-year moving average

[16] Conditional on the growth rate of the working-age population and on the probability of retiring, the steady state version of equation (1), $\psi_m = (1-\omega)/(1+n_m-\gamma_m)$, provides a unique mapping between the dependency ratio of country m and the associated survival probability.

[17] Actual steady-state net foreign asset positions in the model may be positive or negative due to differences in demographic profiles across countries.

Table 3: Discount factors and portfolio-holding costs.

Parameter	Young Economy ($\mathcal{Y}$)	Old Economy ($\mathcal{O}$)	Rest of the World ($\mathcal{W}$)
Discount factor (β_m)	0.992	1.023	1.004
Portfolio-holding costs ($\Lambda_{\mathcal{Y}n0}$)	0	300	19.2
Portfolio-holding costs ($\Lambda_{\mathcal{O}n0}$)	300	0	64.6
Portfolio-holding costs ($\Lambda_{\mathcal{W}n0}$)	19.2	64.6	0

centered in 1990 of the ex-ante short yield (the same data plotted in Figure 1). For external positions, we target gross foreign debt assets and liabilities relative to GDP from Lane and Milesi-Ferretti (2017). The focus on debt aligns well our real interest rate measure with the appropriate asset class in the data. We use gross positions as the empirical counterpart to the model's net foreign asset positions, as the model abstracts from factors that could lead agents in the small young economy to hold foreign assets and agents in the small old economy to borrow from abroad. Given the relative size of the three countries, the bulk of capital flows tend to occur between each small country and the global economy. Hence, for simplicity, we ensure the absence of capital flows between the two small economies by setting high enough portfolio-holding costs between $\mathcal{Y}$ and $\mathcal{O}$. For parsimony, we assume symmetric portfolio-holding cost parameters between each small country and the world economy (*i.e.* $\Lambda_{\mathcal{W}mt} = \Lambda_{m\mathcal{W}t}$, for $m \in \{\mathcal{Y}, \mathcal{O}\}$) and choose their initial values to target the small economies' foreign positions against the global economy in the initial steady state. For the young economy, we match the 80^{th} percentile of the empirical distribution of real interest rates (6.76%) and the 20^{th} percentile of gross foreign debt liabilities relative to GDP (36.76%). For the old economy, we match the 20^{th} percentiles of the distribution of real interest rates (2.37%) and of gross foreign debt assets relative to GDP (15.73%). Finally, for the global economy, we match the median real interest rate (5.12%). Owing to its relative size, the global economy's external position is approximately balanced. Table 3 reports the resulting parameter values.[18]

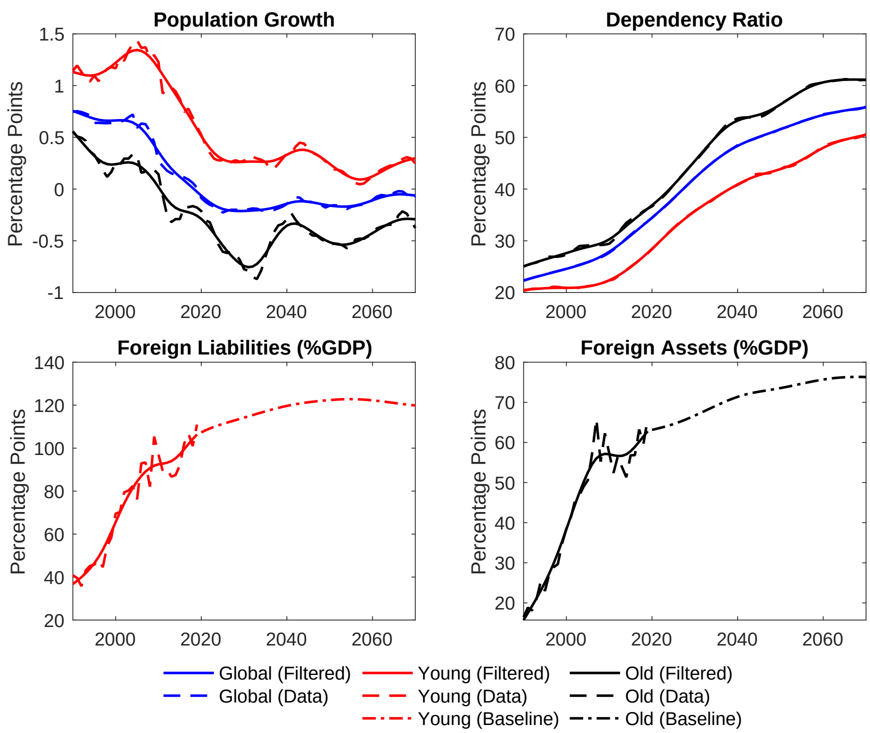
3.2 Experiment

In our baseline experiment, the size of the three economies and the degree of financial integration evolve over time in response to the evolution of the demographic variables and to changes in the portfolio-holding cost parameters. The targets that pin down the paths for these variables are the same as for the steady state (growth rate of working-age population, dependency ratio, and gross foreign debt assets and liabilities relative to GDP). We feed the model with the trend component of an HP filter (Hodrick and Prescott, 1997) applied to the data (see Figure 3).[19] We consider the period 1990-2019 as our "sample." For demographic variables, the simulation uses the trends obtained by combining the sample with U.N. projections until 2070. For foreign assets and liabilities, the in-sample trend comes from the HP filter,

[18] The household problem is well defined even when some discount factors are bigger than one, as long as $\beta_m \gamma_{mt} < 1$ for all regions, which is always the case in our experiments.

[19] We set the HP filter smoothing parameter for both the demographic variables and the financial integration measure to 40, so that the trend component approximates well the evolution of the variables of interest.

Figure 3: Calibration of demographic processes and degree of financial integration.



Note: Growth rate of working-age population (top-left panel), dependency ratio (top-right panel), foreign liabilities (bottom-left), and foreign assets (bottom-right). The red lines correspond to the young economy, the black lines to the old economy, and the blue lines to the rest of the world. The dashed lines are the data, the solid lines are the fitted processes (trends extracted from the data using the HP filter), the dashed-dotted lines in the two bottom panels are projections using the model, under the assumption that portfolio-cost parameters remain constant at their 2019 values.

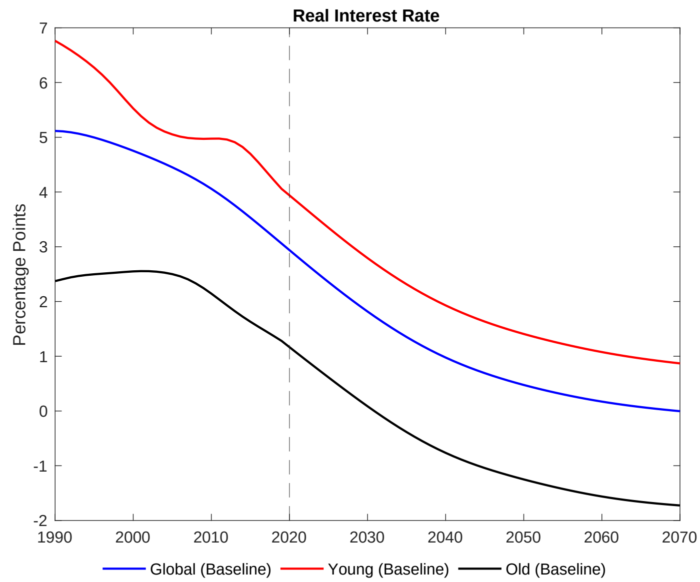
whereas out-of-sample we adjust the last in-sample value of portfolio-holding costs ($\Lambda_{m\ell 2019}$) according to the relative population growth rates of recipient and investor countries (see Section 3.3 below). We refer to the results for the period 2020-2070 as "projections."

3.3 Results

Figure 4 shows the results of the baseline experiment for the period 1990-2070. In sample (1990-2019), the real interest rate of the global economy (solid blue line) falls by more than two percentage points, from its initial value of 5.12% to 2.94%. In the young economy, the decline reaches almost three percentage points (from 6.76% to 3.94%). Over the entire sample, the interest rate also falls in the old economy, by a little more than one percentage point (from 2.37% to 1.17%). Differently from the other two countries, however, the dynamics for the old economy are slightly non-monotonic—before starting to decline, the real interest rate actually increases slightly for over a decade.

These rich dynamics reflect the interaction between the dynamics of demographic variables across countries and financial integration. The initial increase of the real interest rate in the old economy occurs for reasons similar to what happens in a Metzler diagram with two countries that have different discount

Figure 4: Real interest rates in the baseline simulation.



Note: The figure plots the simulated real interest rate from the model (baseline experiment) for the global economy (blue line), the young economy (red line), and the old economy (black line). The vertical line denotes the last period for which data on assets and liabilities in foreign debt are available.

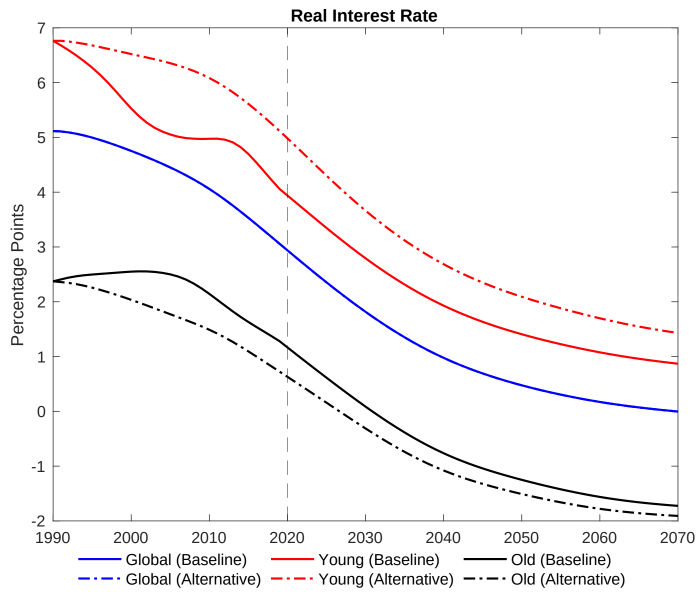
factors and move from autarky to financial integration (Obstfeld and Rogoff, 1996).[20] Initially, when the degree of financial integration is low, real rates in the young economy are higher than in the rest of the world and (even more so) than in the old economy. As the two small open economies become more integrated with the rest of the world, their real interest rates converge toward the global rate, so that the real rate falls for the young economy and increases for the old economy. Over time, as the degree of financial integration becomes sufficiently high, the evolution of domestic and foreign demographic variables start to exert downward pressure on the real interest rate everywhere in the world. Along this process, domestic demographic variables become relatively less important for the determination of the real interest rate in the two small open economies, while foreign demographic variables play an increasingly more central role. Intuitively, the degree of financial integration of the two small economies with the rest of the world represents a time-varying loading on the global demographic factors.

The data on foreign liabilities (bottom left panel of Figure 3) suggest the process of financial integration has slowed down after the global financial crisis of 2008. These dynamics lead to a slight widening of interest rate differentials between the small young economy in the other two regions toward the end of the sample.

Going forward, U.N. projections for demographic variables suggest that the aging process will continue everywhere in the world, with some further decline in the growth rate of the labor force and a progressive

[20]The typical analysis of a Metzler diagram features an instantaneous transition from autarky to full financial integration. In our experiment, such a transition occurs progressively over time, at a pace that is modulated by the portfolio-holding costs.

Figure 5: Real interest rates with constant financial integration.



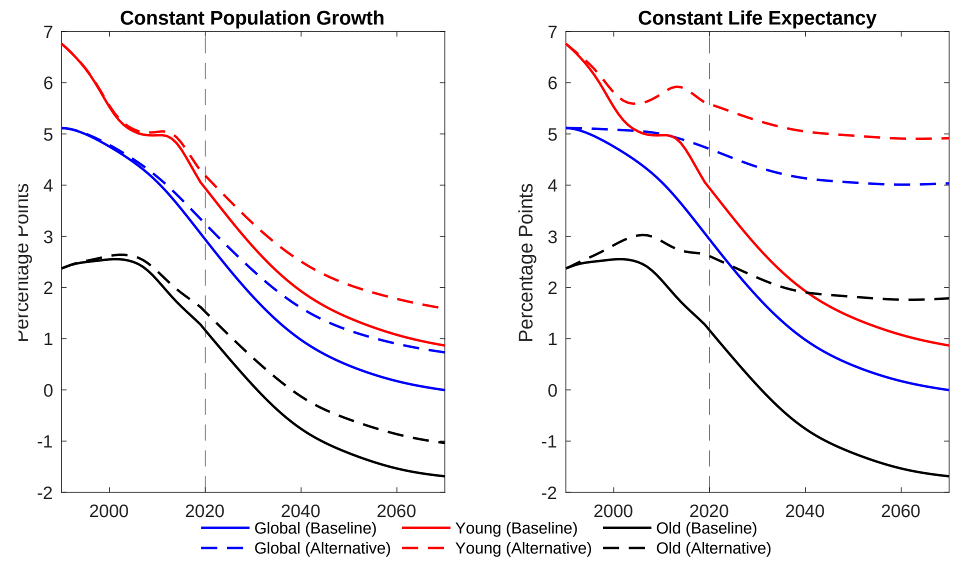
Note: The figure plots the simulated real interest rate from the model for the global economy (blue lines), the young economy (red lines), and the old economy (black lines). Solid lines correspond to the baseline simulation in Figure 4. Dashed-dotted lines are the counterfactual simulation in which the degree of financial integration remains at its initial value.

increase in the dependency ratio. The implication is demographic trends will continue to contribute to lower real interest rates in the foreseeable future. As for financial integration, we can only assess the evolution of foreign assets and liabilities by making further assumptions on the evolution of portfolio-holding costs. Since economies grow at different rates, we choose to adjust the out-of-sample value of these costs based on the relative population growth rates of the two economies.[21] Under this assumption, the model predicts a slight increase in assets and liabilities (dashed-dotted lines in the bottom panels of Figure 3 after 2019). Real rate differentials around the global real interest rate prevailing in 2019 are projected to persist throughout the simulation horizon. Should financial integration experience a renewed acceleration in the future, real interest rates will tend to resume convergence, as observed especially between 1990 and 2005. Conversely, in a “global financial fragmentation” scenario, real rates would tend to diverge.

Figure 5 isolates the role of financial integration in determining the dynamics of real interest rates across regions. Solid lines again depict real interest rates in the three regions from the baseline simulation. Dashed-dotted lines correspond to a simulation in which the initial values of portfolio costs are adjusted to reflect relative population growth rates—*i.e.* the same adjustment applied to the out-of-sample period

[21] Both the values of portfolio-holding costs and relative country sizes influence the degree of openness of each small economy. As small economies grow at different rates compared to the global economy, we adjust portfolio holding costs so as to neutralize this differential growth effect. To that end, costs should decline for the faster-growing young economy and increase for the slower-growing old economy. Formally, if we denote a small economy by m, the adjustment is as follows: $\Lambda_{\mathcal{W}mt} = \Lambda_{\mathcal{W}mt-1} \frac{(1+n_{\mathcal{W}t})}{(1+n_{mt})}$, for $t > 2019$.

Figure 6: Real interest rates in two demographics counterfactuals.



Note: The figure plots the simulated real interest rate from the model for the global economy (blue lines), the young economy (red lines), and the old economy (black lines). The solid lines correspond to the baseline simulation. The dashed-dotted lines are the counterfactual simulations in which the growth rate of the working-age population (left-hand side) and the probability of surviving (right-hand side) remain at their initial values in each country.

of the baseline simulation.[22] As expected, financial integration does not matter for the world economy (country $\mathcal{W}$), and the path of its real interest rate is essentially the same in the two cases. In contrast, financial integration makes a substantial difference for the two small economies. As discussed earlier, the evolution of demographic variables still exerts downward pressure on the real interest rate in both regions. The main difference relative to the baseline simulation is that, with constant financial integration, the real interest rate in the old economy now falls monotonically, and by more. As a result, real interest rates in the two small economies do not converge. With constant (and low) financial integration, domestic demographic developments dominate. Conversely, with increasing financial integration, global demographic trends gain importance over time.

The dashed-dotted lines in Figure 6 show the path of the real interest rate in the three regions in two additional counterfactual simulations. The left-hand side panel presents the result of a simulation in which we fix the growth rate of the working-age population at its initial value. The right-hand side panel displays the experiment in which we hold the probability of surviving constant at its initial value. Focusing on the left-hand side panel, the increase in life expectancy associated with the higher probability of surviving explains the lion's share of the overall effect, with small differences depending on the region. Conversely, in the right-hand side panel, we can see that the lower growth rate of the labor force explains only a small fraction of the overall effect. The intuition for this decomposition is the same as in Carvalho et al. (2016). The increase in life expectancy induces households to save more in anticipation of a longer

[22]See footnote 21 for details.

retirement period. This saving-for-retirement motive is stronger for workers, who face a longer expected lifespan, but also affects retirees since their life expectancy continues to increase even after leaving the workforce.[23] The fall in the growth rate of the working-age population has only a modest effect on the real interest rate because the two effects tend to offset each other. On the one hand, a lower growth rate of the working-age population increases capital per-worker and thus tends to depress asset returns. On the other hand, the reduction in the number of workers implies a change in the composition of the population. Since retirees have a higher marginal propensity to consume, aggregate savings fall and the real interest rate rises. On balance, the first effect associated with the lower growth rate of the working-age population dominates in the simulation, but its quantitative implications are rather small. As a result, the dynamics of real interest rates are dominated by the evolution of life expectancy.

3.3.1 Comparative Statics

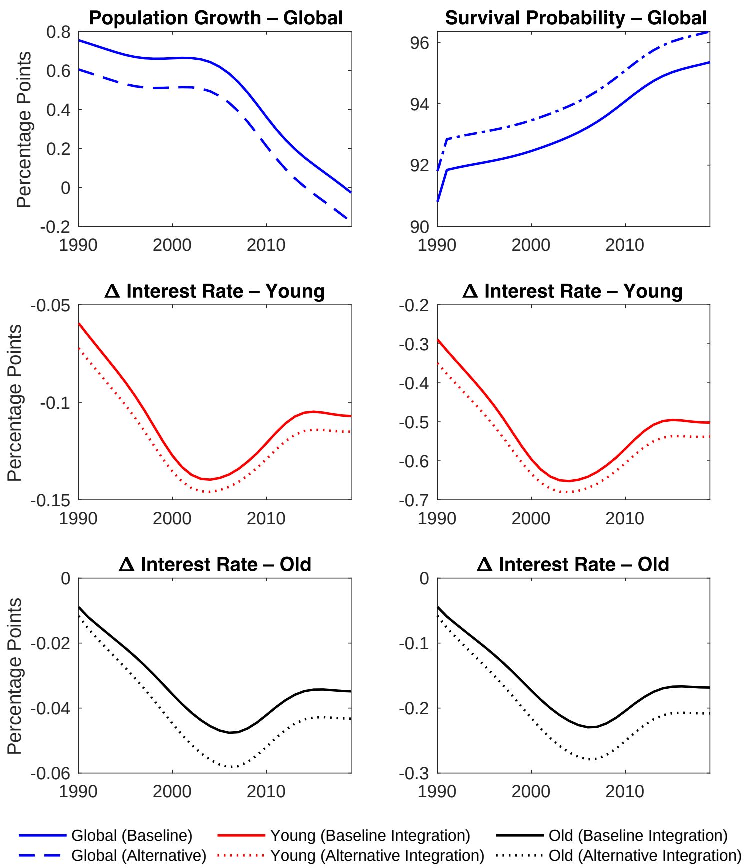
In this section, we perform two comparative-statics exercises to understand how global demographic developments affect the real interest rate in the two small economies, and how this effect depends on the degree of financial integration with the rest of the world. Each exercise perturbs the path of one demographic variable at a time (growth rate of the labor force or probability of surviving) so as to generate an upward shift in the dependency ratio relative to the baseline. We then assess the impact of the perturbed path on the real interest rates of the three regions for different degrees of financial integration.

In the top-left panel of Figure 7, we consider a reduction of the growth rate of the working-age population in the global economy by 15 basis points each year (dashed line) relative to the baseline scenario (solid line). The middle-left panel reports the change of the real interest rate relative to the baseline scenario in the young economy. The solid line keeps the degree of financial integration at the same level as in the main simulation. The dotted line corresponds to a higher degree of financial integration (a 25% reduction of the portfolio-holding cost). The downward shift in the trajectory of global population growth leads to a lower real interest rate in the young economy, and this effect is more pronounced the more financially integrated the country is with the global economy. The bottom-left panel plots the same experiment for the old economy. In this case, the impact of a downward shift of the growth rate of working-age population in the global economy for the domestic real interest rate is smaller. The reason is the old economy is less financially integrated with the global economy than the young one (see Table 3). As a consequence, its sensitivity to demographic developments in the rest of the world is lower.

The top-right panel of Figure 7 shows the effects of an upward shift of the probability of surviving in the global economy by 1 percentage point each year (dashed-dotted line) relative to the baseline scenario (solid line). As in the case of the growth rate of the working-age population, the effect of a higher

[23] An increase in the retirement age would mitigate this effect. In many OECD countries, pension reforms are moving in this direction. In addition, people work for more years, even though the official retirement age has not changed (Scott, 2021). Yet, Carvalho et al. (2016) show that the increase in retirement age necessary to fully offset the consequences of higher life expectancy on the real interest rate is substantial—well above the changes currently being discussed and implemented in most countries.

Figure 7: Real interest rate sensitivity to alternative demographic profiles.



Note: Change in the real interest rate of the small young economy (center) and of the small old economy (bottom) for baseline (solid lines) and high (dotted lines) level of financial integration in response to (i) a larger decline of the growth rate of the labor force in the global economy (dashed line, top-left panel) and (ii) a larger increase of the probability of surviving in the global economy (dashed-dotted line, top-right panel). The solid blue lines in the top panels correspond to the baseline processes for demographic variables.

probability of surviving is stronger in the young economy than in the old economy, and is larger with higher financial integration.

Together with the main counterfactual analysis of Figure 5, both comparative-statics exercises reported in this section reinforce the message that global demographic developments influence the real rate of a small economy progressively more as its financial integration with the rest of the world increases. This point will serve as a guide to the empirical analysis that we present in Section 4.

3.4 Other Determinants of Real Interest Rates

The existing literature has identified a number of factors that contribute to explaining the observed decline of global real interest rates over time. While our analysis focuses on demographic trends, our model features a few additional real interest rate determinants, which we study in this subsection through another set of simple comparative-statics exercises. Perhaps most importantly, these experiments also shed some light on what to expect when we control for those variables in the regressions that we estimate

Table 4: Sensitivity of real interest rate to different factors (small old economy).

Factor	Baseline	Alternative	$\Delta \mathbf{R}^{\mathcal{O}}$ 1990	2020
TFP (x)	0.5%	0.6%	17	11
Debt / GDP (b)	60%	70%	14	8
Government spending / GDP (g)	25%	26%	20	13
Pensions / GDP (e)	7.5%	8.5%	56	36
Retirement age (ω)	65	66	64	31

Note: For each experiment, the first column reports the factor that changes, the second the value in the baseline calibration, the third the value in the alternative calibration, the fourth and the fifth the difference (in basis points) between the real interest rate in the initial steady state (1990) and in 2020, respectively, under the alternative calibration relative to baseline.

in Section 4.

We keep the analysis deliberately simple. The exercises consist of changing the calibrated value of one factor at a time for one of the two small economies and comparing the new real interest rate in the initial steady state (1990) and in 2020 with the one in the baseline simulation, under the same demographic transition. Table 4 reports the results for the old economy.[24] The overarching theme is that the differences tend to dissipate over time. The reason is that, as financial integration increases, domestic factors become less important for the level of the real interest rate.

The first row shows the consequence of increasing TFP growth (x in the model) from 0.5%, as in the baseline calibration, to 0.6%. The real interest rate increases by 17 basis points in the initial steady state and is 11 basis points higher by 2020. The model-implied elasticity of the real interest rate to TFP growth is broadly consistent with some of the recent literature that discusses the importance of the slowdown in trend GDP for the secular decline of the equilibrium real interest rate (Holston et al., 2017; Rachel and Smith, 2017). This experiment could be informative about the potential consequences of the advent of artificial intelligence, which may trigger a dramatic acceleration of TFP growth (Trammell and Korinek, 2023). Policies that incentivize R&D (for example, R&D tax credits) could also boost TFP growth in the future (Minniti and Venturini, 2017).

The next three rows correspond to changes in fiscal variables as a fraction of GDP. In the case of debt (b), we consider a ten percentage point increase relative to the baseline, while for government spending (g) and pensions (e) the increase is one percentage point. In all cases, a more expansionary fiscal policy leads to a higher interest rate.

Following the financial crisis and especially the Covid-19 pandemic, debt-to-GDP ratios have significantly increased in all advanced economies (Romer, 2021). In the model, a 10 percentage point increase in debt/GDP raises the real interest rate by 14 basis points in the initial steady state and 8 basis points

[24] Table A1 in the Appendix reports the results for the small young economy, which are very similar. For completeness, Table A2 presents also the results of similar comparative-statics exercises for population growth and life expectancy for both the old and the young economies.

by 2020. Because of the life-cycle features of the model, government bonds are net wealth for the private sector. Therefore, a higher level of debt relative to GDP supports private sector consumption, and thus contributes to increasing the real interest rate.

The effects of a one percentage point increase in government spending as a fraction of GDP on the real interest rate are slightly larger (20 basis points in the initial steady state and 13 basis points by 2020). In this case, the mechanism works through the crowding out of private consumption and investment. Thus, in the model, expansionary fiscal policy—whether lower taxes or higher spending—raises the real interest rate, as observed in the years that followed the Covid-19 pandemic.

A one percentage point increase in pensions as a fraction of GDP causes a 56 basis points increase in the real interest rate in the initial steady state and a 36 basis points increase by 2020. The increase in pensions has a large effect on the real interest rate (larger than wasteful government spending) because the government transfers resources from agents with lower marginal propensity to consume (workers) to agents with a higher marginal propensity to consume (retirees), and also weakens the incentives to save for retirement.[25] In most advanced economies, reforms to public pensions are in the process of reducing the generosity of existing benefits, given the difficulties in sustaining previous levels of replacement rates (OECD, 2023). Therefore, going forward, the broad global trends associated with pensions may diminish their upward pressure on real interest rates.[26] Overall, our results about fiscal policy are in line with the findings in Rachel and Summers (2019).

The last factor we consider in our analysis of additional real rate determinants is the retirement age.[27] In this experiment, we increase the parameter ω so that the average duration of employment increases to 46 years, compared to 45 in the baseline. This exercise approximates the reforms that governments in many countries are implementing to make their public finances—in particular, their pension systems—more sustainable by decreasing the amounts paid out in pensions and increasing the number of people contributing to the existing pension system. The rise in interest rate in this case is 64 basis points in the initial steady state and 31 basis points by 2020. The intuition for the direction of the effect is that a longer employment span reduces the incentives for workers to save during their working years, and thus diminishes the downward pressure on the real interest rate.

Overall, the message from these comparative statics exercises is that a rebound of TFP growth, different types of fiscal expansions, or an increase in the retirement age are all factors that could contribute to lifting the real interest rate. While the exact magnitudes may depend on the details of the model, we take the direction of the effects and the interaction with financial integration as the main lessons to inform our empirical analysis below.

[25] As we shall see in the next section, pensions are the factor that empirically have contributed the most to higher real interest rates in our sample.

[26] A proper analysis of the effects of pensions on the real interest rate cannot abstract from a full specification of the way in which the government intends to finance any proposed change, and, more broadly, its consequences for the intertemporal government budget constraint. In our experiment, when we change the generosity of the pension system, we keep debt and government spending as a percentage of GDP constant at their steady state value, thus imposing that the fiscal adjustment occurs along the tax margin. We leave a more thorough study of this question for future research.

[27] Carvalho et al. (2016) and Papetti (2021) study the sensitivity of the real interest rate to changes in the retirement age in a closed economy.

4 Empirical Analysis

The analysis presented in the previous section illustrates how demographic variables affect real interest rates in a world of imperfect capital mobility. In particular, the model shows both domestic and global demographics affect the real interest rate of any given country. Furthermore, the relative importance of country-specific and global determinants varies with the degree of financial integration. A more financially integrated country's real rate exhibits a higher sensitivity to global demographic developments. The process of financial integration, coupled with different initial conditions, may generate non-monotonic dynamics in the path of real rates. Finally, size matters too. For given demographics, the larger the relative size of an economy is, the more domestic variables affect the equilibrium of the global economy.

These implications of the calibrated model serve as guidelines for our empirical analysis of the relationship between demographics and real interest rates. To this end, we exploit a panel of countries, thus leveraging variation both across countries and over time.

The model suggests a few features for our panel regressions. First, demographic trends should imply low-frequency movements in real interest rates. We take into account the long-run nature of these relationships by employing a panel error-correction model (panel ECM), which allows for cointegrating relationships between real interest rates and their determinants—in particular, demographic variables.[28] Second, the relative importance of domestic and global factors for a country's real interest rate should vary over time with the degree of financial integration. In light of this consideration, we interact global factors with a measure of the degree of "financial openness" that takes values between zero and one, and domestic factors with its complement (that is, one minus the degree of financial openness).

For the sake of parsimony, we summarize global factors with a measure of the foreign real interest rate faced by each country, interacted with the country's degree of financial openness.[29] We then add country-specific demographic variables and other determinants of real rates, interacted with one minus the degree of financial openness. The resulting panel ECM is

$$\Delta r_{m,t} = \alpha_m + \gamma r_{m,t-1} + \theta\Theta_{m,t-1} r^{\mathcal{W}}_{m,t-1} + \sum_j \psi_j (1-\Theta_{m,t-1}) D_{m,j,t-1} + \sum_k \Psi_k (1-\Theta_{m,t-1}) X_{m,k,t-1}$$
$$+ \lambda\Delta(\Theta_{m,t} r^{\mathcal{W}}_{m,t}) + \sum_j \phi_j \Delta[(1-\Theta_{m,t}) D_{m,j,t}] + \sum_k \chi_k \Delta[(1-\Theta_{m,t}) X_{m,k,t}] + \epsilon_{m,t}, \quad (18)$$

where $r_{m,t}$ is the ex-ante real interest rate of country m, $r^{\mathcal{W}}_{m,t}$ is the global or "world" real interest rate, α_m is a country fixed effect, $\Theta_{m,t}$ is the index of financial openness, $D_{m,j,t}$ includes j demographic variables, $X_{m,k,t}$ collects k other potential determinants of real interest rates, and Δ is the first-difference operator. Based on this specification, the estimated long-run effect of each variable on interest rate corresponds to its estimated coefficient ($\hat{\theta}$, $\hat{\psi}_j$ or $\hat{\Psi}_k$) divided by $-\hat{\gamma}$. The next section explains how we construct the variables that enter the regression.

[28] Our panel ECM approach also shares some features of global VARs (Pesaran et al. 2004, Pesaran et al. 2009).

[29] An alternative would be to include all determinants of global interest rates as regressors, interacted with openness. Due to the relatively small sample, we favor using the foreign real rate as a summary measure.

4.1 Data and Results

We estimate the panel ECM in equation (18) using annual data for a set of 19 OECD countries. Our sample covers the period 1979-2019 and includes Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, the United Kingdom, and the United States.[30] As Figure 1 shows, starting in 1979 allows us to include in the sample a decade during which real interest rates increased, which should mitigate concerns that our results hinge on a continuous downward trend.

We rely on different data sources to construct the variables that enter our regression. The ex-ante short-term real interest rate $r_{m,t}$ equals the difference between the time-t short-term nominal rate $i_{m,t}$ and one-period-ahead expected inflation $\mathbb{E}_t \pi_{m,t+1}$. The data for short-term nominal interest rate are either overnight or three-month official rates (see Table A3 for details). To construct expected inflation, we follow the approach in Hamilton et al. (2016) and calculate the one-year-ahead forecast from AR(1) regressions with rolling 20-year windows.[31] For all countries, we use headline CPI inflation from the OECD.

Using data from Lane and Milesi-Ferretti (2017), we build a measure of financial integration of country m ($LMF_{m,t}$) as the sum of financial assets and liabilities divided by GDP. The index of financial openness we use in the regression ($\Theta_{m,t}$) to weight the global interest rate is a transformation of $LMF_{m,t}$

$$\Theta_{m,t} = \frac{LMF_{m,t}}{100 + LMF_{m,t}},$$

which makes $\Theta_{m,t}$ an index between zero and one, consistent with the specification of equation (18).[32]

The global real interest rate that an individual country faces is a weighted average of all other countries' real interest rates, where the weight associated with each country is the working-age population ($POP_{m,t}$) share adjusted by the index of financial openness

$$r^{\mathcal{W}}_{m,t} = \sum_{\ell \neq m} \left(\frac{\Theta_{\ell,t} POP_{\ell,t}}{\sum_{\ell \neq m} \Theta_{\ell,t} POP_{\ell,t}} \right) r_{\ell,t}.$$

The vector $D_{m,j,t}$ contains two demographic variables: life expectancy ($LE_{m,t}$) and the growth rate of the working-age population. The data source for both variables is the United Nations World Population Prospects 2022. Life expectancy (at age 20) comes straight from the database. We use the data on the number of individuals between 20 and 65 years old to construct the growth rate of the working-age population, given by

$$100 \times \left(\frac{POP_{m,t}}{POP_{m,t-1}} - 1 \right).$$

The model introduced in Section 2 focuses on the effects of demographic trends on real interest rates

[30]The sample excludes countries that experienced episodes of high inflation (above 25% in any given year) between 1970 and 2019.

[31]We estimate the regression $\pi_{m,t} = a_m + b_m \pi_{m,t-1} + \varepsilon_{m,t}$ by OLS and construct expected values as $\mathbb{E}_t \pi_{m,t+1} = \hat{a}_m + \hat{b}_m \pi_{m,t}$.

[32]Appendix C reports results using trade openness as an alternative proxy for international integration (Table A5).

given financial openness. In practice, as discussed in Section 3.4, a number of other forces may contribute to explaining the dynamics of real interest rates. The vector $X_{m,k,t}$ in the panel ECM (18) includes those factors and others studied in the literature.

More specifically, following Holston et al. (2017), the first variable that we consider is TFP growth, which we obtain from the Penn World Tables. Second, in line with the analysis in Rachel and Smith (2017) on the role of fiscal policy, we add government debt as a fraction of GDP (from the AMECO database). Still in relation to fiscal policy, albeit on a different dimension, we also include data on pension spending as a fraction of GDP (from the OECD database) as a separate potential explanatory variable. Papetti (2021) explores the possibility that the generosity of pensions may affect the real interest rate by reducing the incentive of workers to save while employed.[33] Third, we consider versions of our regressions augmented with convenience yields obtained from Del Negro et al. (2019).[34] Since convenience yields are only available for seven major advanced economies, the specifications that include this variable have a smaller sample size. Finally, Mian et al. (2021) have recently highlighted the potential importance of widening inequality in explaining the secular decline of real interest rates. To account for this potential explanation, we also control for the Gini coefficient of each country, which we obtain from the World Bank.

Table 5 presents the main results from the estimation of equation (18). Standard errors are clustered at the country level. The first eight rows report the estimated long-run coefficients from the panel ECM, that is, $\hat{\theta}$, $\hat{\psi}_j$, and $\hat{\Psi}_k$ divided by $-\hat{\gamma}$. Kao cointegration test for panel data (Kao, 1999) suggests strong evidence of cointegration in all specifications.[35] The row 'Lagged real rate' reports the estimated value of $\hat{\gamma}$, which is the coefficient that dictates the speed of adjustment toward the long-run cointegration relationship. For example, an estimated value of $\hat{\gamma} = -0.5$ implies that deviations from the cointegrating relationship have a half-life of one year.

Column (1) considers a basic specification that only includes demographic variables. The global real interest rate is statistically significant at the 1% level, with a point estimate just below 0.7. Life expectancy is statistically significant but enters with the opposite sign relative to the model predictions. The growth rate of working-age population has the right sign but is not statistically significant. Column (2) adds TFP growth, which is not statistically significant and does not meaningfully alter the results, although the sign of its coefficient is consistent with the prediction of our model.

The picture changes once we control for fiscal variables (government debt and pension spending) in column (3). Pension spending, in particular, is significant at the 1% level. The sign is in line with the prediction of the model and the point estimate is economically large and, as we discuss below, consistent

[33] One variable from the comparative-statics exercises in the model that we do not include in the regressions is the retirement age. The reason is that, in each country, the retirement age typically exhibits very little variation over time, so that country fixed effects are likely to pick up its contribution to the real interest rate.

[34] We thank the authors for kindly sharing their series, which are available for Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States.

[35] The test comprises a battery of Dickey-Fuller tests whose null hypothesis is the absence of cointegration among the variables included in the regression. Table A4 in the Appendix reports the values of the test statistic and their respective p-values. See Pedroni (2018) for a recent survey of the literature on panel cointegration.

Table 5: Panel Error Correction Model.

Coefficient on	(1)	(2)	(3)	(4)	(5)	(6)
Global Rate	0.68***	0.66***	0.70***	0.74***	1.00***	1.48***
	(0.17)	(0.17)	(0.13)	(0.14)	(0.20)	(0.20)
Life Expectancy	0.14***	0.14***	-0.24***	-0.17	-0.36***	-0.54*
	(0.04)	(0.04)	(0.06)	(0.19)	(0.09)	(0.28)
Growth Rate of Labor Force	0.24	0.30	6.03***	6.12***	8.95***	11.59***
	(1.02)	(1.01)	(0.98)	(1.10)	(1.51)	(1.49)
TFP Growth		0.49	0.02	-0.14	-0.02	-0.01
		(0.34)	(0.30)	(0.37)	(0.39)	(0.41)
Government Debt			0.03	0.01	0.07**	0.10**
			(0.02)	(0.03)	(0.03)	(0.04)
Pension Spending			2.31***	2.11***	2.12***	2.65***
			(0.41)	(0.59)	(0.53)	(0.80)
Gini Coefficient				-0.05		-0.03
				(0.23)		(0.33)
Convenience Yield					0.67	1.99
					(1.35)	(1.68)
Lagged real rate	-0.31***	-0.32***	-0.46***	-0.50***	-0.53***	-0.68***
	(0.03)	(0.03)	(0.03)	(0.04)	(0.06)	(0.06)
Kao test	R***	R***	R***	R***	R***	R***
R^2	0.24	0.24	0.39	0.36	0.53	0.55
Adjusted R^2	0.21	0.22	0.35	0.31	0.48	0.48
Observations	743	743	505	445	206	169
Clusters	19	19	19	18	7	7

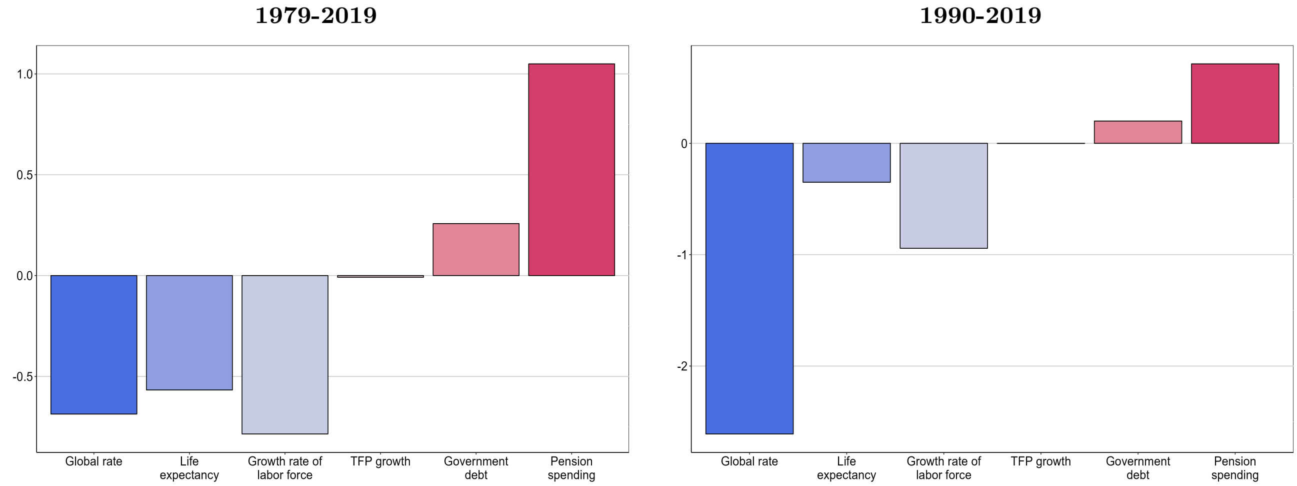
Note: Results from the estimation of equation (18). Standard errors (in parenthesis) are clustered at the country level. The row 'Kao test' reports the result from a battery of Dickey-Fuller tests whose null hypothesis is the absence of cointegration among the variables included in the regression (Kao, 1999). 'R'/'DNR' stands for 'reject'/'do not reject' the null hypothesis, respectively. One (*), two (**) and three (***) asterisks indicate statistical significance at the 10, 5 and 1% level, respectively.

with the quantitative prediction of our calibrated model. More importantly, both life expectancy and the growth rate of the labor force become significant at the 1% level and have the sign predicted by the model.[36] Columns (4) and (5) introduce the Gini coefficient and the convenience yield, respectively. Neither is statistically significant. With the inclusion of the Gini coefficient, life expectancy loses statistical significance, although the sign and magnitude of the coefficient remains comparable to column (3). The inclusion of convenience yields significantly reduces the number of observations, due to a smaller number of countries for which the variable is available.[37] Results, however, remain comparable to those obtained in column (3). The coefficient on life expectancy is negative and again statistically significant at the 1% level. In addition, government debt also becomes significant (at the 5% level), with a coefficient of the expected

[36]The inclusion of fiscal variables reduces the number of observations, as some data are missing for a few countries in the sample. In order to ensure that the difference in the sign and significance of the coefficients do not simply reflect the different sample, we reestimate regressions (1) and (2) with the same sample as in regression (3). Table A6 shows that, qualitatively, the results from this exercise are consistent with those in the baseline specification.

[37]Table A7 shows that the baseline results in specifications (1) to (5) are stable even with this more restricted sample of seven countries.

Figure 8: Decomposition of the change in real interest rates averaged across countries.



Note: Contribution of the explanatory variables (in percentage points) to the change in real interest rates averaged across countries (left panel: full sample; right panel: post-1990 sample) based on specification (3) in Table 5.

sign.[38] Finally, column (6) reports the regression with both the Gini coefficient and the convenience yield, which are again not statistically significant. In this case, the results are largely comparable to column (5). The main difference is that the coefficient on life expectancy is larger in magnitude, although statistically significant only at the 10% level.

Because of the interaction with the degree of financial openness (for the global real interest rate) or with its complement (for country-specific variables), the estimated coefficients in Table 5 literally measure the elasticity of the real interest rate in country m to the interacted factors. However, we can alternatively interpret the same numbers as country-specific and time-varying coefficients that apply to the original (that is, non-interacted) factors. This second perspective is useful because we can more clearly quantify the contribution of each factor to the dynamics of real interest rates over time. Figure 8 provides an illustration of this alternative interpretation. The figure reports the contribution of each factor included in regression (3) of Table 5 to the change in the real interest rate averaged across countries over the full (left panel) and post 1990 (right panel) samples. More precisely, we construct the contribution of each factor in four steps. First, we calculate the change of each factor country by country over the sample period. Second, we compute the average degree of openness and its complement also country by country. Third, we multiply the estimated coefficient in the regression by the numbers obtained in the first two steps so as to obtain a decomposition country by country. In the last step, we take the simple cross-country average of the values obtained in the third step.

The left panel of Figure 8 shows that global factors account for about 75 basis points of the change

[38] One possible explanation for this finding is that the convenience yield may be sensitive to the supply of government debt. This interpretation is arguably consistent with the role of government debt as a safe asset (Caballero et al., 2017). Using a cross-country state-space model, Ferreira and Shousha (2023) find the global supply of safe assets to be a major determinant of real interest rates in a sample of 11 advanced countries between 1960 and 2019.

in real interest rates averaged across countries over the whole sample. In the post-1990 sample (right panel), the contribution of global factors increases to almost three percentage points due to the larger change in real interest rates and the increase in financial integration. The quantitative importance of domestic demographic factors is roughly the same (between one and one and a half percentage points) in the full sample and in the post-1990 period.[39] Government debt, and especially pension spending are the two factors that push real interest rates in the opposite direction. According to our decomposition, the observed change in pension spending as a fraction of GDP contributes to a higher real interest rates averaged across countries—by just above one percentage point over the whole sample and by about 80 basis points post-1990. In both the full sample and the post-1990 period, the contribution of the observed change in government debt to the change in real interest rates averaged across countries is about one quarter of a percentage point.

A simple back-of-the-envelope calculation shows the estimated contribution of pension spending is consistent with the comparative-statics result reported in Table 4. The calculation entails scaling the estimated contribution to account for the fact that changes in pension spending/GDP over our sample period do not coincide with the arbitrary difference used to calculate the impact reported in Table 4. The average across countries of the change in pension spending between 1990 and the end of our sample is around 1.5% of GDP, as opposed to 1% of GDP in the comparative-statics exercise we performed with the model. Therefore, we should scale the estimated contribution of around 80 basis points by 1/1.5, which yields approximately 53 basis points. This number is within the 1990–2020 range of 36–56 basis points reported in Table 4. The same is true for the scaled estimated contribution of government debt. While our comparative-statics analysis assumed a 10 percentage point higher debt/GDP, the cross-country average increase observed in our sample was around 27% of GDP.[40] Hence, the estimated contribution of around 25 basis points scales to approximately 9 basis points, which is also well within the range of 8–14 basis points reported in Table 4.

On balance, the panel error correction model suggests a robust link between the global rate and domestic real interest rates. Once we control for fiscal variables, the coefficients on life expectancy and the growth rate of working-age population become significant and broadly consistent across specifications.[41] TFP growth is never statistically significant, while government debt becomes significant only once we also add the convenience yield. Pension spending is consistently significant in all specifications. Lastly, neither the Gini coefficient nor the convenience yield are statistically significant, whether included separately or together.[42] As in the model, available projections suggest demographic variables will continue to exert

[39]Empirically, the total effect of the decline in the growth rate of the working-age population is larger than the effect of the increase in life expectancy. This finding is in contrast with the model results. A possible reason is that, in practice, individuals may not fully realize (or act upon) the increase in life expectancy predicted by available demographic projections.

[40]The data on government debt are more sparse, so the window over which we computed the change displays some variation across countries. We excluded New Zealand from the calculations because its debt series only starts in 2002 in our dataset.

[41]We have also experimented with a number of additional specifications that include only a subset of the control variables at a time. While the results are broadly robust, we have found cases in which statistical significance or at times the sign of coefficients change. In general, controlling for both government debt and pension spending is important for the statistical significance and the sign of the coefficient on life expectancy, while introducing even just one of those two variables is sufficient to make the coefficient on the growth rate of working-age population positive and statistically significant.

[42]The convenience yield is statistically significant if introduced without fiscal variables. In this case, all other variables

Table 6: Panel ECM without interaction with measure of openness.

Coefficient on	(1)	(2)	(3)	(4)	(5)	(6)
Global Rate	0.32**	0.30**	-0.23	-0.36***	-0.24	-0.21
	(0.13)	(0.13)	(0.15)	(0.13)	(0.21)	(0.17)
Life Expectancy	-0.76***	-0.72***	-1.49***	-1.43***	-0.95***	-1.07***
	(0.14)	(0.13)	(0.16)	(0.16)	(0.30)	(0.31)
Growth Rate of Labor Force	-0.58	-0.41	0.58	0.04	1.64**	1.16*
	(0.35)	(0.36)	(0.41)	(0.36)	(0.74)	(0.63)
TFP Growth		0.31**	0.24*	0.05	0.34*	0.16
		(0.14)	(0.14)	(0.14)	(0.20)	(0.19)
Government Debt			0.01	-0.02**	0.01	-0.02*
			(0.01)	(0.01)	(0.01)	(0.01)
Pension Spending			0.47**	-0.03	0.71**	0.05
			(0.21)	(0.20)	(0.28)	(0.26)
Gini Coefficient				0.08		-0.00
				(0.09)		(0.14)
Convenience Yield					-1.90**	-0.53
					(0.85)	(0.78)
Lagged real rate	-0.33***	-0.34***	-0.42***	-0.53***	-0.48***	-0.66***
	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.06)
Kao test	R***	R***	R***	R***	R***	R***
R^2	0.24	0.24	0.30	0.31	0.50	0.54
Adjusted R^2	0.21	0.21	0.25	0.26	0.44	0.46
Observations	743	743	505	445	206	169
Clusters	19	19	19	18	7	7

Note: Results from the estimation of equation (19). Standard errors (in parenthesis) are clustered at the country level. The row 'Kao test' reports results from a battery of Dickey-Fuller tests whose null hypothesis is the absence of cointegration among the variables included in the regression (Kao, 1999). 'R'/'DNR' stands for 'reject'/'do not reject' the null hypothesis, respectively. One (*), two (**) and three (***) asterisks indicate statistical significance at the 10, 5 and 1% level, respectively.

downward pressure on global real interest rates. Fiscal variables, in particular debt and pensions, are likely to be the main offsetting factors.

The last step in our empirical investigation is to document the importance of accounting for time-varying financial integration in the regressions. Table 6 reports the results for the same specifications as in Table 5, but without interacting the explanatory variables with the degree of financial openness

$$\Delta r_{m,t} = \alpha_m + \gamma r_{m,t-1} + \theta r^{\mathcal{W}}_{m,t-1} + \sum_j \psi_j D_{m,j,t-1} + \sum_k \Psi_k X_{m,k,t-1} + \lambda \Delta r^{\mathcal{W}}_{m,t} + \sum_j \phi_j \Delta D_{m,j,t} + \sum_k \chi_k \Delta X_{m,k,t} + \epsilon_{m,t}. \quad (19)$$

For most variables, the values and statistical significance of the associated coefficients are less stable across specifications. Life expectancy is the only variable that is consistently significant across specifications, with the correct sign. The global rate ceases to be statistically significant once we add fiscal

are not statistically significant, except for life expectancy, which however enters with the wrong sign.

variables and the convenience yield, except in the specification in which we include the Gini coefficient, although in this case the coefficient is actually negative. The growth rate of the labor force is significant only in two out of the six specifications. Among the control variables, TFP growth is significant in three out of five specifications, at the 5% level. The coefficients for pension spending are significant in two out of four specifications and for the convenience yield in one out of two, with signs are in line with our model and the literature. Conversely, government debt is significant in two out of four specifications, but with the wrong sign. The Gini coefficient remains insignificant as in Table 5.

Overall, the comparison between the results reported in Tables 5 and 6 highlights the importance of taking the degree of financial integration into account, as suggested by our model.

5 Conclusions

The demographic trends that most advanced economies are facing are a natural explanation for the prolonged decline of global real interest rates observed between 1990 and 2020. In this paper, we have explored the interaction of these trends with the process of increasing financial integration that took place globally over the same period.

First, we have developed a multicountry, general-equilibrium model with imperfect capital mobility and differential demographic trends. A calibrated three-country version of the model highlights how low-frequency movements in a country's real interest rate depend on its own as well as on global demographic developments. The weight on global demographic variables is increasing in the degree of global financial integration. Conversely, domestic demographic developments matter less for the real interest rate of a highly financially integrated country.

Drawing on the lessons from the model, we have then estimated several specifications of a panel error-correction model that relates real interest rates to demographic variables and other possible drivers, interacted with a measure of financial integration. A "world" real interest rate, which summarizes global factors, is consistently significant in all specifications. Nevertheless, domestic demographic variables remain important determinants of real interest rates, along with pension spending.

Conditional on available projections, both our model and our empirical analysis predict demographics will continue to exert downward pressure on real rates over the next fifty years or so. Fiscal policy represents the main upside risk to the forecasts for real interest rates. The potential discontinuities associated with technological developments make any prediction associated with productivity growth highly uncertain. Finally, the recent wave of geopolitical instability has increased the risks of financial fragmentation, which could hinder further convergence of real interest rates.

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Appendix

A Model Derivations

This section presents the derivations of retirees' and workers' problems.

A.1 Retirees

Retirees maximize (2) subject to (3). After substituting the constraint into the objective function, we can rewrite the unconstrained maximization problem as

$$V_{mt}^{r} = \max_{\{A_{m\ell t}^{r}\}_{\ell=1}^{\mathcal{M}}} \left\{ \left[\frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{r} + E_{mt} - \left(1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{nm} \right)^{2}}{2} \right) \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^{r} \right]^{\frac{\sigma-1}{\sigma}} + \gamma_{mt+1} \beta_{m} (V_{mt+1}^{r})^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}.$$

The first-order condition with respect to foreign assets (A_{mpt}^{r}, $p \neq m$) is

$$\left[\left(1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right)^{2} \right) - \sum_{\ell \neq m} \Lambda_{m\ell t} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right) \eta_{m\ell t}^{r} + \Lambda_{mpt} \left(\eta_{mpt}^{r} - \bar{\eta}_{mp} \right) \left(1 - \eta_{mpt}^{r} \right) \right] \left(C_{mt}^{r} \right)^{-\frac{1}{\sigma}} = \beta_{m} \gamma_{mt+1} \left(V_{mt+1}^{r} \right)^{-\frac{1}{\sigma}} \frac{\partial V_{mt+1}^{r}}{\partial A_{mpt}^{r}},$$

while the first-order condition with respect to domestic assets (A_{mmt}^{r}) is

$$\left[\left(1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right)^{2} \right) - \sum_{\ell \neq m} \Lambda_{m\ell t} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right) \eta_{m\ell t}^{r} \right] \left(C_{mt}^{r} \right)^{-\frac{1}{\sigma}} = \beta_{m} \gamma_{mt+1} \left(V_{mt+1}^{r} \right)^{-\frac{1}{\sigma}} \frac{\partial V_{mt+1}^{r}}{\partial A_{mmt}^{r}}.$$

By the Envelope Theorem, the partial derivatives above are

$$\frac{\partial V_{mt+1}^{r}}{\partial A_{mpt}^{r}} = \left(V_{mt+1}^{r} \right)^{\frac{1}{\sigma}} \left(C_{mt+1}^{r} \right)^{-\frac{1}{\sigma}} \frac{R_{pt}}{\gamma_{mt+1}}, \quad \forall p = 1, ..., \mathcal{M}. \tag{A.1}$$

Substituting (A.1) into the first-order conditions above gives

$$\left[\left(1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right)^{2} \right) - \sum_{\ell \neq m} \Lambda_{m\ell t} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right) \eta_{m\ell t}^{r} + \Lambda_{mpt} \left(\eta_{mpt}^{r} - \bar{\eta}_{mp} \right) \left(1 - \eta_{mpt}^{r} \right) \right] \left(C_{mt}^{r} \right)^{-\frac{1}{\sigma}} = \beta_{m} R_{pt} (C_{mt+1}^{r})^{-\frac{1}{\sigma}}, \tag{A.2}$$

and

$$\left[\left(1+\sum_{\ell\neq m}\frac{\Lambda_{m\ell t}}{2}\left(\eta^{r}_{m\ell t}-\bar{\eta}_{m\ell}\right)^{2}\right)-\sum_{\ell\neq m}\Lambda_{m\ell t}\left(\eta^{r}_{m\ell t}-\bar{\eta}_{m\ell}\right)\eta^{r}_{m\ell t}\right]\left(C^{r}_{mt}\right)^{-\frac{1}{\sigma}}=\beta_{m}R_{mt}(C^{r}_{mt+1})^{-\frac{1}{\sigma}}. \tag{A.3}$$

Dividing (A.2) by (A.3) and rearranging yields:

$$\left[1+\sum_{\ell\neq m}\frac{\Lambda_{m\ell t}}{2}\left(\eta^{r}_{m\ell t}-\bar{\eta}_{m\ell}\right)^{2}\right]\left(R_{pt}-R_{mt}\right)=\Lambda_{mp}\left(\eta^{r}_{mpt}-\bar{\eta}_{mp}\right)R_{mt},$$

which correspond to equation (4) in the main text.

Next, if we multiply equation (A.2) by η^{r}_{mmt} and equation (A.3) by $\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}\eta^{r}_{m\ell t}$, and we add them up, we obtain the Euler equation for the optimal path of consumption of retirees

$$C^{r}_{mt+1}=\left[\frac{\beta_{m}\sum_{\ell=1}^{n}\eta^{r}_{m\ell t}R_{\ell t}}{1+\sum_{\ell\neq m}\frac{\Lambda_{m\ell t}}{2}\left(\eta^{r}_{m\ell t}-\bar{\eta}_{m\ell}\right)^{2}}\right]^{\sigma}C^{r}_{mt}. \tag{A.4}$$

In order to find the difference equation for the marginal propensity to consume out of wealth for retirees, we substitute the retirees budget constraint (3) into the policy function (5). After rearranging, we obtain

$$\frac{1-\xi^{r}_{mt}}{\xi^{r}_{mt}}C^{r}_{mt}=\left[1+\sum_{\ell\neq m}\frac{\Lambda_{m\ell t}}{2}\left(\eta^{r}_{m\ell t}-\bar{\eta}_{m\ell}\right)^{2}\right]\sum_{\ell=1}^{\mathcal{M}}A^{r}_{m\ell t}+\frac{\gamma_{mt+1}S_{mt+1}}{\widetilde{R}_{mt}},$$

where the present discounted value of pension benefits to retirees S_{mt} and the adjusted return $\widetilde{R}_{mt}$ are defined in the text. Replacing for current consumption from the Euler equation (A.4) , we obtain

$$\frac{1-\xi^{r}_{mt}}{\xi^{r}_{mt}}C^{r}_{mt+1}(\beta_{m}\widetilde{R}_{mt})^{-\sigma}=\left[1+\sum_{\ell\neq m}\frac{\Lambda_{m\ell t}}{2}\left(\eta^{r}_{m\ell t}-\bar{\eta}_{m\ell}\right)^{2}\right]\sum_{\ell=1}^{\mathcal{M}}A^{r}_{m\ell t}+\frac{\gamma_{mt+1}S_{mt+1}}{\widetilde{R}_{mt}}$$

Finally, we can substitute the guess of the consumption function at $t+1$ for C^{r}_{mt+1} to obtain

$$\frac{1-\xi^{r}_{mt}}{\xi^{r}_{mt}}\xi^{r}_{mt+1}\left(\frac{1}{\gamma_{mt+1}}\sum_{\ell=1}^{\mathcal{M}}R_{\ell t}A^{r}_{m\ell t}+S_{mt+1}\right)(\beta_{m}\tilde{R}_{mt})^{-\sigma}$$
$$=\left[1+\sum_{\ell\neq m}\frac{\Lambda_{m\ell t}}{2}\left(\eta^{r}_{m\ell t}-\bar{\eta}_{m\ell}\right)^{2}\right]\sum_{\ell=1}^{\mathcal{M}}A^{r}_{m\ell t}+\frac{\gamma_{mt+1}S_{mt+1}}{\widetilde{R}_{mt}}$$

Dividing by $\sum_{\ell=1}^{\mathcal{M}}A^{r}_{m\ell t}$ and using the definition of η_{mpt} allows us to obtain (7) in the text.

Finally, we guess and verify that the value function is linear in the level of consumption:

$$V^{r}_{mt}=\Delta^{r}_{mt}C^{r}_{mt}.$$

Substituting the guess into the functional equation (2) together with the Euler equation (A.4) to eliminate

C^r_{mt+1}, we obtain

$$\Delta^r_{mt} C^r_{mt} = \left[\left(C^r_{mt}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m \gamma_{mt+1} \left(\Delta^r_{mt+1}\right)^{\frac{\sigma-1}{\sigma}} \left(\beta_m \widetilde{R}_{mt}\right)^{\sigma-1} \left(C^r_{mt}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

We can simplify the last expression by eliminating the terms in C^r_{mt}. After rearranging, we obtain

$$\frac{1}{\left(\Delta^r_{mt}\right)^{\frac{\sigma-1}{\sigma}}} = 1 - \gamma_{mt+1} \beta^\sigma_m \widetilde{R}^{\sigma-1}_{mt} \left(\frac{\Delta^r_{mt+1}}{\Delta^r_{mt}} \right)^{\frac{\sigma-1}{\sigma}}. \tag{A.5}$$

Comparing (A.5) with the difference equation for the marginal propensity to consume (7), we can see that

$$\Delta^r_{mt} = \left(\xi^r_{mt}\right)^{-\frac{\sigma}{\sigma-1}}.$$

A.2 Workers

The workers' problem is to maximize (8) subject to (9). After substituting the constraint into the objective, the unconstrained maximization problem becomes

$$V^w_{mt} = \max_{\{A^w_{m\ell t}\}^{\mathcal{M}}_{\ell=1}} \left\{ \left[\sum^{\mathcal{M}}_{\ell=1} R_{\ell t-1} A^w_{m\ell t-1} + W_{mt} - T_{mt} - \left(1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{nm}\right)^2 \right) \sum^{\mathcal{M}}_{\ell=1} A^w_{m\ell t} \right]^{\frac{\sigma-1}{\sigma}} \right.$$
$$\left. + \beta_m \left[\omega_{mt+1} V^w_{mt+1} + (1-\omega_{mt+1}) V^{rt+1}_{mt+1} \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}.$$

The first-order condition with respect to country-p assets (with $p \neq m$) is

$$\left[\left(1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 \right) - \sum_{\ell \neq m} \Lambda_{m\ell t} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right) \eta^w_{m\ell t} + \Lambda_{mpt} \left(\eta^w_{mpt} - \bar{\eta}_{mp}\right) \left(1 - \eta^w_{mpt}\right) \right] \left(C^w_{mt}\right)^{-\frac{1}{\sigma}}$$
$$= \beta_m \left[\omega_{mt+1} V^w_{mt+1} + (1-\omega_{mt+1}) V^r_{mt+1} \right]^{-\frac{1}{\sigma}} \left[\omega_{mt+1} \frac{\partial V^w_{mt+1}}{\partial A^w_{mpt}} + (1-\omega_{mt+1}) \frac{\partial V^r_{mt+1}}{\partial A^r_{mpt}} \right], \tag{A.6}$$

while the first-order condition with respect to domestic assets is

$$\left[\left(1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 \right) - \sum_{\ell \neq m} \Lambda_{m\ell t} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right) \eta^w_{m\ell t} \right] \left(C^w_{mt}\right)^{-\frac{1}{\sigma}}$$
$$= \beta_m \left[\omega_{mt+1} V^w_{mt+1} + (1-\omega_{mt+1}) V^r_{mt+1} \right]^{-\frac{1}{\sigma}} \left[\omega_{mt+1} \frac{\partial V^w_{mt+1}}{\partial A^w_{mmt}} + (1-\omega_{mt+1}) \frac{\partial V^r_{mt+1}}{\partial A^r_{mmt}} \right]. \tag{A.7}$$

As for retirees, we use the Envelope Theorem to calculate the partial derivatives above

$$\frac{\partial V^w_{mt+1}}{\partial A^w_{mpt}} = \left(V^w_{mt+1}\right)^{\frac{1}{\sigma}} \left(C^w_{mt+1}\right)^{-\frac{1}{\sigma}} R_{pt}. \tag{A.8}$$

To solve the workers' problem, we need to guess the functional form of the value function at this stage.

Like for retirees, we conjecture that the value function is linear in consumption and the slope is the same function of the marginal propensity to consume

$$V^w_{mt} = \Delta^w_{mt} C^w_{mt}, \quad \text{with} \quad \Delta^w_{mt} = (\xi^w_{mt})^{-\frac{\sigma}{\sigma-1}}. \tag{A.9}$$

By substituting equation (A.8) and the guess (A.9) into equation (A.7) we get

$$\left[\left(1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right) - \sum_{\ell \neq m} \Lambda_{m\ell t} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right) \eta^w_{m\ell t}\right] (C^w_{mt})^{-\frac{1}{\sigma}}$$
$$= \beta_m \left[\omega_{mt+1} \Delta^w_{mt+1} C^w_{mt+1} + (1 - \omega_{mt+1}) \Delta^r_{mt+1} C^r_{mt+1}\right]^{-\frac{1}{\sigma}} \left[\omega_{mt+1} \left(\Delta^w_{mt+1}\right)^{\frac{1}{\sigma}} + (1 - \omega_{mt+1}) \left(\Delta^r_{mt+1}\right)^{\frac{1}{\sigma}}\right]. \tag{A.10}$$

Multiplying both sides of (A.10) by $(\Delta^w_{mt+1})^{\frac{1}{\sigma}}$ and rearranging yields

$$\left[\omega_{mt+1} C^w_{mt+1} + (1 - \omega_{mt+1}) \frac{\Delta^r_{mt+1}}{\Delta^w_{mt+1}} C^r_{mt+1}\right]^{\frac{1}{\sigma}}$$
$$= \frac{\beta_m \left[\omega_{mt+1} + (1 - \omega_{mt+1}) \left(\frac{\Delta^r_{mt+1}}{\Delta^w_{mt+1}}\right)^{\frac{1}{\sigma}}\right] R_{mt} \left(C^r_{mt}\right)^{\frac{1}{\sigma}}}{\left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] - \sum_{\ell \neq m} \Lambda_{m\ell t} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right) \eta^w_{m\ell t}}.$$

Using the solution for the value function of retirees and the guess for the value function of workers, we can rewirte the last expression as

$$\left[\omega_{mt+1} C^w_{mt+1} + (1 - \omega_{mt+1}) \left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{\sigma}{1-\sigma}} C^r_{mt+1}\right]^{\frac{1}{\sigma}}$$
$$= \frac{\beta_m \left[\omega_{mt+1} + (1 - \omega_{mt+1}) \left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{1}{1-\sigma}}\right] R_{mt} \left(C^r_{mt}\right)^{\frac{1}{\sigma}}}{\left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] - \sum_{\ell \neq m} \Lambda_{m\ell t} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right) \eta^w_{m\ell t}}. \tag{A.11}$$

Following the same steps for equation (A.6), we obtain

$$\left[\omega_{mt+1} C^w_{mt+1} + (1 - \omega_{mt+1}) \left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{\sigma}{1-\sigma}} C^r_{mt+1}\right]^{\frac{1}{\sigma}} =$$
$$\frac{\beta_m \left[\omega_{mt+1} + (1 - \omega_{mt+1}) \left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{1}{1-\sigma}}\right] R_{pt} \left(C^r_{mt}\right)^{\frac{1}{\sigma}}}{\left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] - \sum_{\ell \neq m} \Lambda_{m\ell t} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right) \eta^w_{m\ell t} + \Lambda_{mp}(\eta^w_{mpt} - \bar{\eta}_{mp})}. \tag{A.12}$$

Dividing equation (A.11) by equation (A.12) shows that workers choose asset shares according to the same condition as retirees

$$\left[1+\sum_{\ell\neq m}\frac{\Lambda_{m\ell t}}{2}\left(\eta^{w}_{m\ell t}-\bar{\eta}_{m\ell}\right)^{2}\right](R_{pt}-R_{mt})=\Lambda_{mpt}\left(\eta^{w}_{mpt}-\bar{\eta}_{mp}\right)R_{mt},$$

which implies $\eta^{w}_{mpt}=\eta^{r}_{mpt}=\eta_{mpt}\ \forall p$.

We can find the Euler equation for workers' consumption following the same steps we did for retirees and get

$$\omega_{mt+1}C^{w}_{mt+1}+(1-\omega_{mt+1})\left(\frac{\xi^{r}_{mt+1}}{\xi^{w}_{mt+1}}\right)^{\frac{\sigma}{1-\sigma}}C^{r}_{mt+1}=\left(\beta_m\Omega_{mt+1}\widetilde{R}_{mt}\right)^{\sigma}C^{r}_{mt}, \tag{A.13}$$

with Ω_{mt} defined in (11) in the text.

Next, we substitute the guesses for the policy functions, (5) and (10), for C^{r}_{mt+1} and C^{w}_{mt+1}, respectively, in (A.13) to obtain

$$\omega_{mt+1}\xi^{w}_{mt+1}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t}A^{w}_{m\ell t}+H_{mt+1}+Z_{mt+1}\right)+(1-\omega_{mt+1})\left(\frac{\xi^{r}_{mt+1}}{\xi^{w}_{mt+1}}\right)^{\frac{1}{1-\sigma}}\xi^{w}_{mt+1}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t}A^{r}_{m\ell t}+S^{r}_{mt+1}\right)$$
$$=\left(\beta_m\Omega_{mt+1}\widetilde{R}_{mt}\right)^{\sigma}\xi^{w}_{mt}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A^{w}_{m\ell t-1}+H_{mt}+Z_{mt}\right).$$

Dividing this expression by ξ^{w}_{mt+1} and $\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}R_{\ell t}$ gives

$$\omega_{mt+1}\left(\sum_{\ell=1}^{\mathcal{M}}A^{w}_{m\ell t}+\frac{H_{mt+1}+Z_{mt+1}}{\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}R_{\ell t}}\right)+(1-\omega_{mt+1})\left(\frac{\xi^{r}_{mt+1}}{\xi^{w}_{mt+1}}\right)^{\frac{1}{1-\sigma}}\left(\sum_{\ell=1}^{\mathcal{M}}A^{r}_{m\ell t}+\frac{S_{mt+1}}{\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}R_{\ell t}}\right)$$
$$=\left[\frac{\beta_m\Omega_{mt+1}}{1+\sum_{\ell\neq m}\frac{\Lambda_{m\ell t}}{2}\left(\eta^{w}_{m\ell t}-\bar{\eta}_{m\ell}\right)^{2}}\right]^{\sigma}\left(\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}R_{\ell t}\right)^{\sigma-1}\frac{\xi^{w}_{mt}}{\xi^{w}_{mt+1}}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A^{w}_{m\ell t-1}+H_{mt}+Z_{mt}\right).$$

Note that, for a worker who retires, $\sum_{\ell=1}^{\mathcal{M}}A^{w}_{m\ell t}=\sum_{\ell=1}^{\mathcal{M}}A^{r}_{m\ell t}$. Therefore, we can simplify the previous equation using the workers' budget constraint as to obtain

$$\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A^{w}_{m\ell t-1}+W_{mt}+T_{mt}-C^{w}_{mt}+\frac{\omega_{mt+1}(H_{mt+1}+Z_{mt+1})}{\Omega_{mt+1}\widetilde{R}_{mt}}+\frac{(1-\omega_{mt+1})\left(\frac{\xi^{r}_{mt+1}}{\xi^{w}_{mt+1}}\right)^{\frac{1}{1-\sigma}}S_{mt+1}}{\Omega_{mt+1}\widetilde{R}_{mt}}$$
$$=\beta_m^{\sigma}\left(\Omega_{mt+1}\widetilde{R}_{mt}\right)^{\sigma-1}\frac{\xi^{w}_{mt}}{\xi^{w}_{mt+1}}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A^{w}_{m\ell t-1}+H_{mt}+Z_{mt}\right).$$

Substituting the guess for C^{w}_{mt} and using the recursive definitions of H_{mt+1} and Z_{mt+1}, we get the difference equation for the marginal propensity to consume of workers (12).

The last step to characterize the workers' problem is to verify the guess for the value function. After substituting the guess into equation (8) and rearranging, we get

$$\Delta_{mt}^{w} C_{mt}^{w} = \left\{ \left(C_{mt}^{w}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m \left[\omega_{mt+1} C_{mt+1}^{w} + (1-\omega_{mt+1}) \frac{\Delta_{mt+1}^{r}}{\Delta_{mt+1}^{w}} C_{mt+1}^{r} \right]^{\frac{\sigma-1}{\sigma}} \left(\Delta_{mt+1}^{w}\right)^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}. \quad \text{(A.14)}$$

We can then substitute the Euler equation (A.13) into (A.14) and write

$$\Delta_{mt}^{w} C_{mt}^{w} = \left\{ \left(C_{mt}^{w}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m \left[(\beta_m \Omega_{mt+1} \widetilde{R}_{mt})^{\sigma} C_{mt}^{w} \right]^{\frac{\sigma-1}{\sigma}} \left(\Delta_{mt+1}^{w}\right)^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}.$$

Simplifying C_{mt}^{w} from the equation and rearranging leads to

$$\left(\Delta_{mt}^{w}\right)^{\frac{\sigma-1}{\sigma}} = 1 + \beta_m^{\sigma} \left(\Omega_{mt+1} \widetilde{R}_{mt} \right)^{\sigma-1} \left(\Delta_{mt+1}^{w}\right)^{\frac{\sigma-1}{\sigma}}. \quad \text{(A.15)}$$

Comparing equation (A.15) to equation (12) shows that the guess for the policy function is correct provided that

$$\Delta_{mt}^{w} = \left(\xi_{mt}^{w}\right)^{-\frac{\sigma}{\sigma-1}}.$$

A.3 Wealth Distribution

The heterogeneity between workers and retirees requires keeping track of the distribution of wealth between the two groups.

We start by writing the law of motion of the amount of assets held by retirees

$$\sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^{r} = \frac{\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{r} + E_{mt} - C_{mt}^{r}}{1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2} + (1-\omega_{mt+1}) \frac{\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{w} + W_{mt} - T_{mt} - C_{mt}^{w}}{1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2}. \quad \text{(A.16)}$$

From the workers' aggregate budget constraint, we substitute the total amount of workers' assets into the second term of the right-hand side of equation (A.16). Next, we substitute out retirees' consumption, and rewrite retirees and workers' total value of non-human assets as shares of total assets using the definition in the text

$$\lambda_{mt} \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t} - \frac{1-\omega_{mt+1}}{\omega_{mt+1}} (1-\lambda_{mt}) \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}$$
$$= \frac{\lambda_{mt-1} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} + E_{mt} - \xi_{mt}^{r} \left(\lambda_{mt-1} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} + S_{mt} \right)}{1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2}.$$

After rearranging and using the definition of aggregate assets $A_{mt} \equiv \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}$, we obtain equation (14) in the text.

A.4 Net Foreign Assets

Adding budget constraints across workers gives

$$C^w_{mt} + \left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] \sum_{\ell=1}^{\mathcal{M}} A^w_{m\ell t} = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^w_{m\ell t-1} + N^w_{mt} W_{mt} - T_{mt}.$$

Similarly, for retirees, we obtain

$$C^r_{mt} + \left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] \sum_{\ell=1}^{\mathcal{M}} A^r_{m\ell t} = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^r_{m\ell t-1} + E_{mt}.$$

Adding these two expressions yields a consolidated household budget constraint

$$C_{mt} + \left[1 + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t} = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} + N^w_{mt} W_{mt} - T_{mt} + E_{mt}.$$

Substituting the government budget constraint (15) to eliminate taxes and pensions, we get

$$\begin{aligned} C_{mt} + \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t} + \left[\sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t} + R_{mt-1} B_{mt-1} + G_{mt} \\ - \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2} (\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell mt} - \sum_{\ell \neq m} \sum_{k \neq \ell} \frac{\Lambda_{m\ell t}}{2} (\eta_{m\ell t} - \bar{\eta}_{m\ell})^2 A_{mkt} \\ = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} + N^w_{mt} W_{mt} + B_{mt}. \end{aligned}$$

We then use firms' zero-profit condition to replace wages with output and capital terms, break total assets into capital, government bonds and foreign assets. and simplify portfolio-holding cost terms to arrive at

$$\begin{aligned} C_{mt} + K_{mt} + F_{mt} + \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 A_{m\ell t} + R_{mt-1} B_{mt-1} + G_{mt} \\ - \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2} (\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell mt} = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} + Y_{mt} - R_{mt-1} K_{mt-1} + (1-\delta) K_{mt-1}, \end{aligned}$$

where F_{mt} are country m's net foreign assets (NFA), defined in Section 2.7.

Using the law of motion for capital, we can further simplify the previous expression to

$$\begin{aligned} F_{mt} = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} - R_{mt-1} K_{mt-1} - R_{mt-1} B_{mt-1} \\ - \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 A_{m\ell t} + \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2} (\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell mt} + (Y_{mt} - C_{mt} - I_{mt} - G_{mt}). \end{aligned}$$

Adding and subtracting $F_{mt-1} = \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t-1} - K_{mt-1} - B_{mt-1}$ from the previous expression, we obtain.

$$F_{mt} = F_{mt-1} + \sum_{\ell=1}^{\mathcal{M}} (R_{\ell t-1} - 1) A_{m\ell t-1} - (R_{mt-1} - 1)(B_{mt-1} + K_{mt-1})$$
$$- \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 A_{m\ell t} + \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2} (\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell mt} + NX_{mt}.$$

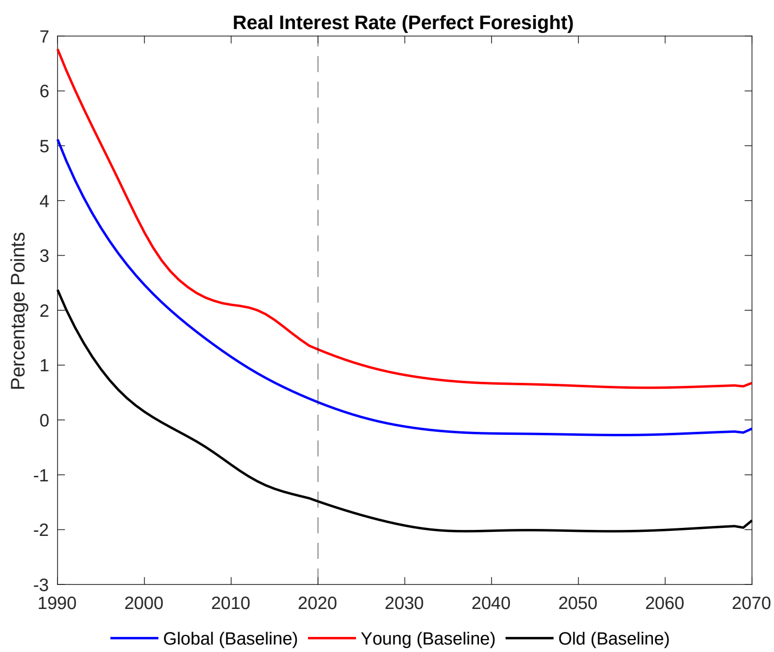
Finally, using the fact that $\sum_{\ell=1}^{\mathcal{M}} A_{\ell mt-1} = K_{mt-1} + B_{mt-1}$, we can write the law of motion of NFA as

$$F_{mt} = F_{mt-1} + \sum_{\ell \neq m} (R_{\ell t-1} - 1) A_{m\ell t-1} - (R_{mt-1} - 1) \sum_{\ell \neq m} A_{\ell mt-1}$$
$$- \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 A_{m\ell t} + \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2} (\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell mt} + NX_{mt},$$

which corresponds to expression (17) in the text.

B Additional Model Results

Figure 9: Baseline simulation under perfect foresight



Note: The figure plots the simulated real interest rate from the model (baseline experiment) for the global economy (blue line), the young economy (red line), and the old economy (black line) under perfect foresight. The vertical line denotes the last period for which data on assets and liabilities in foreign debt are available.

Figure 9 shows the results of the baseline simulation under perfect foresight. With this approach, workers and retirees know the full path of n_{mt} and γ_{mt} at birth. As discussed in the text, solving the model with

Table A1: Sensitivity of real interest rate to different factors (small young economy).

			$\Delta\mathbf{R}^{\mathcal{Y}}$	
Factor	**Baseline**	**Alternative**	**1990**	**2020**
TFP (x)	0.5%	0.6%	14	8
Debt / GDP (b)	60%	70%	22	8
Government spending / GDP (g)	25%	26%	19	7
Pensions / GDP (e)	7.5%	8.5%	63	25
Retirement age (ω)	65	66	64	19

Note: For each experiment, the first column reports the factor that changes, the second the value in the baseline calibration, the third the value in the alternative calibration, the fourth and the fifth the difference (in basis points) between the real interest rate in the initial steady state (1990) and in 2020, respectively, under the alternative calibration relative to baseline.

perfect foresight implies a lot more front-loading. As a consequence, the real interest rate falls faster than in the baseline simulation with the extended path approach. The other key difference is the decline of the real interest rate in the small old economy is monotonic.

Table A1 repeats the exercise reported in Table 4 for the young economy. Table A2 presents the results of similar comparative-statics exercises for population growth and life expectancy for both the small old and young economy. In this case, we design a parallel shift in the paths of each of the two demographic variables (one at a time) relative to the baseline paths discussed in Section 3. In keeping with the spirit of the comparative-statics exercise for the other variables, we choose shifts that induce higher real interest rates: a 0.1% higher growth rate of working-age population, and a lower probability of surviving that decreases life expectancy by one year. The results confirm the relatively low sensitivity of real interest rates to population growth and the high sensitivity to life expectancy.

Table A2: Sensitivity of real interest rate to demographic factors.

		$\Delta\mathbf{R}^{\mathcal{O}}$		$\Delta\mathbf{R}^{\mathcal{Y}}$	
Factor	**Parallel shift**	**1990**	**2020**	**1990**	**2020**
Working-age population growth rate (n)	0.1%	3	-0.2	3	3
Steady state life expectancy ($65 + (1-\gamma)^{-1}$)	-1 year	63	30	62	18

Note: For each experiment, the first column reports the factor that changes, the second the size of the parallel shift in the path relative to the baseline simulations in Section 3, the third and the fourth the difference (in basis points) between the real interest rate in the initial steady state (1990) and in 2020, respectively, under the alternative path relative to baseline for the small old economy, and the fifth and the sixth the same differences for the small young economy.

C Constructing Ex-Ante Real Interest Rates

Table A3: Sources for nominal short-term interest rates used to construct ex-ante real interest rates.

Country	**Source**	**Description**
Australia	World Bank	Lending interest rate
Austria	OECD	1-day central bank yield
Belgium	OECD	3-month interbank yield
Canada	OECD	1-day central bank yield
Denmark	OECD	1-day central bank yield
Finland	OECD	1-day central bank yield
France	OECD	3-month interbank rate
Germany	AMECO	Short term interest rate
Ireland	OECD	3-month interbank rate
Italy	AMECO	Short term interest rate
Japan	OECD	1-day central bank yield
Netherlands	AMECO	Short term interest rate
New Zealand	OECD	3-month bankbill yield
Norway	OECD	3-month interbank yield
Spain	OECD	3-month interbank rate
Sweden	OECD	3-month interbank rate
Switzerland	OECD	3-month interbank loan rate
United Kingdom	OECD	3-month interbank loan rate
United States	IFS	Money market rate

This section describes the sources of the data for short-term nominal interest rates and the way in which we construct ex-ante real interest rates. For each country in our sample, Table A3 reports the source and the maturity of the nominal interest rate i_t used to construct the ex-ante real interest rate r_t, according to

$$r_t = i_t - \mathbb{E}_t \pi_{t+1}.$$

As discussed in the text, we construct expected inflation following Hamilton et al. (2016). Specifically, for each country m, we first estimate a regression of inflation on its own lag with rolling windows of 20 years

$$\pi_{m,t} = a_m + b_m \pi_{m,t-1} + \varepsilon_{m,t}. \tag{C.17}$$

We then calculate the one-year-ahead forecast

$$\mathbb{E}_t \pi_{m,t+1} = \hat{a}_m + \hat{b}_m \pi_{m,t},$$

where $\hat{a}_m$ and $\hat{b}_m$ are the OLS estimates of the coefficients in (C.17). For all countries, we use the headline CPI inflation rate obtained from the OECD.

D Additional Empirical Results

Table A4 reports the statistics of Kao cointegration test (Kao, 1999) with one lag for all regression specifications reported in Tables 5 and 6. The test rejects the null hypothesis of no cointegration at the 1% significance level for all specifications (columns 1 to 6).

Table A4: Kao (1999) cointegration test statistics.

Test	(1)	(2)	(3)	(4)	(5)	(6)
			Table 5			
Modified Dickey–Fuller t	-11.52	-11.51	-9.45	-7.96	-8.63	-6.07
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Dickey–Fuller t	-9.50	-9.47	-7.76	-7.02	-5.38	-4.78
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Augmented Dickey–Fuller t	-7.54	-7.61	-7.18	-6.17	-4.60	-3.53
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Unadjusted modified Dickey–Fuller t	-16.18	-16.14	-10.72	-10.62	-8.31	-7.74
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Unadjusted Dickey–Fuller t	-10.37	-10.34	-8.02	-7.65	-5.34	-5.12
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
			Table 6			
Modified Dickey–Fuller t	-10.16	-12.62	-7.62	-7.81	-6.26	-5.29
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Dickey–Fuller t	-8.76	-9.42	-6.85	-7.06	-4.96	-4.33
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Augmented Dickey–Fuller t	-8.24	-7.61	-5.95	-6.51	-4.18	-3.74
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Unadjusted modified Dickey–Fuller t	-15.57	-16.14	-9.19	-9.81	-7.53	-6.57
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Unadjusted Dickey–Fuller t	-9.91	-10.34	-7.23	-7.53	-5.20	-4.61
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: The rows correspond to the value of the statistics for each test included in Kao (1999)'s cointegration test (with one lag) and their respective p-values (in parenthesis). The columns correspond to the regressions (1)-(6) of Table 5 (top panel) and Table 6 (bottom panel).

Table A5: Panel ECM with trade integration measure.

Coefficient on	(1)	(2)	(3)	(4)	(5)	(6)
Global Rate	2.61***	2.43***	2.48***	2.18***	1.23*	1.21*
	(0.32)	(0.33)	(0.35)	(0.35)	(0.69)	(0.63)
Life Expectancy	0.20*	0.17*	-0.26**	0.03	-0.24*	-0.05
	(0.10)	(0.10)	(0.12)	(0.18)	(0.14)	(0.22)
Growth Rate of Labor Force	0.49	0.58	3.25***	1.75**	4.41***	3.11***
	(0.44)	(0.44)	(0.70)	(0.75)	(1.01)	(1.12)
TFP Growth		0.56**	0.59**	0.39	0.38	0.23
		(0.22)	(0.25)	(0.28)	(0.28)	(0.31)
Government Debt			-0.00	-0.05***	0.01	-0.04*
			(0.01)	(0.02)	(0.02)	(0.02)
Pension Spending			1.57***	1.21***	1.67***	0.87*
			(0.37)	(0.45)	(0.41)	(0.50)
Gini Coefficient				-0.23		-0.34
				(0.19)		(0.24)
Convenience Yield					-3.59***	-3.04***
					(0.90)	(0.92)
Lagged real rate	-0.33***	-0.34***	-0.42***	-0.53***	-0.48***	-0.66***
	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.06)
Kao test	R***	R***	R***	R***	R***	R***
R^2	0.25	0.25	0.31	0.32	0.49	0.49
Adjusted R^2	0.22	0.22	0.27	0.27	0.44	0.41
Observations	742	742	505	445	206	169
Clusters	19	19	19	18	7	7

Note: Results from the estimation of equation (18) with trade integration measured. Standard errors (in parenthesis) are clustered at the country level. The row 'Kao test' reports results from a battery of Dickey-Fuller tests whose null hypothesis is the absence of cointegration among the variables included in the regression (Kao, 1999). 'R'/'DNR' stands for 'reject'/'do not reject' the null hypothesis, respectively. One (*), two (**) and three (***) asterisks indicate statistical significance at the 10, 5 and 1% level, respectively.

Table A5 reports coefficient estimates for the same six specifications as in the baseline analysis, but with a measure of openness based on trade integration. It is defined as

$$\widetilde{\Theta}_{m,t} \equiv \frac{TO_{m,t}}{(100 + TO_{m,t})},$$

where $TO_{m,t}$ is the sum of exports and imports of goods and services as a share of GDP (from the World Bank).

We then re-run our baseline regression replacing $\Theta_{m,t}$ with $\widetilde{\Theta}_{m,t}$

$$\Delta r_{m,t} = \alpha_m + \gamma r_{m,t-1} + \theta \widetilde{\Theta}_{m,t-1} r^{\mathcal{W}}_{m,t-1} + \sum_j \psi_j (1 - \widetilde{\Theta}_{m,t-1}) D_{m,j,t-1} + \sum_k \Psi_k 1 - \widetilde{\Theta}_{m,t-1}) X_{m,k,t-1}$$
$$+ \lambda \Delta(\widetilde{\Theta}_{m,t} r^{\mathcal{W}}_{m,t}) + \sum_j \phi_j \Delta[(1 - \widetilde{\Theta}_{m,t}) D_{m,j,t}] + \sum_k \chi_k \Delta[(1 - \widetilde{\Theta}_{m,t}) X_{m,k,t}] + \epsilon_{m,t}.$$

The results are largely in line with those in the main text. The global rate is always significant. The sign of the coefficient on life expectancy remains mostly in line with the model predictions and the findings of Table 5. This is also the case for the coefficient on the growth rate of the working-age population. TFP growth is significant in two specifications, compared to one in the baseline, but loses statistical significance once we introduce fiscal variables, the Gini coefficient and the convenience yield. Pension spending is always significant and enter with the expected sign. The Gini coefficient is not statistically significant while convenience yield becomes statistically significant and with the expected sign.

Tables A6 and A7 report the results from the baseline specification (Table 5) restricting the number of observations to be the same as in columns (3) and (6), respectively. As columns (1) and (2) of Tables 5 and A6 show, results are qualitatively the same, reinforcing the importance of considering other drivers of real rates in our empirical assessment. Results reported in columns (1) to (4) of Tables 5 and A7 also depict a qualitatively similar message, despite the much more limited number of countries included in the latter table.

Table A6: Panel ECM with number of observations as in specification (3) of Table 5.

Coefficient on	(1)	(2)	(3)	(4)	(5)	(6)
Global Rate	0.15	0.18	0.70***	0.74***	1.00***	1.48***
	(0.18)	(0.18)	(0.13)	(0.14)	(0.20)	(0.20)
Life Expectancy	0.18***	0.18***	-0.24***	-0.17	-0.36***	-0.54*
	(0.04)	(0.04)	(0.06)	(0.19)	(0.09)	(0.28)
Growth Rate of Labor Force	1.98*	1.94*	6.03***	6.12***	8.95***	11.59***
	(1.05)	(1.07)	(0.98)	(1.10)	(1.51)	(1.49)
TFP Growth		0.06	0.02	-0.14	-0.02	-0.01
		(0.38)	(0.30)	(0.37)	(0.39)	(0.41)
Government Debt			0.03	0.01	0.07**	0.10**
			(0.02)	(0.03)	(0.03)	(0.04)
Pension Spending			2.31***	2.11***	2.12***	2.65***
			(0.41)	(0.59)	(0.53)	(0.80)
Gini Coefficient				-0.05		-0.03
				(0.23)		(0.33)
Convenience Yield					0.67	1.99
					(1.35)	(1.68)
Lagged real rate	-0.35***	-0.35***	-0.46***	-0.50***	-0.53***	-0.68***
	(0.03)	(0.03)	(0.03)	(0.04)	(0.06)	(0.06)
Kao test	R***	R***	R***	R***	R***	R***
R^2	0.33	0.33	0.39	0.36	0.53	0.55
Adjusted R^2	0.30	0.29	0.35	0.31	0.48	0.48
Observations	505	505	505	445	206	169
Clusters	19	19	19	18	7	7

Note: Results from the estimation of equation (18) with baseline sample as in specification (3) of Table 5. Standard errors (in parenthesis) are clustered at the country level. The row 'Kao test' reports results from a battery of Dickey-Fuller tests whose null hypothesis is the absence of cointegration among the variables included in the regression (Kao, 1999). 'R'/'DNR' stands for 'reject'/'do not reject' the null hypothesis, respectively. One (*), two (**) and three (***) asterisks indicate statistical significance at the 10, 5 and 1% level, respectively.

Table A7: Panel ECM with number of observations as in specification (6) of Table 5.

Coefficient on	(1)	(2)	(3)	(4)	(5)	(6)
Global Rate	0.95***	0.97***	1.32***	1.32***	1.49***	1.48***
	(0.20)	(0.20)	(0.17)	(0.17)	(0.20)	(0.20)
Life Expectancy	-0.01	0.00	-0.43***	-0.41	-0.57***	-0.54*
	(0.05)	(0.05)	(0.10)	(0.26)	(0.14)	(0.28)
Growth Rate of Labor Force	7.71***	7.59***	10.78***	10.95***	11.61***	11.59***
	(1.28)	(1.31)	(1.35)	(1.34)	(1.50)	(1.49)
TFP Growth		-0.37	-0.05	-0.06	-0.01	-0.01
		(0.45)	(0.39)	(0.39)	(0.41)	(0.41)
Government Debt			0.06*	0.06*	0.09**	0.10**
			(0.03)	(0.04)	(0.04)	(0.04)
Pension Spending			2.33***	2.33***	2.70***	2.65***
			(0.54)	(0.71)	(0.60)	(0.80)
Gini Coefficient				-0.04		-0.03
				(0.31)		(0.33)
Convenience Yield					1.79	1.99
					(1.67)	(1.68)
Lagged real rate	-0.59***	-0.58***	-0.68***	-0.69***	-0.67***	-0.68***
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
Kao test	R***	R***	R***	R***	R***	R***
R^2	0.47	0.47	0.56	0.55	0.55	0.55
Adjusted R^2	0.43	0.42	0.50	0.49	0.49	0.48
Observations	169	169	169	169	169	169
Clusters	7	7	7	7	7	7

Note: Results from the estimation of equation (18) with baseline sample as in specification (3) of Table 5. Standard errors (in parenthesis) are clustered at the country level. The row 'Kao test' reports results from a battery of Dickey-Fuller tests whose null hypothesis is the absence of cointegration among the variables included in the regression (Kao, 1999). 'R'/'DNR' stands for 'reject'/'do not reject' the null hypothesis, respectively. One (*), two (**) and three (***) asterisks indicate statistical significance at the 10, 5 and 1% level, respectively.