FAQ: How do I extract the output gap?

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Newest version of the paper at:
https://drive.google.com/file/d/15EGsbxp1g54SB4afKneLv-nAInjPkwsm/view

Introduction

• Great interest in star variables (potential output, natural rate of interest, NAIRU, etc.) and in the post-COVID nature of cyclical fluctuations.

• Policymakers want (i) to respond to gap fluctuations but not to potential changes; (ii) to know the state of the economy.

• Measurement of star variables elusive:

- Gap=$y - y^{Pot}$? $y - y^{Perm}$? $y - y^{trend}$?

- Measurement tools have no links to models used to interpret the dynamics of star variables.

• Policy analyses whimsical.

Plenty of academic discussion about behavior of latent variables:

• What is potential output post-2008? Coibion et al. (2018);

• Secular fall in the natural real rate? Laubach and Williams (2015); Del Negro et al. (2019).

• Properties of NAIRU? Crump et al. (2019).

• Cyclical dynamics of hours: Beaudry et al. (2019).

• Permanent and transitory exchange rates drivers? Schmitt-Grohe and Uribe (2019).

• Permanent effects of demand shocks? Jorda et al (2020); Furlanetto et al. (2019).

• The trend generates the cycle, Aguiar and Gopinath (2007); or the cycle drives the trend? Heathcote et al. (2020).

• How to extract cyclical fluctuations? Hamilton (2018); Hodrick (2020).

• V, U or L-shaped post-COVID recovery, see e.g. https://www.brookings.edu/blog/up-front/2020/05/04/the-abcs-of-the-post-covid-economic-recovery/
or
https://www.weforum.org/agenda/2020/04/alphabet-soup-how-will-post-virus-economic-recovery-shape-up/

• Which models match business cycle facts? Angeletos et al. (2019).

This paper

• Investigates the relationship between **theoretical gaps** (or transitory fluctuations) and cycle estimates using a lab experiment.

• Use standard NK models as DGP. Simulate:

- Potentials and gaps.

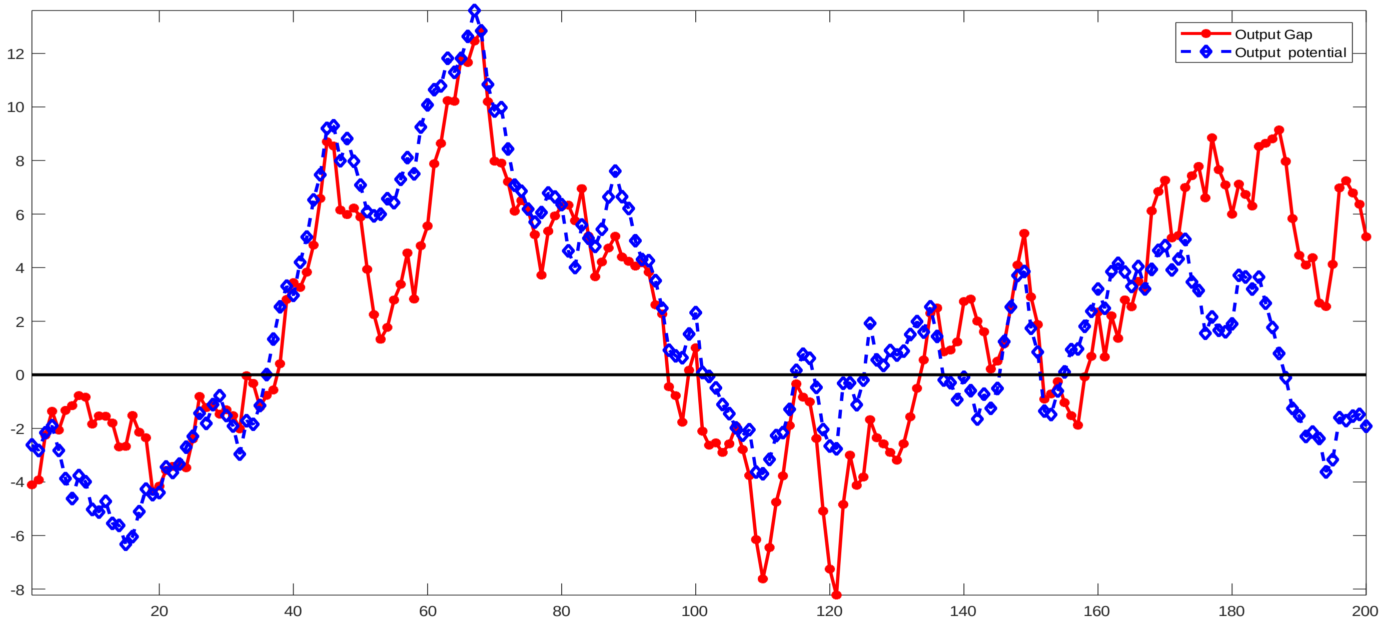
- Permanent and transitory components.

• Apply a number of filters to simulated data sets. Rank filtering procedures and explain the outcomes.

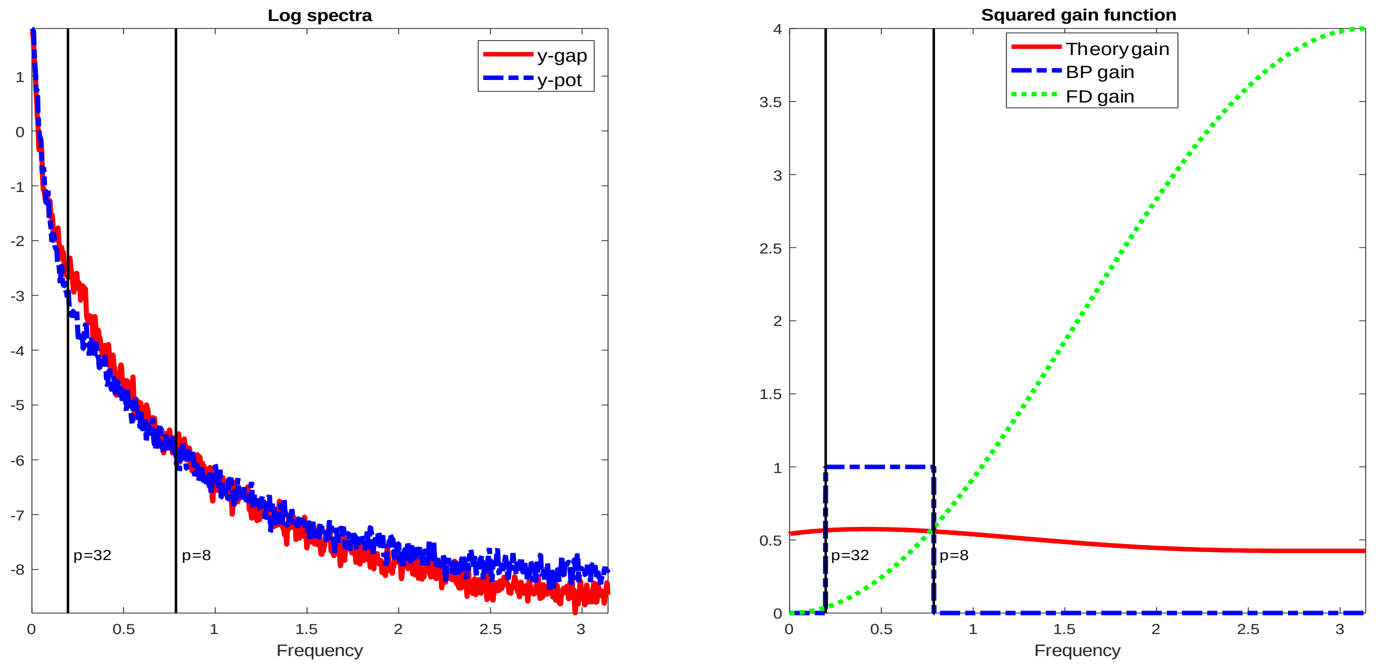
• Design a new filter for gaps extraction that uses basic information from NK models.

General points

- Models driven by persistent shocks produce gap and potentials with similar properties (and they are correlated).



• No filter assumes that latent components have similar spectral properties. Large distortions.



Horse race results 1: Gap extraction

• The *least distorting* is **Polynomial filtering**. Why?

• With Polynomial filtering the frequency distribution of the variance of the gaps undistorted. Estimated cycles display some low frequency variations.

• Conclusions independent of sample size and filters' parameters.

Horse race results 2: Transitory fluctuations extraction

• The *least distorting* is **differencing**, **Polynomial filtering** close second.

• Distortions larger because at business cycle and high frequencies permanent fluctuations matter a lot.

• Small samples affect the ranking; the parameters of the filters do not.

What do we take home?

- If standard NK models are credible, standard filters inappropriate.
- All distort, some more some less.
- Use models to measure time path of latent components.
- Design alternative filters which exploit information models provide

The design of the experiment

- Standard NK model with equations for level and potential variables.

- Baseline setup: all disturbances stationary. Alternatives:

- i) TFP has a unit root; ii) TFP has a unit root and "the trend (unit root) creates the cycle" (Aguiar-Gopinath, 2007); iii) TFP has a unit root and there are no government spending and investment shocks; iv) TFP has a unit root and there are financial frictions (SW-FF, CMR); v) TFP has a unit root but the model is semi-structural (ECB-base).

- Sample sizes: T=150,750; replications: N=100.

Filters

- Polynomial regression (order 2).

- HP ($\lambda = 1600$).

- Short (1 quarter) and long (24 quarters) differencing.

- BP (non-symmetric time varying Christiano and Fitzgerald, 2003, 8-32q cycles); Wavelets (8-64q cycles).

- Hamilton (2018) local projections: h=8, d=4.

- UC: trend is a random walk; cyclical an AR(2), potentially correlated disturbances (MCMC implementation).

- BN/BQ VAR based measures (with output growth and hours)

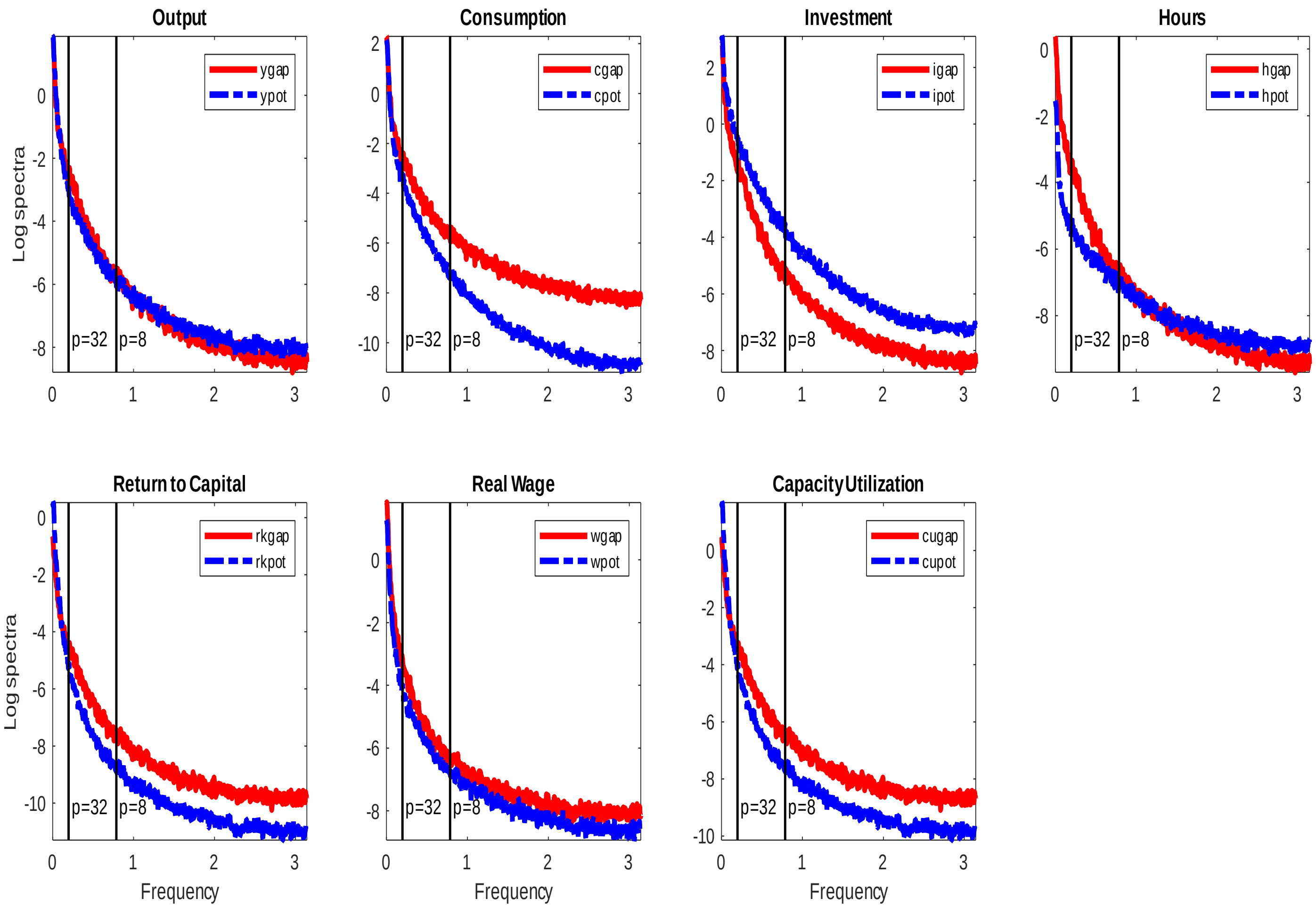
Properties of the DGP

• How does the spectrum of gaps and potentials look like in a baseline NK model?

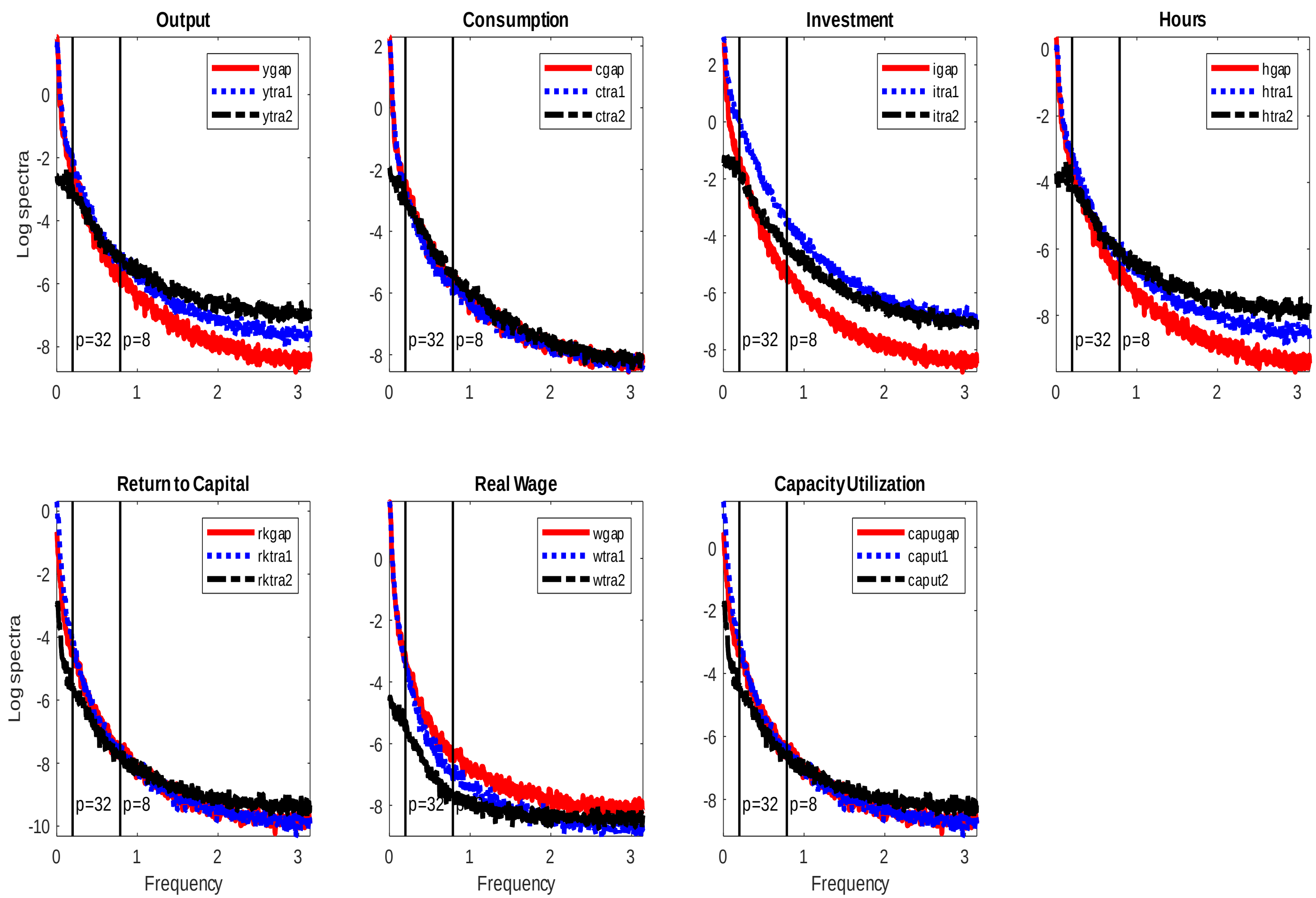
• How different are the spectrum of gaps and transitory components?

• Do the spectral properties of gaps depend on the calibration/features of the model (e.g. presence of shocks, absence of financial frictions)?

• Do they depend on modeling principles?



Output
Consumption
Investment
Hours
Return to Capital
Real Wage
Capacity Utilization
ygap
ypot
cgap
cpot
igap
ipot
hgap
hpot
rkgap
rkpot
wgap
wpot
cugap
cupot
p=32
p=8
Log spectra
Frequency


Output
Consumption
Investment
Hours
Return to Capital
Real Wage
Capacity Utilization
ygap
ytra1
ytra2
cgap
ctra1
ctra2
igap
itra1
itra2
hgap
htra1
htra2
rkgap
rktra1
rktra2
wgap
wtra1
wtra2
capugap
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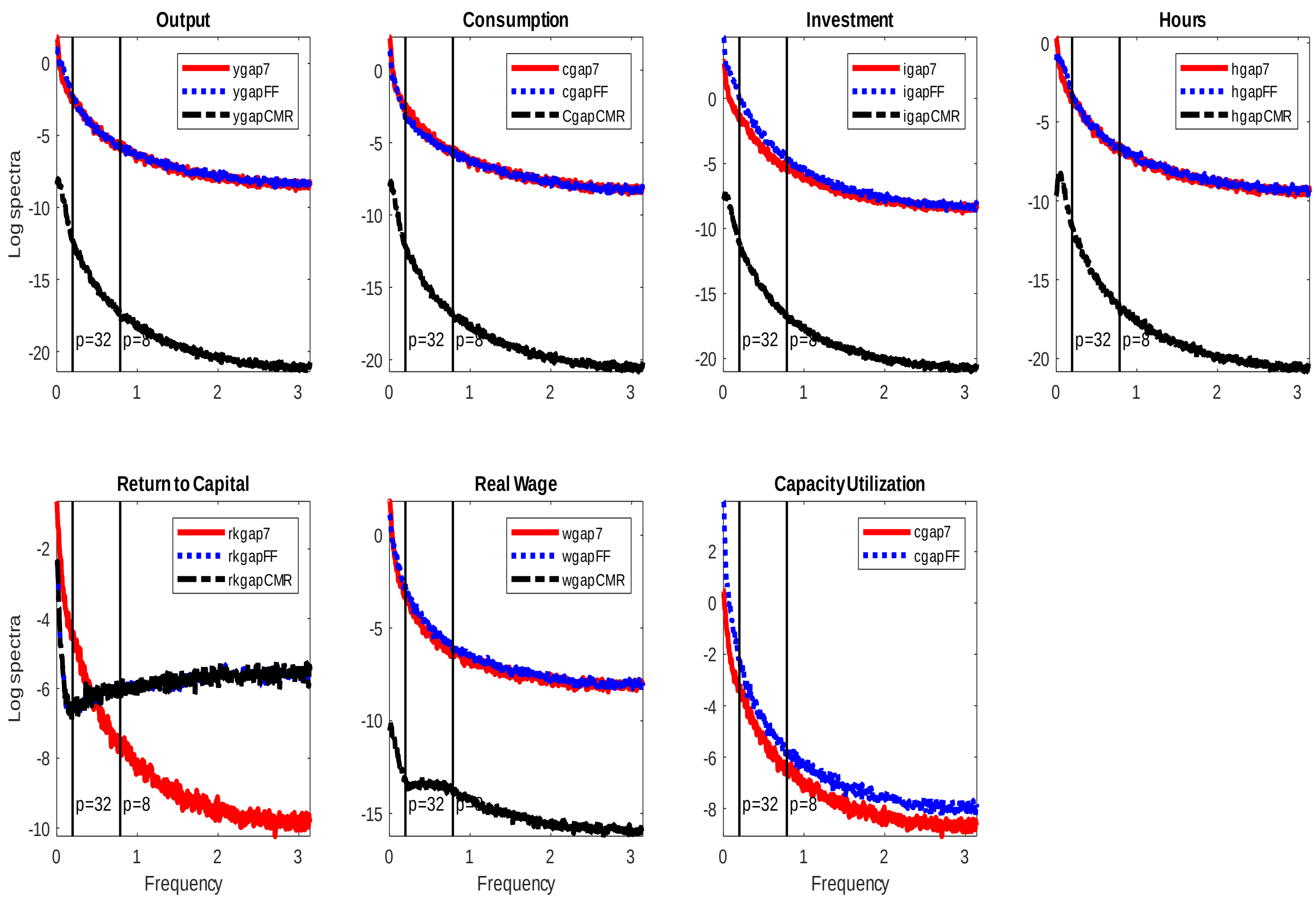
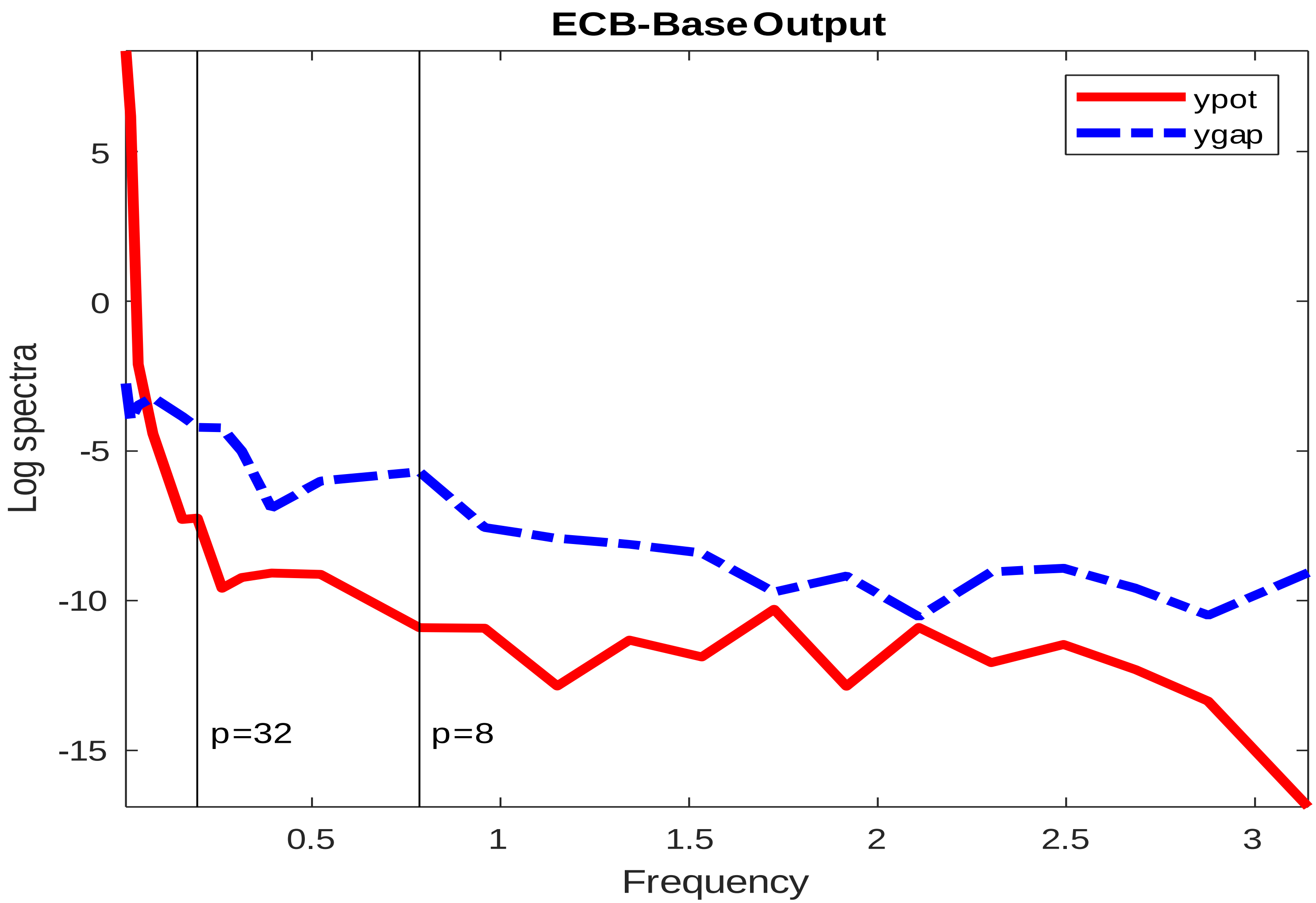
p=32
p=8
Log spectra
Frequency


Output
ygap7
ygapFF
ygapCMR
Consumption
cgap7
cgapFF
CgapCMR
Investment
igap7
igapFF
igapCMR
Hours
hgap7
hgapFF
hgapCMR
Return to Capital
rkgap7
rkgapFF
rkgapCMR
Real Wage
wgap7
wgapFF
wgapCMR
Capacity Utilization
cgap7
cgapFF
Log spectra
Frequency
p=32
p=8


ECB-Base Output
ypot
ygap
5
0
-5
-10
-15
Log spectra
p=32
p=8
0.5
1
1.5
2
2.5
3
Frequency

Keys to understand the results 1: Gap and Potential

- Gaps have important low frequency variability; potentials significant business cycle variability.

- Potentials and gaps have similar distribution of variance by frequency.

- They are driven by the same shocks and have similar persistence.

- Details of the generating economy not important (shocks driving the economy, financial frictions, etc.).

- Patterns in models with different (micro-) foundations similar.

Key to understand results 2: Permanent and Transitory

- Permanent/transitory components display similar distribution of the variance by frequency but they are **uncorrelated**.

- Permanent component: non-stationary, driven by TFP only. Variability at business cycle frequencies important

- Transitory component: stationary, driven by all other shocks. Low frequency variability more important than business cycle variability.

- With unit roots, gaps not interesting (they have both permanent and transitory components). Gaps "never close".

- Gaps$\neq$Transitories; Potentials $\neq$ Permanents.

Table 2: Summary results across variables, SW DGPs, T=750

Statistic	POLY	HP	FOD	LD	BP	Wa	Ham	UC	BN	BQ
	Gap									
MSE	5	3						1	0.5	0.5
Corr	9								0.5	0.5
AR1	4			3		3				
Var	4			2		3	1			
TP	1.5	5	2	1.5						
RT-MSE		1				3	2	3	0.5	0.5
PC	2									
OL				1			1			
Total	25.5	9	2	8.5	0	9	4	4	1.5	1.5
	Transitory									
MSE			9					1		
Corr										
AR1	4					5.5		0.5		
Var	3			6			1			
TP	4	4		2						
RT-MSE			4					6		
PC				1						1
OL				2						
Total	11	4	13	11	0	5.5	1	7.5	0	1

Focusing on output gap/transitory output

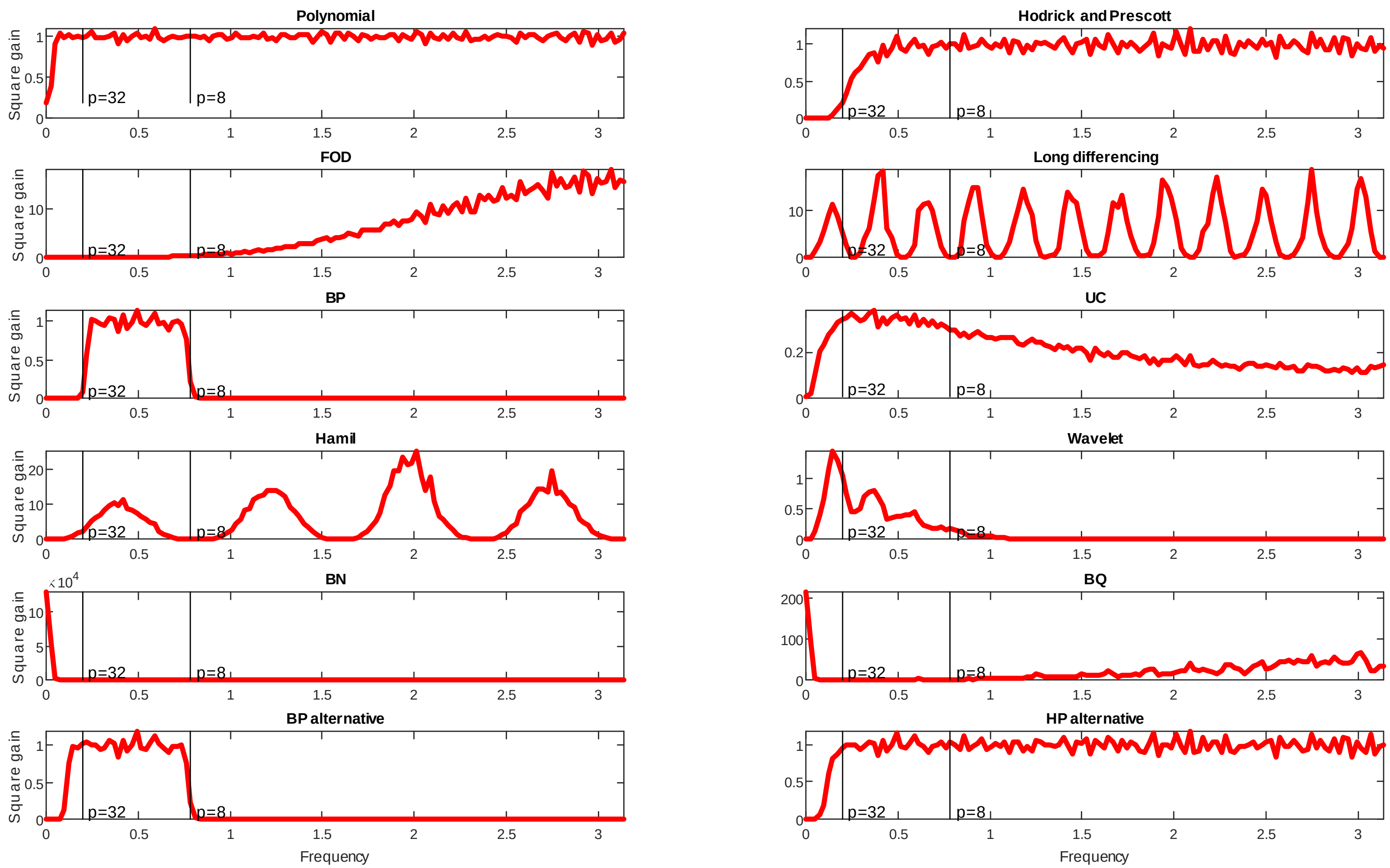
Table 3: Summary results across statistics, different DGPs, T=750

	Output gap									
DGP	POLY	HP	FOD	LD	BP	Wa	Ham	UC	BN	BQ
SW	6	3	1	1			2	1		
SW_FF	10	1	1	2						
CMR	2	2	1	2	0.5	2		1		2.5
SW_5	4	6	1	1				1		1
Total	22	12	4	6	0.5	2	2	3		3.5
	Transitory Output									
SW(unitroot)	2	3	2	4	1					1
SW(trendcycle)	1	4	2			2	1		3	
Total	3	7	4	4	1	2	1		3	1

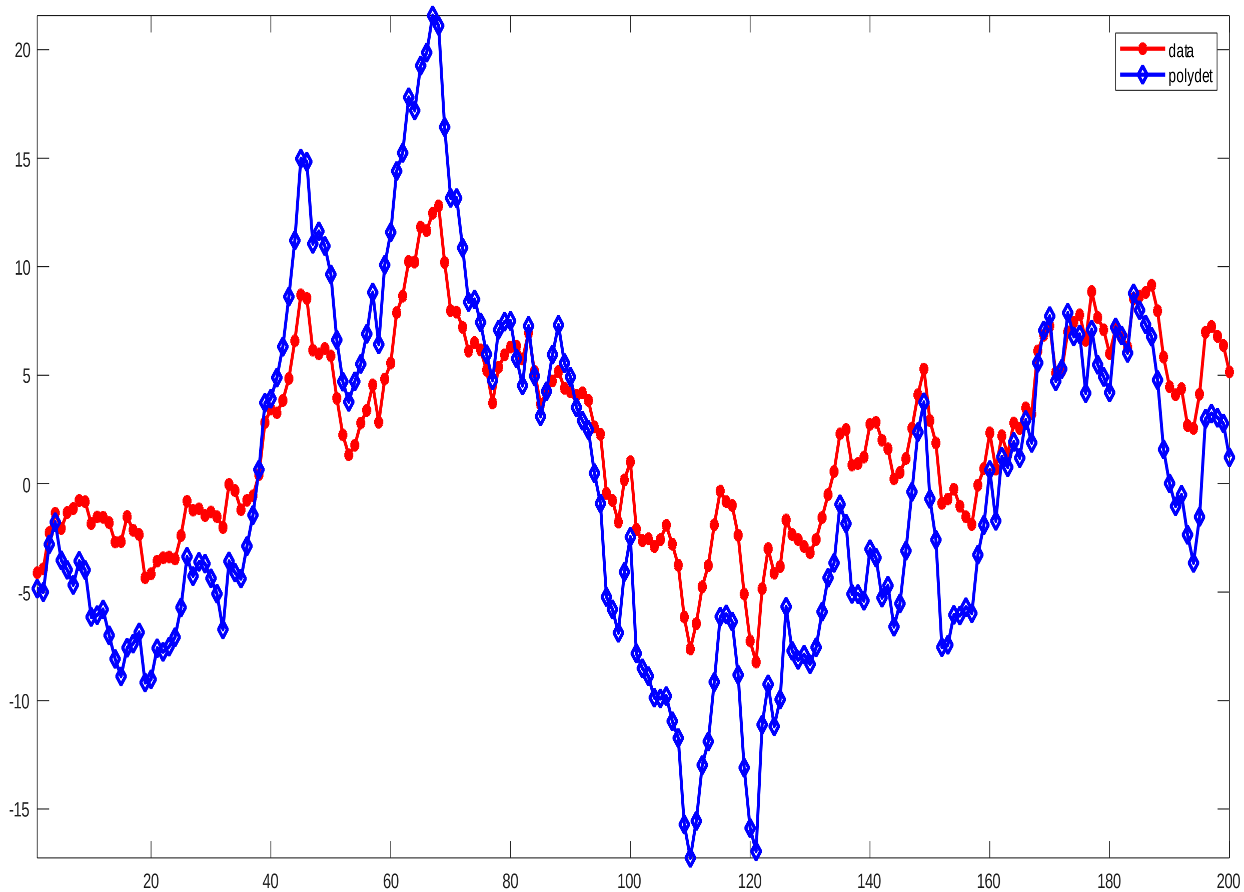
Why do we get these results? Filters properties.

- What are the squared gains of the filters?

- Can any filter mimic the DGP of the gap?



Polynomial
Hodrick and Prescott
FOD
Long differencing
BP
UC
Hamil
Wavelet
BN
BQ
BP alternative
HP alternative
Square gain
p=32
p=8
Frequency



data
polydet
20
15
10
5
0
-5
-10
-15
20
40
60
80
100
120
140
160
180
200

Key to understand results 3: Filter properties

• All low frequency variance typically attributed to trend; all BC (and high frequency) variance attributed to cycle. Exceptions: Polynomial, UC, and Wavelets.

• Persistence of the trend larger than persistence of the cycle.

• Components assumed uncorrelated (exceptions UC, BN).

• Assumptions to identify components do not hold in the DGP.

Zooming in on popular filters: HP and BP

• HP (and BP) poor: why? They leave too much low frequency variations in trend.

• With $\lambda = 51200$ (lower ω_1) cycle has approximately the right amount of low frequency variations. Still the trend has too little business cycle power.

• HP λ is not var(cycle)/ var(Δ(Δ(trend))) when components have similar spectral properties, are correlated, and gap not iid. Low UC/LP estimated inapplicable (see Hamilton, 2018).

Raising star: Hamilton's LP

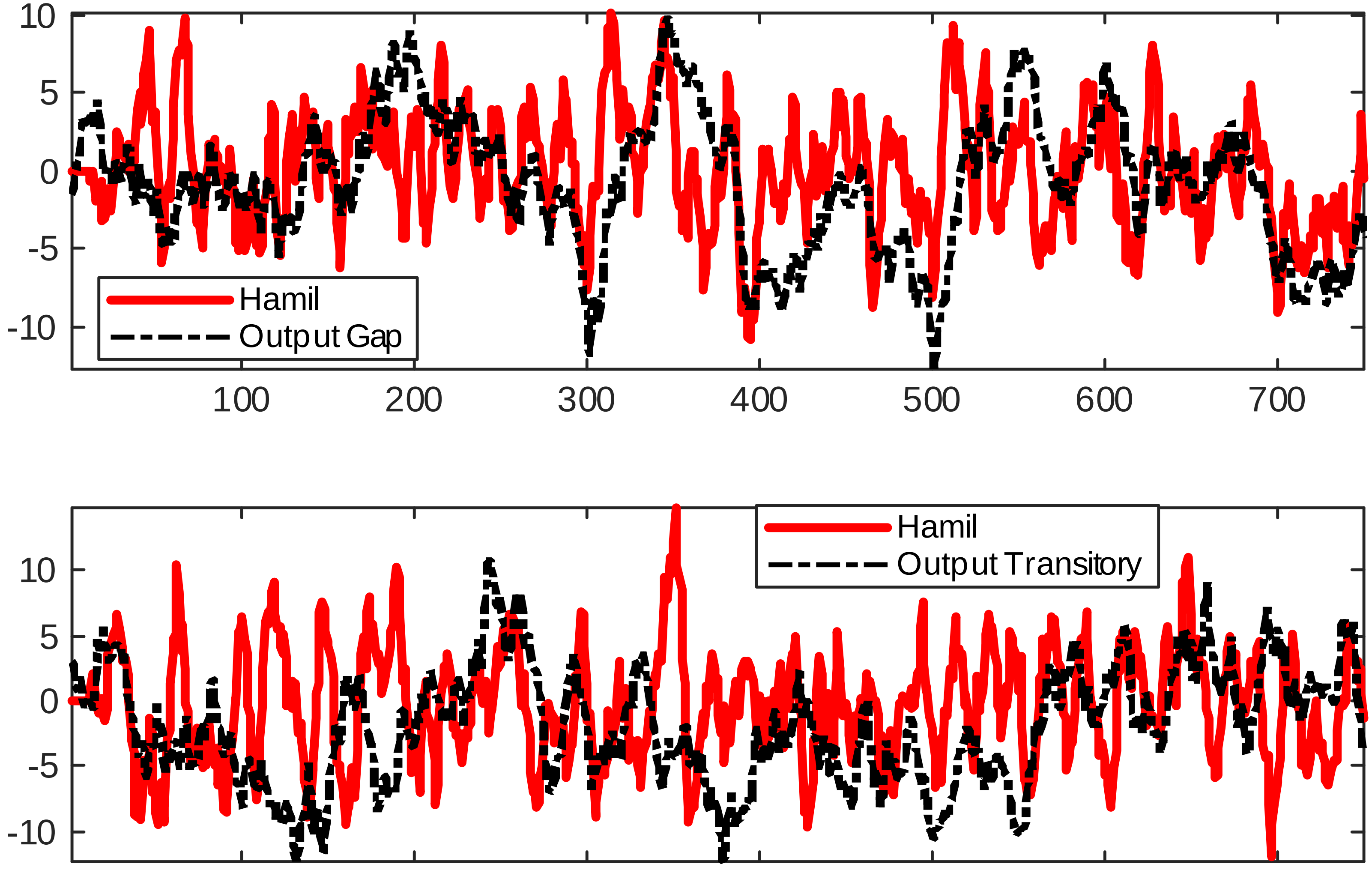
• LP not much better than standard HP. Potential reasons:

• Estimated cycle has too much high frequency and too little low frequency variations (crucial).

• Estimated squared gain has zero in correspondence of the horizon of the projection (minor).

• Gaps and potentials are assumed to be uncorrelated (minor).

• LD has the same latter two problems, but much better performance.


10
5
0
-5
-10
Hamil
Output Gap
100
200
300
400
500
600
700
10
5
0
-5
-10
Hamil
Output Transitory

VAR approaches

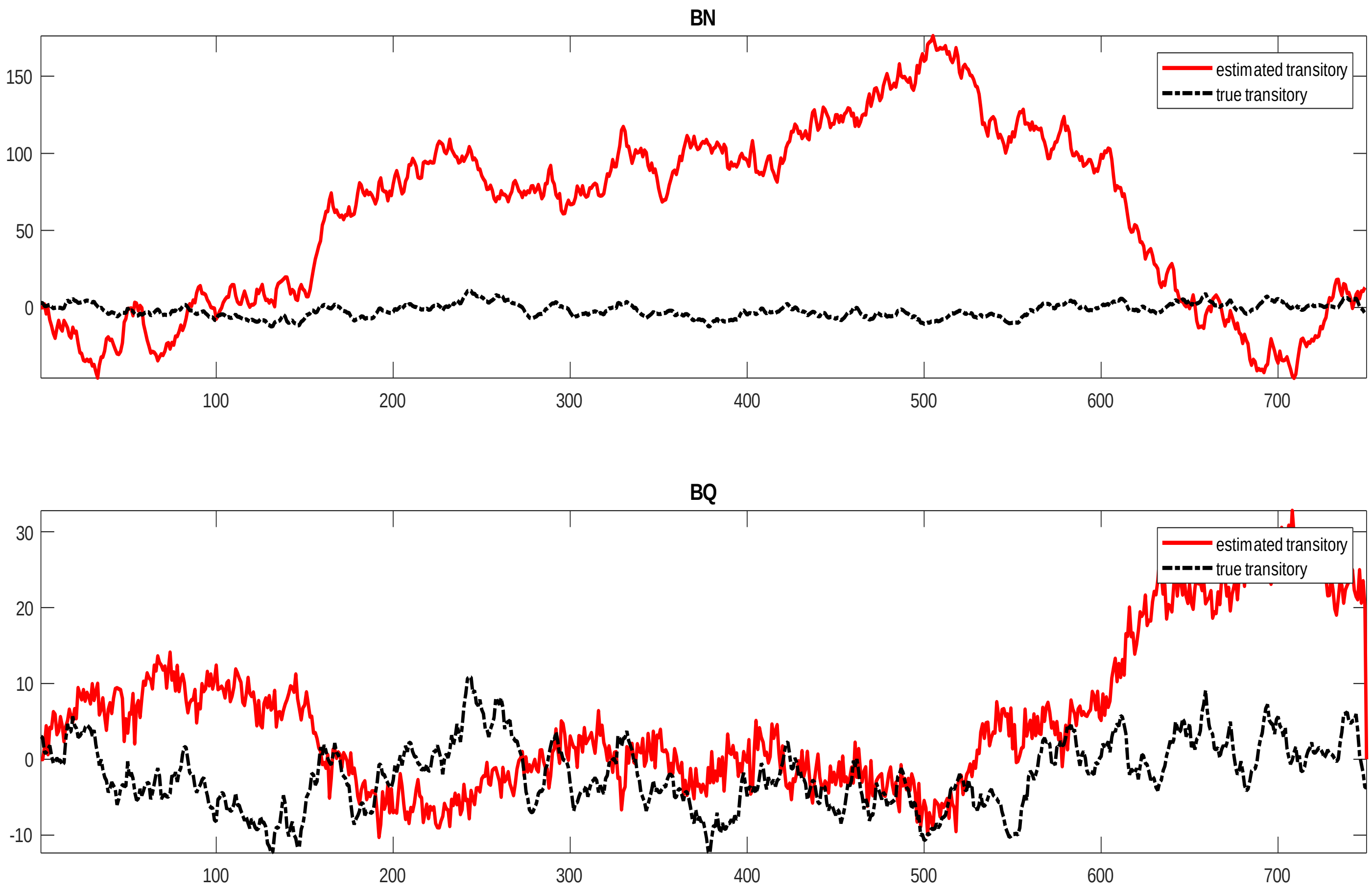
• VAR-based procedures: ok for hours, bad for output.

• For gaps extraction: too much low frequency variability is attributed to the transitory output component. Why?

• Misspecification: output is overdifferenced.

• Assumption that components are uncorrelated (BQ) or correlated (BN) makes little qualitative difference.

• For transitory component extraction: no overdifferencing; still too much low frequency variations in the cycle.



BN
150
100
50
0
100
200
300
400
500
600
700
estimated transitory
true transitory
BQ
30
20
10
0
-10
100
200
300
400
500
600
700
estimated transitory
true transitory

- Coibion et al. (2018): post 2008 measures of potential bad; use BQ to get output potential.

- Is it better? Potentials/Permanent component have very long swings. Permanent has little business cycle fluctuations. Why?

- Deformation (Canova and Ferroni, 2021) 7 structural disturbances compressed in 2 (3) innovations; states of model are missing in VAR.

- Persistence of components increased; correlation between true and estimated structural shocks low (e.g. estimated supply and TFP shocks is 0.43).

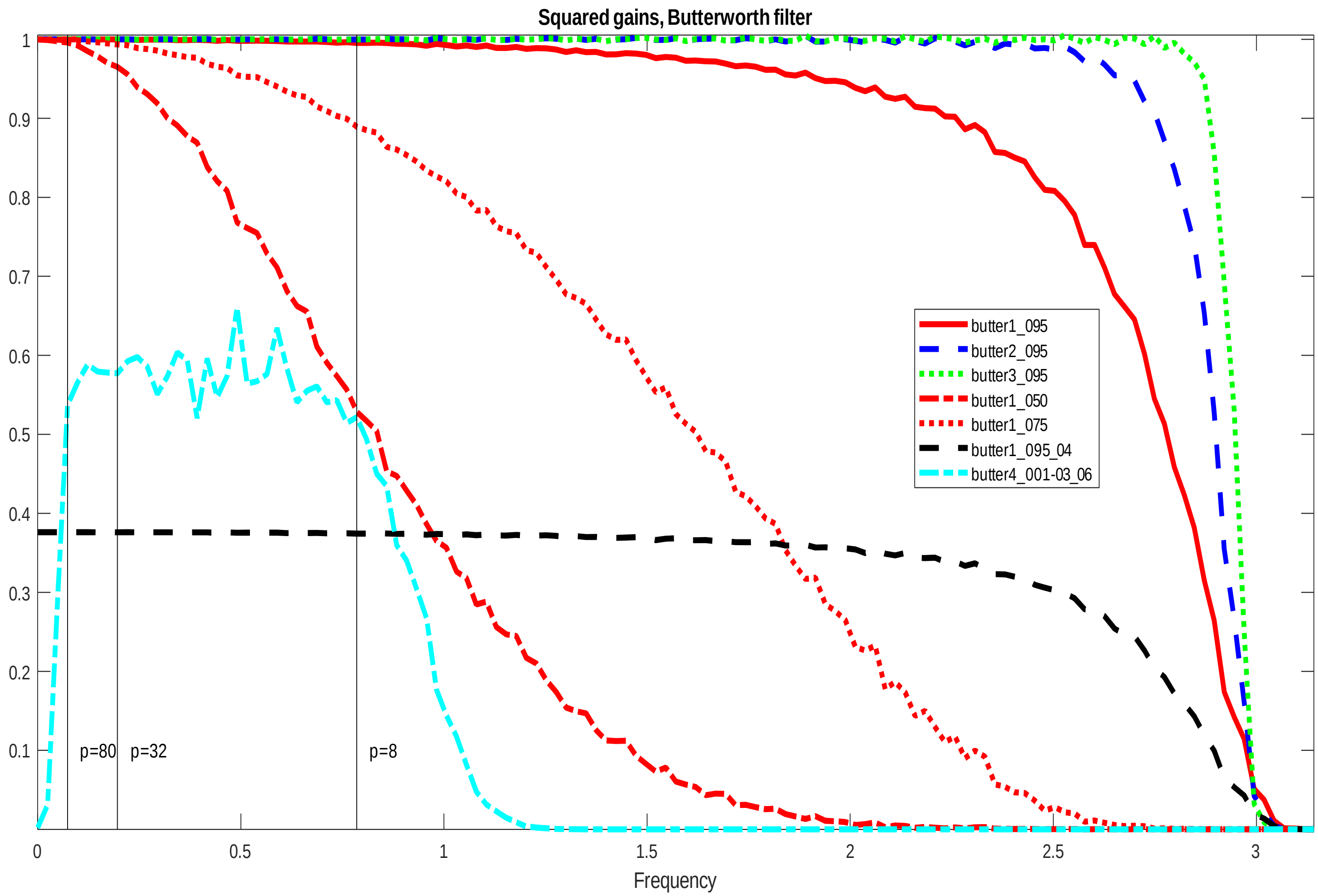
- Short samples will add to the problems, see Erceg et al., 2005.

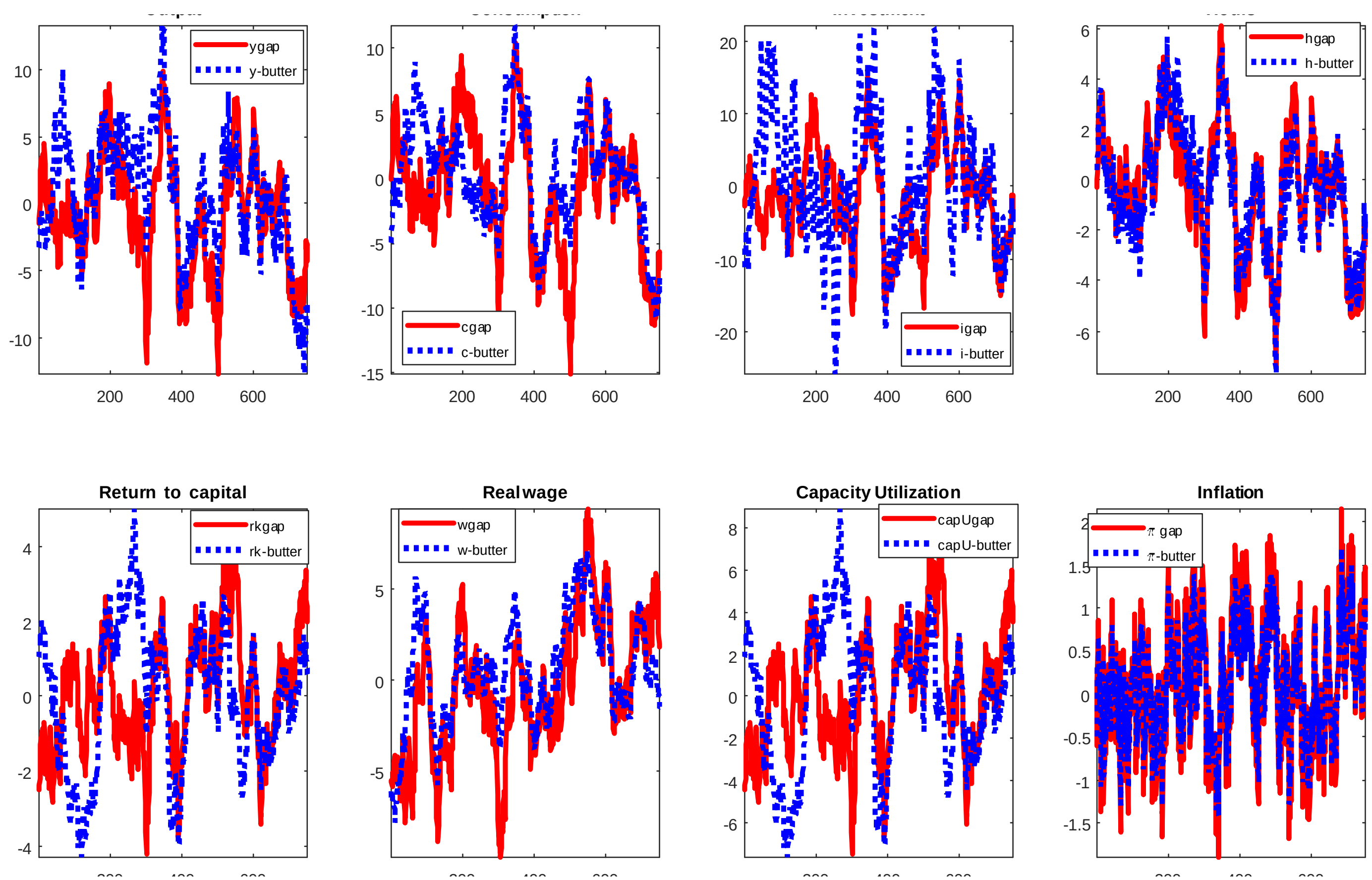
Where to go next?

- Estimate a structural model and construct model-based latent components, e.g. Christiano et al. (2010), Justiniano, et al. (2013); Furlanetto et al. (2020).

- If misspecification is a concern use Canova and Matthes (2021a, b) **composite posterior approach** to robustify inference.

- Design a filter with better properties, given this type of DGP.


Squared gains, Butterworth filter
butter1_095
butter2_095
butter3_095
butter1_050
butter1_075
butter1_095_04
butter4_001-03_06
p=80
p=32
p=8
Frequency
0
0.5
1
1.5
2
2.5
3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



ygap
y-butter
cgap
c-butter
igap
i-butter
hgap
h-butter
Return to capital
rkgap
rk-butter
Real wage
wgap
w-butter
Capacity Utilization
capUgap
capU-butter
Inflation
π gap
π-butter

Table 2: Summary results across variables, SW DGP, T=750

Statistic	POLY	HP	FOD	LD	BP	Wa	Ham	UC	BN	BQ	BW
	Gap										
MSE	5	3						1	0.5	0.5	8
Corr	9								0.5	0.5	8
AR1	4			3		3					6
Var	4			2		3	1				
TP	1.5	5	2	1.5							3
RT-MSE		1				3	2	3	0.5	0.5	8
PC	2										2
OL				1			1				
Total	25.5	9	2	8.5	0	9	4	4	1.5	1.5	35
	Transitory										
MSE			9					1			
Corr											
AR1	4					5.5		0.5			1
Var	3			6			1				1
TP	4	4		2							3
RT-MSE			4					6			
PC				1						1	
OL				2							
Total	11	4	13	11	0	5.5	1	7.5	0	1	5

Table 3: Summary results across statistics, different DGPs T=750

	Output Gap										
DGP	POLY	HP	FOD	LD	BP	Wa	Ham	UC	BN	BQ	BW
SW	6	3	1	1			2	1			7
SW_FF	10	1	1	2							7
CMR	2	2	1	2	0.5	2		1		2.5	
SW_5	4	6	1	1				1		1	7
Total	22	12	4	6	0.5	2	2	3		3.5	21
	Transitory output										
SW(unitroot)	2	3	2	4	1					1	3
SW(trendcycle)	1	4	2			2	1			3	7
Total	3	7	4	4	1	2	1		3	1	10

Conclusions and additional open questions

• If standard models driven by persistent shocks generate the macroeconomic data we observe, toolkit of filters for star (latent) variable extraction inappropriate.

• Could go structural. Do policymakers want to do so? Despite 20 years of NK-DSGEs, little consensus on the model to be used.

• Could design better filters: BW could be one.

• What are the properties of trend/cycles of real data when a BW filter is used? Can a standard NK model match the stylized facts that are produced?

• Are real and financial cycles really different? In SWFF and CMR models they are not. And in the data?