Estimating the Output Gap in Real Time: A Factor Model Approach

Knut Are Aastveit[1] Tørres Trovik [2]

[1]Norges Bank and BI Norwegian Business School

[2]KLP

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  - Competing methods.

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- We reduce the total errors of the real time gap to 25 percent of the standard approach.

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  - Main idea: The common, forecastable component of a large data set is captured by a few factors, F.

$$X_t = \Lambda F_t + \xi_t, \qquad \xi_t \sim i.i.d\ N(0, \Psi) \tag{1}$$

$$F_t = AF_{t-1} + Bu_t, \qquad u_t \sim i.i.d\ N(0, I), \tag{2}$$

where $t = 1, ..., T$. $\xi_t = \begin{pmatrix} \xi_{1t}, & \dots, & \xi_{nt} \end{pmatrix}'$, is a vector of non-forecastable idiosyncratic components, Λ is a $(n \times r)$ matrix of factor loadings and r denotes the number of factors.

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  - Estimate Eqs. (1) and (2) using a two-step procedure.
    1. Parameters are estimated by OLS using principal components on balanced part of data
    2. Factors are re-estimated by applying the Kalman filter and smoother to the entire data set

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   - Apply the Mariano and Murasawa (2003) filter: $Z_{it} = (1 + 2L + 3L^2 + 2L^3 + L^4)z_{it}$
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2. Project quarterly GDP growth on the factors

$$\widehat{\Delta y}_{\tau}^{q_0} = \widehat{\alpha} + \widehat{\beta}' \widehat{F}_{\tau}^{q_0} \quad (3)$$

Transform the estimated GDP growth series to log levels, i.e.,

$$\widehat{y}_{\tau}^{q_0} = y_0^{q_0} + \sum_{j=1}^{\tau/3} \widehat{\Delta y}_{3\times j}^{q_0} \quad (4)$$

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3. Obtain an estimate of the output gap by detrending the estimated log level series for GDP, $\widehat{y}_\tau^{q_0}$ using the HP filter

Data set and Empirical exercise

- Data
  - 55 monthly real time indicators for US for period 1970M1-2006M10
    - Real time data from Philadelphia Fed, see Croushore and Stark (2001).
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- Exercise
  - Calculate real time output gaps with and without a factor model.
  - Compare the gaps computed recursively on real time data up to the relevant point in time with a gap using the full sample of data.
  - Real time out-of-sample evaluation from 1984q1 to 2006q4.
    - Performance measured by relative MSFE.
    - Use the vintage of 2010Q3 as "Final vintage"

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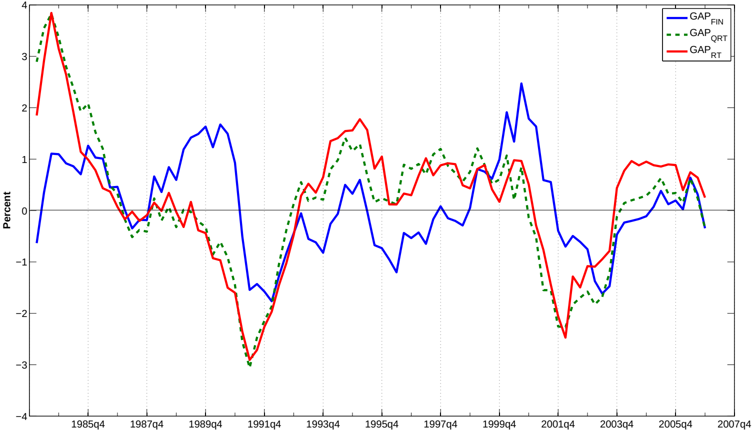
- Total revisions = $\texttt{GAP}_{\texttt{FIN}}$ - $\texttt{GAP}_{\texttt{RT}}$
- Data revisions = $\texttt{GAP}_{\texttt{QRT}}$ - $\texttt{GAP}_{\texttt{RT}}$

Output gaps with standard approach



Benchmarks and Gaps

- **Factor model approach**
  - $\texttt{GAP}_{\texttt{RT-FM}}$: Estimated real time gap
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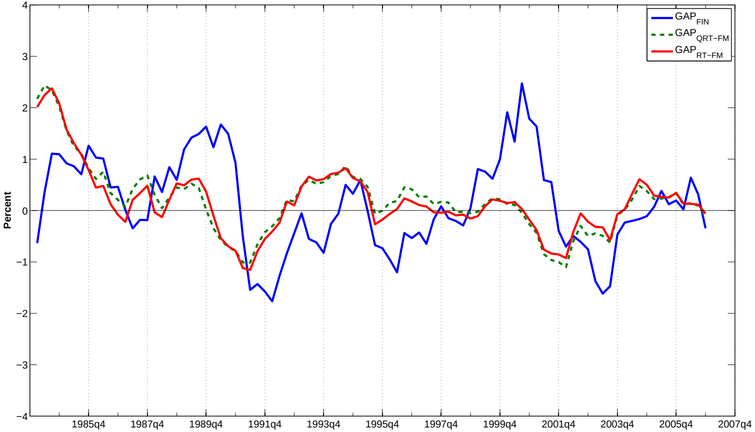
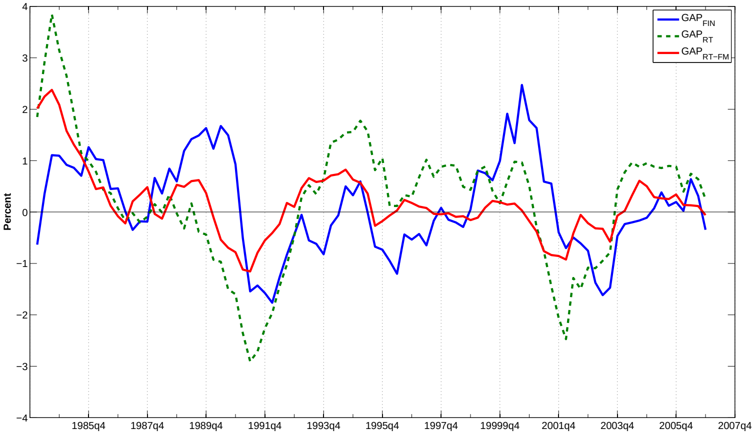
Output gaps with Factor model approach



Real time Output gaps Standard vs Factor model



Relative Mean Squared Errors

Measure	Formula	$\lambda = 1600$	$\lambda = 400$	$\lambda = 100$
True real-time performance	$\frac{mean((\mathtt{GAP}_{\mathtt{RT\text{-}FM}} - \mathtt{GAP}_{\mathtt{FIN}})^2)}{mean((\mathtt{GAP}_{\mathtt{RT}} - \mathtt{GAP}_{\mathtt{FIN}})^2)}$	0.59	0.69	0.64
Data revision performance	$\frac{mean((\mathtt{GAP}_{\mathtt{RT\text{-}FM}} - \mathtt{GAP}_{\mathtt{QRT\text{-}FM}})^2)}{mean((\mathtt{GAP}_{\mathtt{RT}} - \mathtt{GAP}_{\mathtt{QRT}})^2)}$	0.10	0.04	0.06
Quasi real-time performance	$\frac{mean((\mathtt{GAP}_{\mathtt{QRT\text{-}FM}} - \mathtt{GAP}_{\mathtt{FIN}})^2)}{mean((\mathtt{GAP}_{\mathtt{QRT}} - \mathtt{GAP}_{\mathtt{FIN}})^2)}$	0.63	0.76	0.72

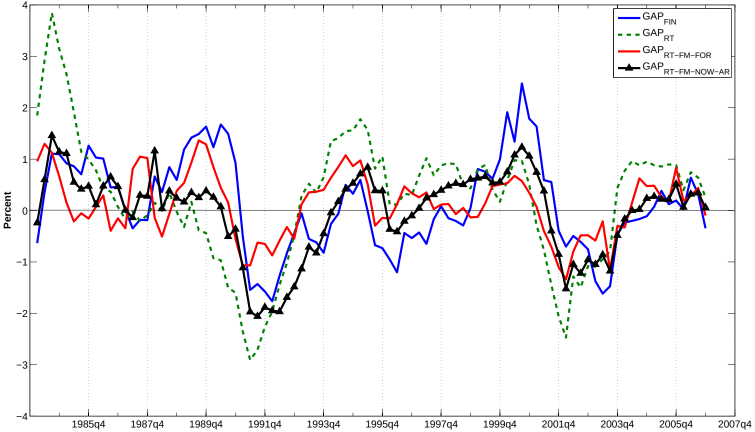
End-of-sample problem

- Future data contains information about trend.
- Add forecast from AR(1) to the data series when computing the cycle (as in Mise, Kim and Newbold (2005)).

Relative Mean Squared Errors

Output gap measure	Real-time performance			Quasi real-time performance		
	$\lambda = 1600$	$\lambda = 100$	$\lambda = 400$	$\lambda = 1600$	$\lambda = 100$	$\lambda = 400$
$\mathtt{GAP}_{\mathtt{RT-FM}}$	0.59	0.69	0.64	0.63	0.76	0.74
$\mathtt{GAP}_{\mathtt{RT-FM-FOR}}$	0.42	0.55	0.45	0.44	0.63	0.53
$\mathtt{GAP}_{\mathtt{RT-FM-NOW-AR}}$	0.27	0.44	0.13	0.26	0.39	0.10

Real time output gaps including forecasts



Inflation forecasts based on real-time output gap estimates

Follow Stock and Watson (1999) and Orphanides and van Norden (2005) and specify the following Phillips curve regression:

$$\pi^4_{\tau+h} = \alpha + \sum_{i=0}^{n} \beta_i \pi^4_{\tau-i} + \sum_{i=0}^{m} \gamma_i gap_{\tau-i} + e_{\tau+h} \quad (5)$$

where π^4_τ denote inflation over 4 quarters ending in quarter τ.

Relative Mean Squared Errors

Output gap measure	**Forecast horizon h=1**			**Forecast horizon h=4**		
	$\lambda = 1600$	$\lambda = 100$	$\lambda = 400$	$\lambda = 1600$	$\lambda = 100$	$\lambda = 400$
$\mathtt{GAP}_{\mathtt{RT}}$	1.39	1.02	1.26	0.95	0.91	0.94
$\mathtt{GAP}_{\mathtt{RT-FM}}$	1.12	0.94	1.08	0.93	0.92	0.94
$\mathtt{GAP}_{\mathtt{RT-FM-FOR}}$	0.90	0.80	0.88	0.87	0.84	0.86
$\mathtt{GAP}_{\mathtt{RT-FM-NOW-AR}}$	0.95*	0.80**	0.90**	0.92	0.91*	0.92*

Summary

- We found that a factor model can substantially improve the reliability of real-time output gap estimates through two mechanisms
  - The data revision problem is considerable reduced as a factor model extract only the common component and disregards the idiosyncratic (noisy) component.
  - The end-of-sample problem is considerably reduced by combining a nowcast from a factor model with long term forecasts from an AR(1).

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- Newer alternative methods:
  - Non-stationary factor model approach (Barigozza and Luciani (2021))
  - Beveridge-Nelson decomposition based on a BVAR (Morley and Wong (2020) and Berger, Morley and Wong (2021))
  - Suite of models approach (Barbarina et al. (2020), Furlanetto et al. (2020))
  - Alternative detrending methods (Hamilton (2018), Mueller and Watson (2017))