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Abstract

This paper looks at whether the recent sharp spike in inflation can be beneficial for public debt sustainability by eroding the real value of nominal debt. Simulations with the European Commission's QUEST model suggest that if the source of inflation is an adverse terms-of-trade shock, then it leads to a rising public debt-to-GDP ratio. In this case, the debt-reducing effect of higher inflation is outweighed by the adverse effects of slower real growth, a declining primary budget balance, and higher interest rates as an active monetary policy tightens to fight inflationary pressures. The results are highly policy-dependent: shorter consolidated debt maturity (brought about by past QE programs) would speed up the rise in interest expenditures, while a more accommodative monetary policy would delay them, also supporting nominal growth. The reaction of the primary fiscal balance (via automatic stabilisers, inflation indexation and debt-stabilisation rules) also matters. However, the baseline result that the debt-to-GDP ratio rises in response to an adverse terms-of-trade shock is fairly robust across all but the most extreme alternative policy scenarios.

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Keywords: terms-of-trade shock, monetary-fiscal policies, DSGE modelling, open economies.

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1 Introduction

In 2021-22 Europe experienced a surge in inflation. While high inflation is usually regarded as economically costly, this has also raised the question whether the situation can be beneficial for public finances and debt sustainability. The real value of nominal (i.e. not inflation-indexed) local currency debt can be eroded by faster price growth, which could *ceteris paribus* lower public debt-to-GDP ratios.

However, debt dynamics depend on a host of other factors as well, which are unlikely to remain unchanged. As a matter of accounting identity, the dynamics of the debt-to-GDP ratio b_t are described by equation (1.1):

$$\Delta b_t = \underbrace{-pb_t + \underbrace{\frac{i^g_{t-1}}{(1+\pi^Y_t)(1+g_t)}\, b_{t-1}}_{\text{interest costs}}}_{-bb_t} - \underbrace{\frac{g_t}{(1+\pi^Y_t)(1+g_t)}\, b_{t-1}}_{\text{real growth effect}} - \underbrace{\frac{\pi^Y_t}{1+\pi^Y_t}\, b_{t-1}}_{\text{inflation revaluation}} \tag{1.1}$$

In general equilibrium, these individual components[2] interact with each other as various shocks propagate through the economy, affecting not just inflation but other things as well. Moreover, the precise way of their interaction depends also on the nature of the shock and how economic policy responds.

Therefore, when trying to assess "the effect of inflation" on debt dynamics, it is essential to identify the underlying structural economic shock which caused inflation to rise. There is no such thing as a "pure" inflation shock, which leaves the rest of the economy unchanged. Inflation can increase due to a positive domestic demand shock, which stimulates growth and puts upward pressure on prices (demand-pull inflation). Alternatively, inflation can also be a result of a negative supply shock, like a rise in domestic markups or a hit to productivity, which depress real growth while also leading to higher prices (cost-push inflation). At a minimum, GDP growth outcomes are different across these two inflationary scenarios, but monetary and fiscal policies are also likely to react in very different ways, affecting not just interest payments and the primary balance, but also feeding back to inflation and real growth. The final dynamics of debt-to-GDP are a result of a complex interaction of all the above.

As a result of surging imported energy prices, over the course of 2021-22 Europe saw its terms-of-trade deteriorate by almost 9% cumulatively, which was undoubtedly an important source of the increase in European inflation.[3] To investigate the resulting general equilibrium fiscal dynamics

[2] pb_t is the primary budget balance, while bb_t is the headline budget balance as a percent of GDP, i^g_{t-1} is the *effective* nominal interest rate on the outstanding government debt stock, π^Y_t is inflation (GDP-deflator) and g_t is the growth rate of real GDP. See the Appendix for a derivation, at equation (A.22).

[3] Of course, the actual evolution of inflation in Europe was driven by a combination of various economic shocks, not only the terms-of-trade shock. Most notably, the post-pandemic reopening with the associated pent-up demand of households, supply-chain disruptions and tightening supply bottlenecks have constituted a complex mixture of demand and supply shocks. However, rather than conducting a historical shock decomposition exercise, the analysis in this paper attempts to isolate the effect of the terms-of-trade shock only, in order to capture its transmission channels without confounding the picture with other disturbances.

in a model-consistent way, this paper explores simulations for the European Union (EU27) done with the Commission's QUEST model, in response to an adverse terms-of-trade shock. Such a shock raises import prices relative to export prices, and thereby drives a wedge between the CPI and the GDP-deflator. At the same time, a deteriorating terms-of-trade depresses real GDP, as imported intermediate inputs for domestic production become more costly (supply effect), and the erosion of domestic households' purchasing power lowers demand not just for imports but also for domestically produced goods (demand effect). The fall in domestic real incomes is larger than implied by lower GDP, due to the terms-of-trade loss which makes the domestic economy as a whole inevitably poorer. Domestic agents cannot all escape the fall in their real income, but their struggle to *try*, could also fuel domestically generated inflation on top of imported inflation (second round effects), as they try to allocate purchasing power losses among themselves.

The main finding from this exercise is that despite its inflationary effect, an adverse terms-of-trade shock leads to a *rising* public debt-to-GDP ratio. The reason is that the debt-reducing effect of higher inflation is outweighed by the detrimental effects of slower real growth, a declining primary budget balance, and higher interest rates as monetary policy tightens to fight inflationary pressures. The fall in the primary balance as a share of GDP, without considering any *discretionary* fiscal response, is driven mainly by CPI-indexed expenditure items at a time when real gross domestic *income* (i.e. CPI-deflated nominal GDP) is falling. In contrast, a positive demand shock with the same inflationary impact would have qualitatively opposite fiscal effects – highlighting the importance of the source of inflation.[4]

Another finding is that the quantitative results depend crucially on various monetary, fiscal and debt management policy settings. When monetary policy raises interest rates less aggressively in response to inflationary pressures, it slows debt dynamics not only directly via financing costs on government bonds, but also indirectly by supporting aggregate demand, and therefore real growth and inflation. In the extreme case of completely unresponsive monetary policy, debt-to-GDP would actually *decline*. In contrast, a shorter average maturity of the outstanding debt stock increases the speed with which rising short term policy rates pass through into effective government financing costs – a highly relevant scenario when the duration of the *consolidated* government's liabilities has been drastically shortened by QE.

Regarding fiscal policy, the primary balance can react in a more debt-stabilising way depending on fiscal rules, income tax progressivity, and the indexation of primary expenditures. A larger share of nominally fixed expenditures would lead to more "benefit erosion" amid an eventually rising nominal GDP, supporting public finances, while more progressive tax systems generate a larger "fiscal drag" effect as the nominal wage distribution shifts into higher (nominally fixed) tax brackets, thereby increasing fiscal revenues. On the other hand, higher primary balances also feed back into weaker real activtiy and inflation via aggregate demand (due to a non-Ricardian economy) and supply side effects (due to distortionary taxation). Overall, our baseline result

[4]This also suggests that, to the extent that inflation was driven by both terms-of-trade and demand shocks, the *actual* evolution of public finances reflected a mixture of these two clean effects.

that the debt-to-GDP ratio rises in response to an adverse terms-of-trade shock is fairly robust across all but the most extreme alternative scenarios.[5]

It is worth emphasising that the analysis in this paper is strictly descriptive: it does not aim to suggest that rising debt ratios in the face of an adverse shock are undesirable from a normative aspect, nor that more inflationary monetary policies or more austere budgetary consolidation would be called for, just so that debt-to-GDP does not increase as much. In fact, far from being unambigously bad, public debt can be a very useful tool in the hands of fiscal policy if it is not overused. In contrast, alternative policies that can achieve more benign debt dynamics are no free lunch, as they impose losses either on domestic long term bondholders (via higher inflation), or on taxpayers and benefit recipients (via stronger fiscal consolidation). Moreover, the general equilibrium consequences of such policies in the form of higher inflation or slower growth and higher unemployment are costly from a welfare perspective in their own right. Rather than deriving *optimal* policies along these complex trade-offs, this exercise just analyses the evolution of debt-to-GDP and maps out the transmission channels in a descriptive way.

This paper joins numerous recent studies that explore the macroeconomic effects of the recent rise in imported energy costs. Auclert et al. (2022) consider a heterogeneous agent model with imported energy in consumption, while Chan, Diz and Kanngiesser (2022) build a two-agent New Keynesian (TANK) model with imported energy as a production input. Both point out that negative aggregate demand effects from the implied real income losses are amplified in the presence of households with high marginal propensity to consume (MPC) – a New Keynesian Cross mechanism that is also at work in QUEST. In addition, similarly to Bachmann et al. (2022), they emphasise the importance of import substitution elasticities for the transmission of these shocks, showing that stronger complementarities can exacerbate the adverse effects – something this paper also explores.[6]

The adverse terms-of-trade shock also revived the discussion about the potential for a "wage-price spiral" to be set off, i.e. to what extent the initial shock could trigger persistent second round effects by spilling over into domestic price pressures. Blanchard (1986) has shown how such a mechanism can operate in a simple New Keynesian model, which Lorenzoni and Werning (2023) recently confirmed in a richer setup. Albrizio et al. (2022) and Alvarez and Dizioli (2023) point out the amplifying potential of more backward-looking expectation formation. Battistini et al. (2022) argue that the behaviour of the labour income share, i.e. the extent to which workers can resist real income losses, can be a good indicator of the risk for second round effects.[7] We also

[5]Potential discretionary fiscal measures in response to the rise in cost-of-living to support the private sector are not considered in this analysis. Depending on the precise design and targeting, this would lead to an even more pronounced rise in public debt.

[6]The potential negative demand side effects of supply shocks are investigated by Guerrieri et al. (2022) and Guerrieri et al. (2021). For further discussion on the macro effects of the current energy shock, see Blanchard and Pisani-Ferry (2022), Gunnella and Schuler (2022), Olivi, Sterk and Xhani (2022), Pieroni (2022).

[7]For a further discussion of wage-price spirals, and historical evidence, see Alvarez et al. (2022), Arce, Hahn and Koester (2023), Baba and Lee (2022), Benassy-Quere (2022*a*), Bodnár et al. (2022), Boissay et al. (2022).

explore the question in our current simulations, and find that the emergence of domestic price pressures depends a lot on the extent to which aggregate demand is weakened, which in turn is mainly affected by substitution elasticities and the strictness of monetary policy. The latter point speaks to the concerns of monetary policymakers who are anxious to keep inflation expectations anchored and prevent second round effects (Broadbent, 2022; Tenreyro, 2022; Schnabel, 2022).

As for public finances in the face an adverse terms-of-trade shock, Bénassy-Quéré (2022*b*) presents simulations by the French Treasury that caution against expecting beneficial fiscal effects from such an inflationary episode, mainly on the same grounds as argued in this paper. A similar simulation by the ECB's EAGLE model shown in Bankowski et al. (2023) confirms this, and illustrates the stark contrast between the fiscal consequences of terms-of-trade and domestic demand driven inflation.[8] Relative to these studies, this paper provides a more detailed analysis of transmision channels through the lenses of QUEST, and provides sensitivity analysis for various policy settings. Reis (2017) discusses how "inflating away" the public debt is harder when debt maturities are short and there is less scope for financial repression. He also points out that large past asset purchases by central banks create fiscal risks when rising interest rates make their short term reserve liabilities costlier – another point that we also explore in our simulations.[9]

The rest of the paper is organised as follows. Section 2 briefly describes some highlighted model features. Section 3 explores the baseline results and the transmission channels of a terms-of-trade shock in more detail, while Section 4 presents the sensitivity analysis with other potential monetary-fiscal policy settings. Section 5 concludes.

2 Model

The model used for the simulation exercise in this paper is a core version of QUEST, an open economy New Keynesian DSGE model developed and maintained by the European Commission. It features two regions, calibrated for the European Union (EU-27) and the rest of the world, using national accounts data, input-output tables and international trade matrices. On the production side there is a tradeable and a non-tradeable sector populated by monopolistically competitive firms whose price setting is subject to nominal rigidities. There are two types of households: liquidity-constrained and unconstrained Ricardian agents. All of them consume goods and supply labour, while Ricardians also invest into domestic physical capital, government bonds and international bonds, and also receive firm profits. Monetary policy controls the short term nominal interest rate, while fiscal policy finances various transfers and public spending by levying taxes on domestic agents and issuing nominal bonds.

[8] In contrast, Poplawski-Ribeiro et al. (2023) report empirical estimates that a rise in inflation can ease debt-burdens, but these estimates are unconditional on the type of shocks.

[9] For the fiscal effects of monetary policy and the impact of debt maturities see also Andreolli and Rey (2022), Andreolli (2021), Hilscher, Raviv and Reis (2022), Claeys and Guetta-Jeanrenaud (2022), Darvas (2022), Gros (2022).

Importantly from the perspective of an adverse terms-of-trade shock, imported goods appear not only directly in the consumption basket of households, but are also used as intermediate inputs in the production process.

The model here is a two-sector, two-region version of the one in Burgert et al. (2020). For the sake of brevity, a full detailed description will not be repeated here, but some key features are highlighted to facilitate the subsequent discussion. The interested reader is referred to that paper for more details.

2.1 Goods market structure

There are two domestic sectors $j \in \{NT, TD\}$ making non-tradeable and domestically produced tradeable goods. Gross output O_t^j in each sector is produced with a CES technology, by combining intermediate inputs INT_t^j and domestic value added Y_t^j:

$$O_t^j = \left[(1 - s_{in}^j)^{\frac{1}{\sigma_{in}}} \left(Y_t^j \right)^{\frac{\sigma_{in}-1}{\sigma_{in}}} + (s_{in}^j)^{\frac{1}{\sigma_{in}}} \left(INT_t^j \right)^{\frac{\sigma_{in}-1}{\sigma_{in}}} \right]^{\frac{\sigma_{in}}{\sigma_{in}-1}} \tag{2.1}$$

where the elasticity of substitution between them is σ_{in}, and their steady state shares are governed by s_{in}^j. Domestic value added, in turn is produced with a Cobb-Douglas technology using labour (L_t), private and public capital (K_t and KG_t):

$$Y_t^j = A_t^j \left[\left(u_t^j K_t^j \right)^{1-\alpha} \left(L_t^j - fcl \right)^{\alpha} (KG_t)^{\alpha_g} - fcy \right] \tag{2.2}$$

where A_t^j, u_t^j, fcl, and fcy denote total factor productivity, capacity utilisation, overhead labour and fixed costs, respectively.

Gross sectoral output O_t^j is then used in bundles for private and public consumption (C_t, G_t), investments (I_t^j, I_t^G), exports (X_t) as well as for intermediate goods (INT_t^j). Final goods composites for these purposes are a nested CES bundle of non-tradeable Z_t^{NT} and tradeable goods Z_t^T, with the latter consisting of both domestically produced Z_t^{TD} and imported goods Z_t^M:

$$Z_t = \left[\left(1 - s^T\right)^{\frac{1}{\sigma_{TNT}}} \left(Z_t^{NT} \right)^{\frac{\sigma_{TNT}-1}{\sigma_{TNT}}} + \left(s^T\right)^{\frac{1}{\sigma_{TNT}}} \left(Z_t^{T} \right)^{\frac{\sigma_{TNT}-1}{\sigma_{TNT}}} \right]^{\frac{\sigma_{TNT}}{\sigma_{TNT}-1}} \tag{2.3}$$

$$Z_t^T = \left[\left(1 - s^M\right)^{\frac{1}{\sigma_x}} \left(Z_t^{TD} \right)^{\frac{\sigma_x-1}{\sigma_x}} + \left(s^M\right)^{\frac{1}{\sigma_x}} \left(Z_t^{M} \right)^{\frac{\sigma_x-1}{\sigma_x}} \right]^{\frac{\sigma_x}{\sigma_x-1}} \tag{2.4}$$

where $Z_t \in \{C_t, G_t, I_t^G, I_t^j, INT_t^j, X_t\}$, with steady state shares governed by s^T and s^M, while σ_{TNT} and σ_x denote elasticities of substitution. For (sector specific) private investment goods I_t^j the tradeable share is set to one $s^T = 1$, while for (sector specific) intermediate goods INT_t^j instead of s^T there is a different steady state share of tradeables versus non-tradeables: s_{INT}^j. Exporters only use domestically produced tradeable goods, so for X_t we have $s^T = 1$ and $s^M = 0$.

It can be seen that imports enter in several ways into the multi-layered and overlapping goods market structure described above. First, imports are used directly for the purposes of consumption, investment and assembling intermediate inputs, as they are part of the final goods bundles via (2.4). Second, since intermediate inputs therefore have some import content, imports enter the domestic production process as well, via (2.1). Finally, due to this import content of domestic output, they show up in final consumption bundles also indirectly, via domestically produced goods.

Demand for each of the above types of goods depends on their relative prices and the substitution elasticities within composite bundles (see Burgert et al. (2020) for details). Therefore, apart from directly affecting the terms-of-trade, import price movements can have rich cascading effects through the domestic economy. The specific nature of those effects depend crucially on all the relevant substitution elasticities, which in the baseline calibration are set to $\sigma_x = 1.1, \sigma_{in} = 0.5, \sigma_{TNT} = 0.5$.

2.2 Price and wage setting

The above CES structures imply the following price indices for final consumption P_t^C (with the appropriate alternative shares used for private investments and intermediate inputs):

$$P_t^C = \left[(1-s^T)(P_t^{NT})^{1-\sigma_{TNT}} + (s^T)(P_t^T)^{1-\sigma_{TNT}}\right]^{\frac{1}{1-\sigma_{TNT}}} \tag{2.5}$$

$$P_t^T = \left[(1-s^M)(P_t^{TD})^{1-\sigma_x} + (s^M)(P_t^M)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}} \tag{2.6}$$

where P_t^T is the price index for tradeable goods. Domestic output prices P_t^{NT} and P_t^{TD} are set by domestic firms operating in the two sectors, subject to nominal rigidities, while import prices P_t^M depend on prices set by foreign exporters. These are analogous to domestic exporters who differentiate domestically produced tradeable goods, and set export prices P_t^X that are based on P_t^{TD}, but are also influenced by export market conditions and exchange rate movements, facing price stickiness in foreign currency.

The terms-of-trade is defined as

$$TOT_t \equiv \frac{P_t^X}{P_t^M} \tag{2.7}$$

Firms in both domestic sectors are subject to nominal rigidities captured by γ_p. This gives rise to a standard New Keynesian Phillips Curve, whereby sectoral output price inflation $\pi_t^j \equiv \frac{P_t^j}{P_{t-1}^j} - 1$ reacts sluggishly to deviations of the firm's real marginal costs (i.e. inverse markups) from their desired level:

$$\pi_t^j = E_t\left(\Lambda_{t,t+1}^r \frac{O_{t+1}^j}{O_t^j}\right)\pi_{t+1}^j + \frac{1}{\gamma_p}\left[\eta_t^j - \left(\frac{\sigma^j - 1}{\sigma^j} - \varepsilon_t^{\eta,j}\right)\right] \tag{2.8}$$

where $\Lambda_{t,t+1}^r$ is the stochastic discount factor of Ricardian households who own the firms, and η_t^j summarizes the real marginal costs of the firm, including wage, capital, and intermediate input costs. Real marginal costs are the inverse of markups, which firms would like to stabilise at their desired level, by trying to raise output prices whenever marginal costs increase. The desired steady state markup is $\frac{\sigma^j}{\sigma^j-1}$, stemming from market power under monopolistic competition, and can be subject to exogenous shocks $\varepsilon_t^{\eta,j}$. The expectation term $E_t\pi_{t+1}^j$ includes some backward-looking element as well.

Monopolistically competitive labour unions set nominal wages W_t, subject to nominal rigidities γ_w, giving rise to a wage Phillips Curve determining nominal wage inflation $\pi_t^w \equiv \frac{W_t}{W_{t-1}} - 1$:

$$\pi_t^w = E_t\left(\Lambda_{t,t+1}^{tot}\frac{L_{t+1}}{L_t}\right)\pi_{t+1}^w + \frac{\theta^r}{\gamma_w}\left[mrs_t^{L,C} - \frac{\theta^r-1}{\theta^r}\frac{(1-\tau_t^L)W_t}{P_t^C} + \frac{BEN_t}{P_t^C}\right] \tag{2.9}$$

which captures labour supply decisions. Workers would like to stabilise the after-tax real consumption value of their wages, with a desired "markup" $\frac{\theta^r}{\theta^r-1}$ over the marginal rate of substitution between leisure and consumption $mrs_t^{L,C}$.[10] The expectation term $E_t\pi_{t+1}^w$ includes some backward-looking element as well.

Wages are part of the firm's real marginal costs, influencing price pressures – while at the same time domestic output prices affect the real wage of workers, thereby influencing wage pressures. Since not all price and wage decisions are taken simultaneously, this can give rise to a "wage-price spiral", as shown by Blanchard (1986) and Lorenzoni and Werning (2023). Even under rational expectations, after a shock there is essentially disagreement between firms and workers about the desired real wage, and reaching the new equilibrium will be a drawn out process due to out-of-sync nominal rigidities. With partially backward-looking expectations, like in this model, the mechanism is further amplified (Alvarez and Dizioli, 2023). Real wage rigidities, as in Blanchard and Galí (2007), would also work in this direction, but those are not considered in the current version of the model.

Real marginal costs are also affected by import prices (via intermediate input costs), which also directly influence consumer prices, and thereby the purchasing power of workers' real wages. Therefore, a change in import prices can set off the above wage-price mechanism starting both from firms and workers. Import prices P_t^M are governed by the following equations:

$$P_t^M = \rho^{im}P_{t-1}^M + \left(1-\rho^{im}\right)\left(\mathcal{E}_t P_t^{X*}\right) \tag{2.10}$$

$$\pi_t^{x*} = E_t\left(\Lambda_{t,t+1}^{r*}\frac{X_{t+1}^*}{X_t^*}\right)\pi_{t+1}^{x*} + \frac{1}{\gamma_X}\left[\frac{P_t^{TD*}}{P_t^{X*}} - \left(\frac{\sigma^x-1}{\sigma^x} - \varepsilon_t^{\eta,x*}\right)\right] \tag{2.11}$$

where $\mathcal{E}_t$ is the nominal exchange rate, and $P_t^{x*} \equiv (1+\pi_t^{x*})P_{t-1}^{x*}$ denotes prices set by foreign exporters in a foreign currency, subject to nominal rigidites γ_x. They also face price stickiness in their destination markets, i.e. in the domestic currency (ρ^{im}). Their marginal costs depend on the prices of foreign tradeable goods that they differentiate. Introducing a markup shock to foreign exporters $\varepsilon_t^{\eta,x*}$ can push up domestic import prices and lead to a terms-of-trade deterioration, thereby constituting an adverse terms-of-trade shock.

[10] Unemployment benefits BEN_t also enter the labour supply decision, pushing wages upwards, as they raise the opportunity cost of working. $\Lambda_{t,t+1}^{tot}$ is the aggregate stochastic discount factor for all households in terms of the consumption good, while L_t denotes hours worked.

2.3 Monetary policy

The central bank follows a standard Taylor rule, setting the short term nominal interest rate i_t that reacts to deviations of consumer price inflation π_t^C and the output gap $\hat{y}_t$, subject to the effective lower bound $\underline{i}$, and also featuring interest rate smoothing via $\rho_i > 0$:

$$i_t = \max\left\{ \underline{i}; \quad \rho_i \, i_{t-1} + (1-\rho_i)\Big[r + \pi + \phi^\pi(\pi_t^c - \pi) + \phi^y \, \hat{y}_t\Big] + \varepsilon_t^m \right\} \tag{2.12}$$

The policy rule satisfies the Taylor principle of $\phi^\pi > 1$, meaning the monetary policy actively manages the *real* interest rate in order to stabilise inflation around its target π.

2.4 Fiscal policy

2.4.1 Government budget constraint and debt management

The government finances nominal primary deficits $-PB_t$ by issuing nominal bonds B_t with an *effective* nominal interest rate of i_t^g, which leads to the following government budget constraint:

$$B_t = (1 + i_{t-1}^g)B_{t-1} - PB_t$$

$$b_t = -pb_t + \underbrace{\left(\frac{1 + i_{t-1}^g}{1 + \pi_t^Y}\right)}_{\equiv 1 + r_{t-1}^g} \frac{1}{1 + g_t} \, b_{t-1} \tag{2.13}$$

where $b_t \equiv \frac{B_t}{P_t^Y Y_t}$ denotes the debt-to-GDP ratio, $pb_t \equiv \frac{PB_t}{P_t^Y Y_t}$ is the primary budget balance as a percentage of GDP, while $\pi_t^Y \equiv \frac{P_t^Y - P_{t-1}^Y}{P_{t-1}^Y}$ and $g_t \equiv \frac{Y_t - Y_{t-1}}{Y_{t-1}}$ are GDP deflator inflation and real growth rates, respectively. The *ex post* effective real interest rate $r_{t-1}^g \equiv \frac{1+i_{t-1}^g}{1+\pi_t^Y} - 1$ determines the real burden of nominal public debt, which can be reduced by surprise inflation π_t^Y as it revalues nominally fixed debt. The differential $(r_{t-1}^g - g_t)$, in turn is a crucial determinant of debt-to-GDP dynamics, capturing the so called "snowball effect". First differencing (2.13) gives us the decomposition of debt-to-GDP dynamics shown in (1.1).[11]

While formally the above formulation implies one-period debt, we can capture long-term public debt via an effective nominal government interest rate i_t^g to which the pass-through of current short term nominal interest rates is only gradual. In every period, only $(1 - \rho_d)$ portion of the outstanding debt stock matures and needs to be rolled over at the current interest rates – for the rest, past nominal effective rates remain locked in. This implies an average weighted maturity on the outstanding public debt stock equal to $\frac{1}{1-\rho_d}$.

$$i_t^g = \rho_d \, i_{t-1}^g + (1-\rho_d)\Big[i_t + \Psi_t - \chi\Big] \tag{2.14}$$

$$\Psi_t = \psi\left(b_t - \bar{b}\right) + \varepsilon_t^\psi \tag{2.15}$$

[11] For a more detailed derivation of this decomposition of debt-to-GDP dynamics, see Appendix A.1.8.

Current interest rates are mainly influenced by the short term safe rate i_t set by monetary policy, and by a risk premium Ψ_t that the government pays depending on the deviation of debt-to-GDP from an exogenous target $\bar{b}$, which is also the steady state debt ratio (calibrated at 0.85 in the baseline scenario). At the same time, the government also enjoys a constant convenience yield χ, relative to the private sector, due to the liquid safe asset status of its debt.

The nominal primary budget balance PB_t is the difference between the government's nominal revenues REV_t and non-interest nominal expenditures EXP_t:

$$PB_t = REV_t - EXP_t \tag{2.16}$$

2.4.2 Primary expenditures

Nominal primary expenditures EXP_t of the government consist of the following items:

$$EXP_t = P_t^C(G_t + I_t^G) + TR_t + BEN_t \underbrace{(1 - npart_t - L_t)}_{u_t} \tag{2.17}$$

where government consumption G_t and public investment I_t^G have their GDP shares (captured by $\overline{gs_t}$ and $\overline{igs_t}$, respectively) evolve exogenously according to the following processes:

$$G_t = \overline{gs}_t \frac{P_t^Y Y_t}{P_t^C} + \varepsilon_t^{gs} \tag{2.18}$$

$$\overline{gs}_t = (1 - \rho_{gs})\overline{gs} + \rho_{gs}\overline{gs}_{t-1} + \varepsilon_t^g \tag{2.19}$$

$$I_t^G = \overline{igs}_t \frac{P_t^Y Y_t}{P_t^C} + \varepsilon_t^{igs} \tag{2.20}$$

$$\overline{igs}_t = (1 - \rho_{igs})\overline{igs} + \rho_{igs}\overline{igs}_{t-1} + \varepsilon_t^{ig} \tag{2.21}$$

Nominal transfers TR_t are indexed to consumer prices in the baseline scenario, with $\overline{trys}_t$ being exogenous:

$$TR_t = \overline{trys}_t P_t^C + \varepsilon_t^{tr} \tag{2.22}$$

Nominal benefits BEN_t (per unemployed person u_t) provide a constant replacement rate $\overline{benr}$ of current nominal wages W_t:

$$BEN_t = \overline{benr}\, W_t \tag{2.23}$$

Manipulating the exogenous variables $\overline{gs_t}$, $\overline{igs_t}$ and $\overline{trys}_t$ allows us to explore alternative indexation rules to these expenditure items in Section 4.2.

2.4.3 Primary revenues and fiscal rule

Nominal tax revenues REV_t are collected through labour (τ_t^L, $\tau^{ssc,j}$) and corporate income taxes (τ^k), consumption taxes (τ^{VAT}) and lump-sum taxes (T_t):

$$\begin{aligned} REV_t = & \tau_t^L(W_t)\, W_t L_t + \sum_j \tau^{ssc,j}\, W_t L_t^j + \\ & + \tau^{VAT} P_t^C C_t + \\ & + \tau^k \sum_j \left[P_t^j O_t^j - P_t^{INT,j} INT_t^j - (1+\tau_t^{ssc,j})\, W_t L_t^j - \delta^j P_t^{I,j} K_t^j \right] + \\ & + T_t \end{aligned} \tag{2.24}$$

The fiscal feedback rule for debt stabilisation is implemented via lump sum taxes T_t, that react to deviations of the debt-to-GDP ratio from its steady state $\bar{b}$:

$$T_t = T_{t-1} + \phi^b \left(b_t - \bar{b} \right) + \phi^{def} \Delta b_t + \varepsilon_t^T \tag{2.25}$$

This fiscal rule, via sufficiently high coefficients ϕ^b and ϕ^{def} ensures that public debt is eventually stabilised by raising primary surpluses, i.e. that fiscal policy is ultimately passive, and inflation can be pinned down by monetary policy.

Fiscal drag effect :

Without explicitly modelling the nominal wage distribution, we assume a progressive labour tax system which makes the average effective tax rate an increasing function of nominal wages $\frac{T_t^L}{W_t L_t} \equiv \tau_t^L(W_t)$. While fiscal policy eventually aims to stabilise $\tau_t^L(W_t)$ around its steady state τ^L, in the short run nominal tax brackets are not instantaneously adjusted in line with nominal wage inflation, so rising nominal wages could temporarily raise the average labour tax rate, as the nominal wage distribution shifts upwards into higher nominally fixed brackets. This effect is referred to as fiscal drag, or tax bracket creep.[12]

To capture this phenomenon, we approximate $\tau_t^L(W_t)$ with the following functional form:

$$\tau_t^L(W_t) = \iota_t \left[\bar{\tau} - \psi_1 + \psi_1 \left(\frac{W_t}{W} \right)^{\psi_2} \right] + (1-\iota_t)\, \bar{\tau} \tag{2.26}$$

$$\iota_t = \rho_\iota \iota_{t-1} + \varepsilon_t^\iota \tag{2.27}$$

where $\iota_t \neq 0$ means that the fiscal drag effect is operative, with ψ_1 and ψ_2 capturing the steepness and curvature of tax progressivity, respectively. In the steady state $\iota = 0$, i.e. the tax brackets are eventually adjusted to whatever level shift occured in nominal wages, such that we have a stable steady state for $\tau^L = \bar{\tau}$.

[12] For a simple illustration on a stylised wage distribution, see Appendix A.2.3.

3 Baseline results: adverse terms-of-trade shock

Appreciating the precise fiscal implications of terms-of-trade shocks first requires an understanding of how it affects the broader macro economy. A terms-of-trade shock is rather unique in how the associated real income loss triggers negative aggregate demand effects, and how it affects domestic price pressures. In particular, what matters for fiscal dynamics is the evolution of the GDP-deflator, which could initially be more muted despite sharply rising inflation in consumer prices – while second-round effects might be stronger later on. Section 3.1 discusses these macroeconomic effects before turning to their fiscal implications in Section 3.2.

3.1 Macroeconomic effects

3.1.1 Terms-of-trade loss and real domestic income

The baseline shock that we consider in this simulation exercise is a series of adverse terms-of-trade shocks, which were important sources of the recent spike in European inflation. These are modelled as persistent markup shocks to foreign exporters $\varepsilon_t^{\eta,x*}$ in (2.11), which drive up import prices for the domestic economy, resulting in a deterioration of its terms-of-trade. The shock series is assumed to hit Europe in the first and third quarter of 2022, and is calibrated such that annual consumer price inflation for 2022 rises by 1 percentage points relative to the steady state.[13]

Rising import prices have a *direct* first round effect on consumer price inflation as imported goods are part of the final consumption basket – as well as an *indirect* first round effect via imported intermediate inputs in domestic production (see top right panel of Figure 1). To the extent that domestic producers can pass on their rising imported input costs, the price of domestically produced goods in the consumption basket also increases, even before the price index for domestic value added (GDP-deflator) would rise amid second round effects.[14]

The deteriorating terms-of-trade erodes the purchasing power of the European economy as its imports become more expensive relative to the products it exports. This *terms-of-trade loss* can also be captured by the wedge that is driven between final consumer prices (P_t^C) and the GDP-deflator (P_t^Y). Essentially, the value added Europe produces, expressed in terms of the basket of goods it consumes (real gross domestic income or $rGDI_t$), declines even if the volume of production (real GDP or Y_t) remained unchanged. This divergence between real domestic

[13]The underlying decline of the model economy's terms-of-trade $ToT_t = \frac{P_t^X}{P_t^M}$ is 7% in the first year, and 10% in the second year (relative to the steady state), before it gradually converges back to zero (see Figure 30). The size of the shock is illustrative, chosen such as to arrive at a 1 pp. increase in CPI inflation on impact. That said, it is close to the observed deterioration in the EU's terms-of-trade of a cumulative 9% over 2021-22, which also makes it a significant disturbance to affect inflation.

[14]Initially, the GDP-delfator actually *declines*, as is explained later in this section.

incomes and real domestic production is shown by the terms-of-trade gain θ_t^{ToT}:

$$rGDI_t \equiv \frac{P_t^Y}{P_t^C} Y_t \;=\; Y_t + \underbrace{\left(\frac{P_t^Y}{P_t^C} - 1\right) Y_t}_{\theta_t^{ToT}} \tag{3.1}$$

$$\Delta NX_t = \underbrace{\Delta Y_t - \Delta D_t}_{\Delta NX_t^{vol}} + \underbrace{\left(1 - \frac{1}{P_t^C}\right) NX_t}_{\Delta NX_t^{price}} + \underbrace{\left(\frac{P_t^Y}{P_t^C} - 1\right) Y_t}_{\theta_t^{ToT}} \tag{3.2}$$

As shown by the upper left panel of Figure 1 (and of Figure 11), the terms-of-trade loss contributes significantly to the decline of Europe's real domestic income, which therefore falls more than real GDP. It also has a major impact on the evolution of the trade balance (see bottom left panel Figure 1), where adverse relative price effects θ_t^{ToT} initially dominate beneficial volume effects ΔNX_t^{vol}, resulting in a characteristic J-curve pattern: even though Europe starts importing relatively less goods than it exports already from the second year onwards, the (nominal) value of trade balance NX_t will stay lower for several more years, raising the external financing needs of the domestic economy.[15]

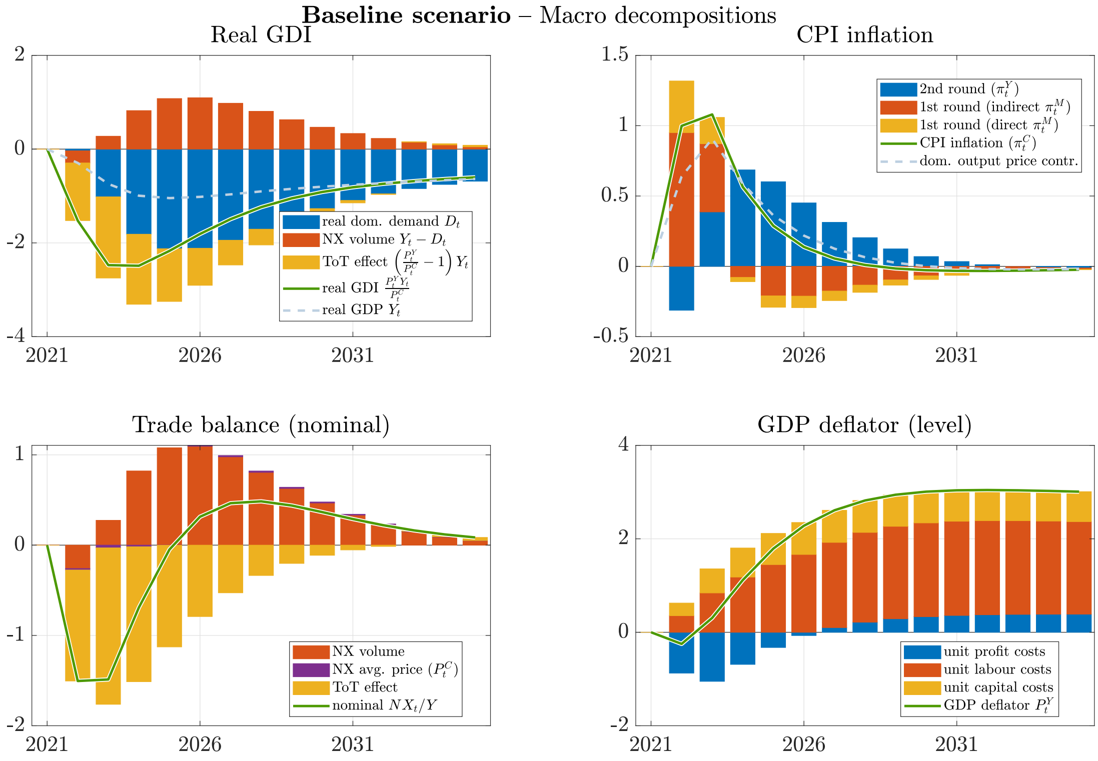

Figure 1: Baseline scenario impulse responses to unexpected ToT shocks, such that CPI inflation would rise by 1 pp. rGDI and and GDP deflator are expressed as percentage deviations from their steady state, CPI inflation as percentage point deviation from steady state, while the nominal trade balance is in level deviations expressed as a percentage of steady state GDP. Bars depict contributions to those changes. Underlying equations are (A.3), (A.20), (A.1), (A.15), respectively. *Source: Commission services.*

[15] The decomposition in (3.2) is inspired by Oblath (2019) and Oblath (2022). D_t denotes CPI-deflated real domestic demand, for more details see the Appendix, at equation (A.1).

In addition to lowering real incomes through the terms-of-trade loss, the shock also leads to a fall in real GDP. Domestic production is hit both via demand and supply channels. On the demand side, there are two counteracting forces. On the one hand, the declining real income of households depresses their overall consumption, since several of them are liquidity constrained and cannot smooth their consumption in the face of fluctuating real incomes. If consumed in unchanged proportions, this would lower demand not just for imports but also for domestically produced goods, *hurting* GDP. On the other hand, the increase in the relative price of imports induces some substitution away from them and towards domestic goods, which *supports* GDP. In other words, the deteriorating terms-of-trade erodes the purchasing power of the domestic economy, while at the same time also making it more competititve: the negative income effects are counteracted by beneficial *expenditure switching* effects. As Auclert et al. (2022) show, the balance of these forces depends on the degree to which consumption smoothing is available (e.g. the share of liquidity constrained households) and on the elasticity of substitution between imported and domestically produced goods.[16] In our baseline simulations domestic demand is also hurt via intertemporal substitution channels, as an active monetary policy raises real interest rates to fight inflationary pressures, discouraging consumption smoothing even by non-constrained households.

On the supply side, more expensive and imperfectly substitutable imported intermediate inputs raise marginal costs, acting as a cost-push shock, and encouraging domestic firms to scale back production.[17] As Chan, Diz and Kanngiesser (2022) show, the strength of this channel depends on the degree of nominal rigidities and the elasticity of substitution between imported inputs and domestic factors of production. There is expenditure switching also on the supply side, which can constrain the rise in overall marginal costs by prompting firms to shift away from more expensive imported inputs towards relatively cheaper domestic ones, labour and capital. Therefore, even if gross *final output* declines, real domestic *value added*, i.e. real GDP does not need to.[18] But with relatively high complementarities in production, such subtitution is not strong enough to avoid adverse supply side effects on GDP. Chan, Diz and Kanngiesser (2022) also point out the interaction of these supply side channels with aggregate demand in the presence of liquidity-constrained households. Under high enough complementarities, rising imported input costs lower the income share of domestic production factors in total output,

[16]In their representative agent (RA) model, negative income effects are very weak due to low MPCs, and expenditure switching leads to rising GDP even with low elasticities. In their heterogeneous agent (HA) model, negative income effects are strong, but sufficiently high substitution elasticities can offset this such that GDP would not fall, and might even expand. With unitary elasticities GDP increases in the HA model by the same amount as in the RA model: the boost to incomes from expenditure switching exactly offsets the effect of the terms-of-trade loss on real incomes, such that the negative income effect vanishes, overal consumption remains unchanged, and only expenditure switching remains from imports towards domestic goods (raising overall GDP).

[17]Under nominal rigidities, rising marginal costs lead to falling profit markups, which firms will try to recover by gradually increasing their prices. This would directly help their markups, while also curtailing their demand-determined output, which in turn reduces marginal costs.

[18]Domestic value added (GDP) is defined as the difference between gross final output and intermediate inputs.

reducing the income of liquidity-constrained domestic workers and thereby hurting aggregate demand.[19]

As a result of these demand and supply side forces, in our baseline simulations with QUEST real GDP declines, so real domestic incomes take a further hit on top of the erosion of purchasing power. Expenditure switching effects are not strong enough to offset the negative income effects stemming from the terms-of-trade loss, the demand-cooling effects of monetary tightening via intertemporal substitution, and the detrimental effects of rising marginal costs on production. This is illustrated on the upper left panel of Figure 1: the fall in real domestic incomes (green line) pulls down domestic demand (blue bars), which is not fully offset by rising net export volumes (red bars), resulting in lower real GDP levels (dashed line). Increasing net export volumes reflect the expenditure *switching* effects of substituting away from relatively more expensive imported consumption goods and intermediate inputs, as well as the expenditure *changing* effects of an overall decline in spending, on imports and domestic goods alike (i.e. even if there were no substitution effect).[20] Consumption smoothing is captured by an initially widening trade deficit (sum of red and yellow bars), which allow domestic spending (blue bars) to fall less than real domestic income (green line) – even if it is not shielded perfectly.

3.1.2 Real income distribution

As we have seen, the terms-of-trade loss erodes the purchasing power of the domestic economy *as whole*, making it inevitably poorer as the real consumption value of what it collectively produces declines. Domestic agents might try to protect their real incomes, but at an aggregate level all they can do is shift purchasing power losses among themselves – unless the underlying terms-of-trade shock reverses, shifting losses back to foreigners, somebody must end up bearing them. This struggle might result in higher domestic price pressures in the process (second round effects, see later), without managing to raise *aggregate* real domestic income. However, the *distribution* of that real domestic income within the economy, i.e. the outcome of the struggle, is less obvious and depends on several factors.

Figure 2 maps the distribution of real incomes across domestic factors of production. As the upper right panel shows, workers, capital owners and firm profits alike take a hit from the shock, facing lower real incomes in absolute terms. However, the upper left panel also demonstrates that they do so to varying degrees. The income share of firm profits declines, mirroring a rise in

[19] Sticky prices, however, mean that firm profits also take a hit, so the labour share *in domestic value added* (GDP) might still increase (see later). It depends on the relative strength of nominal rigidities and elasticities of substitution in production. In general, more flexible prices can protect firm profits from rising marginal costs – at the expense of the real income of production factors.

[20] Note that following this shock, European exports are not becoming more competitive, so expenditure switching mainly works via imports. The reason is that the markup shock we consider here only raises the prices set by foreign exporters, but not the prices of foreign output and consumption in general, so for foreign households European products are not becoming relatively cheaper. In fact, European exporters even *lose* competitiveness as imported inputs become costlier.

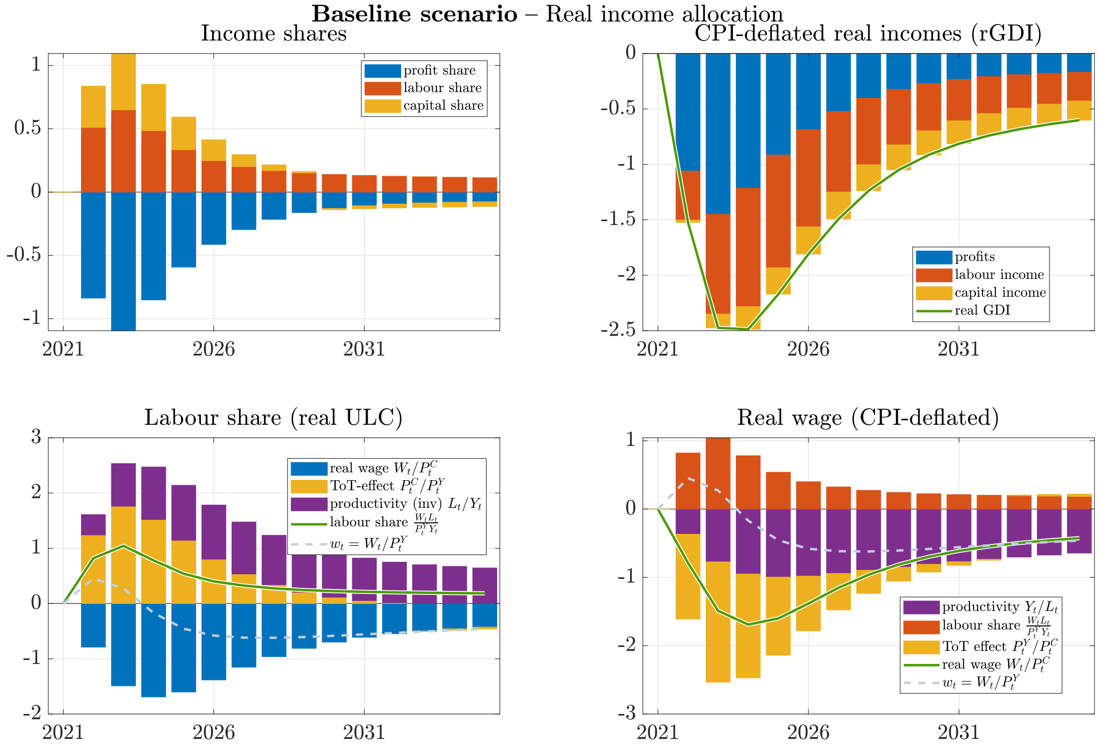

Figure 2: Baseline scenario impulse responses to an adverse terms-of-trade shock. Green lines show percentage deviations from steady state, while bars capture contributions to that change. Income shares (out of GDP) are shown as percentage point deviations from their steady state. Underlying equations are (A.10), (A.11), (A.12), (A.14). *Source: Commission services.*

labour's and capital's share of income. One reason behind this is that under nominal rigidities rising marginal costs depress the markups of firms, since they cannot fully pass on the higher cost of their imported inputs into sticky final output prices. Lower average markups directly translate into lower profit share out of *gross output*. If the share of domestic factors of production (labour and capital) out of rising *total costs* was not suppressed too much by imported inputs, then lower markups also translate into a lower profit share out of *domestic value added*, which is the sum of the incomes of all domestic agents (i.e. profits, labour and capital) – that is what the chart displays.[21] The flipside of these declining profit shares is the rising income share of labour and capital.[22]

[21] A stylised illustration of the relationship between gross output $P_t O_t$, value added $P_t^Y Y_t$ and total costs TC_t, with Ω_t denoting nominal profits. See Appendix A.2.1 and equation (A.26) for more details.

$$P_t O_t = \Omega_t + \underbrace{W_t N_t + i_t^K K_t + P_t^M Int_t^M}_{TC_t} =$$

$$= \underbrace{\Omega_t + W_t N_t + i_t^K K_t}_{P_t^Y Y_t} + P_t^M Int_t^M$$

[22] In light of the recent discussions about *rising* profit shares in Europe (e.g. Arce, Hahn and Koester (2023)), recall that the simulations here isolate the effect of an adverse terms-of-trade shock only. In other words, they could still be consistent with the actually observed rise in the profit share in Europe, as that is the outcome of also several other types of shocks hitting the economy (e.g. post-pandemic reopening).

With sufficiently stronger complementarities between domestic production factors and imported inputs, the share of labour and capital out of total production costs would drop enough in response to rising import prices, such that labour and capital incomes are hurt even more than profits, leading to a *rising* profit share out of domestic value added (i.e. GDP). In other words, the above described pattern of income distribution is not a universal feature of an adverse terms-of-trade shock, but depends crucially on the interplay between substitution elasticities, and nominal price and wage rigidities (as also pointed out by Lorenzoni and Werning (2023) and Chan, Diz and Kanngiesser (2022)). Indeed, for this reason we explore some alternative scenarios in Section 4.4 (see also Figure 27). However, in our baseline scenario, even with the relatively low elasticities (under which expenditure switching effects were not strong enough to offset the terms-of-trade loss), the share of labour income out of GDP increases.

Despite a rising labour share, workers are not better off though. As the bottom right panel of Figure 2 shows, the rising labour share is counteracted by lower labour productivity and an erosion of purchasing power due to the adverse terms-of-trade effect, resulting in a marked decline of CPI-deflated real wages. A rising labour share would normally mean that workers are moderating their wage demands as real wage growth outstrips productivity growth. However, with large adverse terms-of-trade effects this is no longer true for the *consumption value* of wages, which can still decline more than labour productivity, prompting workers try and recover their purchasing power. The corollary to this is illustrated on the bottom left panel of Figure 2: despite the consumption value of wages falling more than productivity, once we adjust for the terms-of-trade effect, *GDP-deflated* real wages are higher, explaining the rising labour share.

In summary, despite the (by construction) offsetting income share fluctuations out of GDP, both firms and workers have an incentive to set higher prices and wages. Firms would aim to rebuild their reduced markups, while workers would like to recover the consumption value of their salaries. This collective attempt to offset the aggregate purchasing power losses brought by the terms-of-trade loss is one of the channels which explains the evolution of domestic prices pressures and second round effects.[23]

3.1.3 Domestic price pressures and second round effects

As already touched upon, rising import prices exert *first round* upward pressures on domestic consumer price inflation i) directly via imported consumption goods, and ii) indirectly via imported intermediate inputs in production, depending on their respective shares in the consumption basket and in gross final output. The latter indirect first round effects imply that the price of domestically produced consumption goods faces upward pressure even before we

[23]Note that this effect (i.e. trying to recover real income losses by raising prices/wages) is a charachteristic of the terms-of-trade shock. Alternatively, if real domestic incomes were to fall by a similar amount, but due to a negative demand shock, we would not see this incentive to raise prices. Falling real marginal costs under nominal rigidities would lead to rising markups, prompting firms to gradually *lower* prices, while the mirrored rise in the labour share would also constrain nominal wage inflation.

consider any change in the price index of domestic value added, i.e. in the GDP deflator. This is the decomposition that the top right panel of Figure 1 displays.[24] But the price of domestic value added does change, which is referred to as iii) *second round* effects in the chart. It is these domestic price pressures via the GDP deflator that we turn to discuss now, given its highligted role played in fiscal dynamics.

Final domestic output prices are subject to nominal rigidities and price setting is governed by a New Keynesian Phillips Curve (NKPC). The price of domestic value added is an implicit residual, after subtracting the effect of imported input costs. As such, it is influenced by 3 counteracting forces:

a) mechanical effect, due to the changing share of import costs in nominal output

b) aggregate demand effects, due to domestic slack

c) attempt to recover real income loss ("wage-price spiral")

These channels all depend on a complex interaction of many features of the economy, including nominal rigidities, import substitution, expectation formation and monetary policy reaction, while also taking into account the evolution of the real economy discussed above. Note that often it is only the last channel that is meant by "second round effects", but here we use that term more broadly, referring to domestic price pressures on top of (first round) imported inflation, as captured by the GDP deflator.

a) Mechanical effect: This channel is analogous to the relative price effect in the trade balance (terms-of-trade loss).[25] To the extent that rising imported input prices do not perfectly pass through to final output prices, the difference must be mechanically absorbed by a lower price of domestic value added. On the one hand, the strength of this channel depends on the degree of nominal rigidities. As domestic firms cannot fully compensate for higher import costs by rising their sticky output prices, unit profit costs fall, pulling the GDP deflator downwards (see bottom right panel of Figure 1). On the other hand, import elasticities also play a role to the extent that they affect how other components of the firm's marginal cost evolve. With strong enough complementarities between domestic value added and imported inputs, high import prices are putting a downward pressure on wages and capital rental rates. Then the firm does not need to increase final output prices as much (even under fully flexible prices), as rising import costs are absorbed by cheaper domestic production factors, but that would be pulling the GDP deflator downwards all the same (see Figure 26).

[24] For a detailed derivation of this decomposition, and the steady state shares of imported inputs and domestic value added in final output, see Appendix A.1.7. See also Battistini et al. (2022) for a decomposition which combines first round indirect and second round effects.

[25] For a more detailed illustrative example, see Appendix A.2.2. Keeping quantities fixed, if the nominal value of imported input costs rises by more than the nominal value of gross output, it mechanically implies a falling nominal GDP, and therefore (with fixed quantities and real GDP) also a falling GDP deflator.

b) Aggregate demand effects: As we have seen above, domestic demand falls due to a combination of negative income effects from the terms-of-trade loss and monetary tightening, that are not fully counterbalanced by higher net external demand through expenditure switching. Thereby real GDP declines and aggregate demand becomes weaker. Larger slack in the economy moderates domestically generated price pressures, as lower production levels imply lower real marginal costs for firms, mainly via softening wage requests from the labour supply side. This is the standard textbook mechanism in the New Keynesian Phillips Curve.

c) Attempt to recover real income loss: As we have seen, domestic agents cannot all escape the fall in their real income imposed by the terms-of-trade loss – but the struggle to *try*, generates domestic price pressures, as they attempt to shift purchasing power losses to others. On the one hand, firms aim to rebuild their lower markups hurt by rising imported input costs, which prompts them to raise prices as much as nominal rigidities allow. On the other hand, from the labour supply side, workers also want to recover losses in the consumption value of their wages, prompting them to ask for higher nominal wages (as much as nominal wage rigidities allow). Higher wages then raise marginal costs of the firm further, bringing markups back down and fuelling more price increases – which in turn erode the real wage again, fuelling more wage inflation, and so on. The emergence of such "wage-price spiral" (or more precisely, wage-price *persistence*), where price pressures increasingly come from domestic sources and stay persistent, depends on several factors.

Lorenzoni and Werning (2023) interpret the New Keynesian wage-price persistence mechanism fundamentally as a disagreement about the real wage between firms and workers, thereby pointing to a *distributional conflict* as the proximate cause of inflation. As Blanchard (1986) has shown, these inconsistent real wage targets can exist even under fully forward-looking rational expectations, as long as not all price and wage decisions are taken simultaneously – a feature of every New Keynesian model with staggered price (and/or wage) setting.[26] Instead of reaching the new equilibrium real wage instantaneously, the process can be drawn out due to out-of-sync nominal rigidities (with its exact pattern depending on relative price and wage stickiness). More backward-looking expectation formation also increases price inertia, a point emphasised by Albrizio et al. (2022) and Alvarez and Dizioli (2023). *Real* wage rigidities are another feature that can result in more wage-price persistence, and eventually lead to higher nominal price and wage

[26] A simplified price and wage NKPC for illustration, where firm real marginal costs η_t depend on producer real wages w_t and the relative price of imported inputs p_t^M, while the labour supply of workers depends on the CPI-deflated real wage $w_t - p_t^C$:

$$\begin{aligned} \pi_t &= \beta\,\pi_{t+1} + \lambda\,\hat{\eta}_t + \varepsilon_t \\ \pi_t^w &= \beta^w\,\pi_{t+1}^w + \lambda_w\Big[(\sigma+\varphi)\hat{y}_t - (\hat{w}_t - \hat{p}_t^C\uparrow)\Big] + \varepsilon_t^w \\ \hat{w}_t &= \hat{w}_{t-1} + (\pi_t^w - \pi_t) \\ \uparrow\hat{\eta}_t &= f(\hat{w}_t, \hat{p}_t^M\uparrow) \end{aligned}$$

Price and wage setting in QUEST has the same mechanisms operating at its core. See Phillips curves (2.8), (2.9).

levels (with the same real wage) than otherwise, as demonstrated by Auclert et al. (2022).[27]

The final dynamics of the GDP deflator are the result of the balance of the three channels described above. As the right panels of Figure 1 shows, initially the mechanical downward effect of incomplete price pass-through and the price moderating effect of weak aggregate demand dominates, and the GDP deflator actually *falls* in the first year of the shock. Later on, however, the attempt by domestic agents to recover their real income losses becomes the main driver of domestic price pressures, pushing the GDP-deflator up, even as first round imported inflation subsides.

The bottom right panel of Figure 1 takes another approach to decomposing the dynamics of the GDP deflator, to look at the underlying distribution of profit, labour and capital incomes. This shows that the initial decline of the GDP deflator is driven mainly by a fall in unit profit costs, while nominal unit labour costs (ULC) contribute positively.[28] This uneven contribution is the flipside of the income distribution dynamics already discussed and shown in Figure 2,[29] where we are also reminded that despite trying, none of the domestic agents managed to avoid lower real incomes. The increase of nominal ULC occurs in spite of the marked fall in CPI-deflated real wages: nominal wages are actually increasing, while labour productivity drops (see bottom right panel of Figure 12).

Taken together with the decline in real GDP, the just discussed evolution of the GDP deflator implies that nominal GDP is lower in the first two years of the simulation, and becomes higher only thereafter (see bottom left panel of Figure 12). It is important to understand the forces behind nominal GDP dynamics, as it is a very consequential variable for fiscal indicators, given that many of those are expressed as a percentage of nominal GDP – while their numerator might evolve independently.

3.2 Fiscal effects

3.2.1 Debt-to-GDP ratio

The terms-of-trade trade shock leads to rising inflation, which raises the question whether the real value of nominally fixed public debt could be inflated away. First, if we are interested in how public debt evolves as a share of GDP, then the relevant inflation measure is that of the GDP deflator which, as discussed above, exhibits more muted dynamics than consumer prices following a terms-of-trade shock. Second, the path of the debt-to-GDP ratio does not only depend on inflation, but also on the other factors already described by (1.1), namely real GDP growth, nominal interest rates and the primary budget balance. The final effect is a balance of all these forces.

[27]Their open economy model also illustrates that the wage-price mechanism does not only raise the domestic price level, but due to nominal exchange rate depreciation, works also via raising import prices, so that it does not manage to alter the terms-of-trade either, similarly to not being able to persistently alter the real wage.

[28]For the underlying equation see (A.15).

[29]Profit vs labour shares could also go in the opposite direction, as shown in Section 4.4 and Figures 26-27.

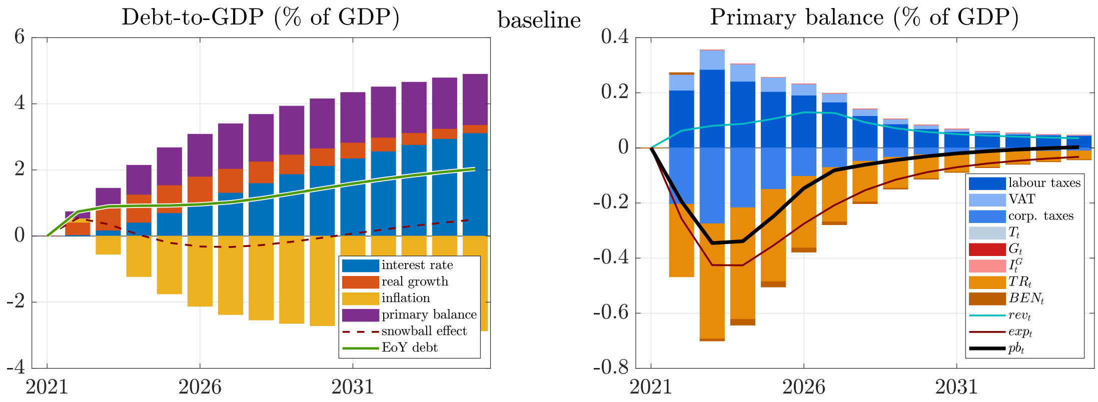

Figure 3: Fiscal dynamics in response to a series of unexpected ToT shocks. Debt and primary balance (expressed as a share of GDP) are shown in percentage point deviation from steady state. Contribution to debt dynamics are based on the cumulated version of (1.1): $b_t - b_0 = \sum_{k=1}^{t} \Delta b_k$ as in (A.23). *Source: Commission services.*

As the left panel of Figure 3 shows, the public debt-to-GDP ratio *increases* in response to an adverse terms-of-trade shock. The reason is that the beneficial effect of (eventually) higher inflation is outweighed by the detrimental effects of higher interest rates, slower real growth and a deteriorating primary balance. Monetary tightening in response to rising consumer price inflation raises interest costs on public debt, even though it does so only gradually due to rather long average maturities of 7 years. Rising real interest rates depress real economic growth further beyond the direct recessionary impact of the shock, while also mitigating the rise in inflation itself, all of which contribute to increase debt-to-GDP.

The so called "snowball effect" due to the interest-growth differential $r - g = (i - \pi^Y) - g$ is the sum of the blue, red and yellow bars on the left chart of Figure 3. As one can see, it mostly takes a positive value which is the opposite of the debt reducing effect that the inflation term alone would imply: higher nominal interest rates and lower real growth counterbalance that. That said, the term is not too high, either, and temporarily even turns negative. The reason for this is mainly long-term public debt, which can protect its issuer against rising short term interest rates, and at the same time impose losses from inflation revaluation on bondholders. Although monetary policy makes sure to raise *short term* real interest rate throughout the simulation horizon, the *effective* real interest rate for long term government debt can initially decline (see Figure 4), as due to longer maturities the pass-through from higher short term nominal rates is only gradual, while the effect of higher inflation is felt immediately.

Therefore, in our baseline scenario "inflating away" the public debt, in the sense of negative effective real interest rates on government bonds, does happen, even if only to a limited extent and for a contained period of time. It is just more than offset by slower real growth and a declining primary budget balance, such that the debt-to-GDP ratio is higher throughout the simulation.

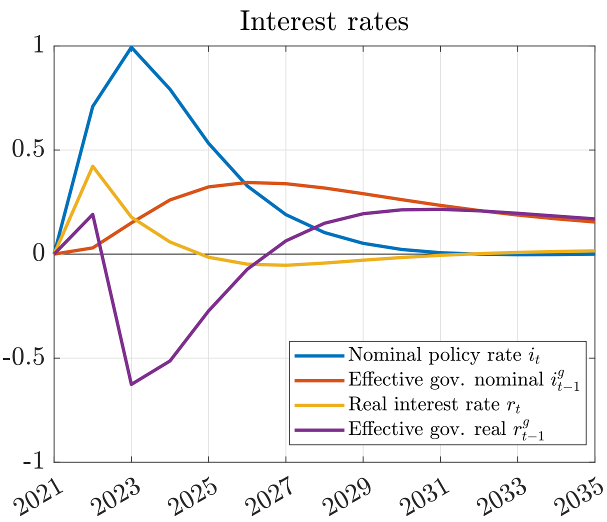

Figure 4: IRFs of interest rates in response to a series of unexpected ToT shocks, in terms of percentage point deviations from steady state. *Source: Commission services.*

3.2.2 Primary budget balance

Zooming in on the behaviour of the primary balance, an adverse terms-of-trade shock leads to a widening deficit as a share of GDP. Importantly, the baseline scenario assumes no discretionary reaction from the side of fiscal policy in response to the increasing debt-to-GDP ratio (nor in response to the developing recession), in order to isolate the effect of the shock from these fiscal decisions. In other words, the debt-stabilising motive in the fiscal rule, that would automatically raise primary balances in response to a rising public debt ratio, is switched off for the first 20 years of the simulation.[30]

This means that even in the absence of any fiscal decisions, the government budget is influenced by the indirect effects of the shock. As the right panel of Figure 3 shows, the decline in the primary balance is driven mainly by expenditure items. In particular, CPI-indexed transfers (e.g. pensions) increase as a share of GDP, when real gross domestic *income* (i.e. CPI-deflated nominal GDP) is falling. To put it another way, expenditure items that are assumed to be fixed in CPI-deflated real terms, increase as a share of GDP when real GDP declines and a wedge opens up between the CPI and GDP-deflator. Automatic stabilisers such as unemployment benefits also increase as the real economy weakens. Government consumption and public investments are assumed to be fixed as a share of GDP in our baseline simulation, therefore they do not contribute to the change in the primary balance. The above effects depend crucially on the indexation rules of expenditure items.[31]

Fiscal revenues come mainly from taxes levied on the incomes of domestic agents as well as on consumption. To a first approximation, under unchanged tax *rates* (and relatively stable

[30] Beyond this point, a debt stabilising *passive* fiscal rule is restored, in order to prevent conflict with an *active* inflation targeting monetary policy rule, thereby ensuring non-explosive stable model dynamics, amid a coordinated monetary-fiscal policy regime.

[31] A sensitivity analysis in Section 4.2 explores this further and shows alternative indexation rules.

consumption-to-income ratios) this would imply that tax revenues change roughly in line with incomes and stay stable as a share of GDP. Therefore, the "denominator" effect coming from changing nominal GDP would not apply to tax revenues very much. The *composition* of these tax revenues would of course change to reflect the fluctuating income shares shown on Figure 2, with declining corporate tax shares mirroring the fall in profit shares and higher labour tax shares driven by a higher labour income share – which is indeed the pattern seen on the left panel of Figure 3. Despite this first pass intutition, however, the chart also shows that overall revenue changes contribute *positively* to the change in primary surplus-to-GDP ratios. There are two main reasons behind this.

First, an important characteristic of the the terms-of-trade shock is that it drives a wedge between the CPI and the GDP deflator. Therefore, absent a large decline in the real consumption-to-output ratio, consumption taxes would increase as a share of GDP due to a beneficial relative price effect: $\tau^C \frac{P_t^C}{P_t^Y} \frac{C_t}{Y_t}$. In this sense, while the terms-of-trade loss erodes the real income of the economy as a whole, it contributes positively to the primary balance as the government collects taxes on these relatively more expensive consumption goods. Another way to see this, is to consider the consumption smoothing mechanism reflected in the widening trade deficit that basically captures the difference between nominal GDP and nominal domestic spending (much of which is private consumption): households borrow externally to be able to afford a more expensive consumption basket in spite of their nominal incomes not keeping up, which results in an increasing *nominal* consumption-to-GDP ratio $\frac{P_t^C C_t}{P_t^Y Y_t}$.[32]

Second, in our baseline scenario labour tax revenues do not only track the labour income share, but the average labour tax *rate* also increases. This phenomenon is often referred to as the "fiscal drag" effect, that occurs in progressive tax systems as the nominal wage distribution shifts into higher (nominally fixed) tax brackets, that are not automatically adjusted in line with inflation. As discussed above, in the baseline scenario nominal wages increase (see bottom right panel of Figure 12), which means that the average labour tax rate also rises (which are modelled for 5 years, as a function of nominal wages capturing European tax progressivity in a reduced form way). Therefore labour tax revenues as a share of GDP increase more than implied by a higher labour income share: $\tau^L(W_t)\frac{W_t L_t}{P_t^Y Y_t}$.

As a balance of the above forces, the primary deficit widens, contributing to the rise in public debt-to-GDP. These results are mostly in line with Bénassy-Quéré (2022*b*).

[32] The *real* consumption-to-GDP ratio, however, declines due to expenditure switching as households substitute away from imports towards domestic goods: real GDP declines less than real consumption, due to rising net export volumes (see top left panel of Figure 1).

4 Sensitivity analysis

As mentioned in the introduction, changing various features of economic policy can have major implications for how an adverse terms-of-trade shock affects debt dynamics. Accordingly, in this section we explore various alternative policy settings, and see how differently the very same shock process propagates through the economy, and what it means for public debt sustainability.

We consider the following policy settings:

- **monetary policy:** reaction parameters to inflation (ρ_i and ϕ^π) in the the Taylor rule (2.12) for the short term nominal interest rate i_t
- **debt management:** average maturity of $\frac{1}{1-\rho_d}$ on the total outsanding debt stock, affecting the pass-through speed of short term interest rates i_t into effective government financing costs i_t^g via (2.14)
- **fiscal policy:**
  - expenditure side: various indexation rules for primary expenditures (see Section 4.2)
  - revenue side (Section 4.3):
    * debt stabilisation motive (ϕ^b) in the fiscal rule for primary balance (2.25), governing the reaction of lump sum taxes T_t to debt ratio deviations
    * progressivity of the labour tax system, governed by ψ_1 and ψ_2 in (2.26)

Beyond these alternative policy environments we also look at the role of substitution elasticities between imported and domestic goods (Section 4.4), and finally we also consider the markedly different consequences of a positive demand shock which has the same inflationary effect as our baseline terms-of-trade shocks (Section 4.5).

4.1 Monetary and debt management policies

Regarding parameters for monetary and debt management policies, this sensitivity analysis will explore changing ϕ^π, ρ_i and ρ_d. Their values are summarised by Table 1.

	a) baseline	**b) stricter IT**	**c) ZLB**	**d) short maturity**
ϕ^π inflation reaction	1.2	3	1.6	1.2
ρ_i interest smoothing	0.8	0.8	1 (for 5 quarters)	0.8
$\frac{1}{1-\rho_d}$ debt maturity	7	7	7	2

Table 1: Sensitivity analysis with respect to monetary and debt management policy settings.

For monetary policy, we consider two alternative scenarios relative to the baseline. While in one of them monetary policy reacts more strongly to deviations from its inflation target ("stricter Inflation Targeting"), in the other the central bank is more accommodative, not responding immediately to inflationary pressures ("ZLB"). We could try to associate our scenarios with the following simplified monetary policy regimes:

- active monetary policy: $\phi^{\pi} > 1$ (Taylor principle) means that the central bank raises short term *real* interest rates in response to inflationary pressures and pins down the price level. In such a regime inflation could not stabilise short term public debt by construction, but rather taxpayers would have to raise primary budget surpluses in the future (or grow it out). (The picture is more nuanced with long term debt).

- passive monetary policy: $\phi^{\pi} < 1$ means that the central bank is not (very) responsive to inflationary pressures. In this case, inflation has the potential to stabilise even short term real debt at the expense of bondholders, by lowering *ex post* real interest rates (even more so in the case of long term debt).

Of course, in all of our scenarios monetary policy remains ultimately active.[33] However, the ZLB scenario *temporarily* suspends this by keeping the policy rate completely unresponsive for 5 quarters, delaying the policy reaction, and bringing in some of the features of a passive monetary policy regime. Considering these alternative scenarios might be relevant, given the discussion around the ongoing euro area monetary policy normalisation, where some commentators blamed the ECB for acting too late, while others warn about the risks of too excessive monetary tightening.

For debt maturity, the rather long weighted average maturity $\frac{1}{1-\rho_d}$ of outstanding government bonds might be a misleading indicator. One reason is that it is biased upwards by a few very long maturity bonds, such that the median ("interest-rate half life") is much shorter, which means that near term financing costs could increase faster than expected (The Economist, 2022). Another reason is that quantitative easing (QE) by central banks has shortened the average maturity of the *consolidated* public balance sheet (i.e. central bank and treasuries combined). QE effectively swapped long-term government liabilities (bonds) for very short-term monetary liabilities (central bank reserves) in the hands of the private sector: while these long bonds still count towards average maturity indicators, they have in effect been bought back by the consolidated government, which issued very short term liabilities in their stead. Another way to see how rising short term interest rates can impact the fiscal balance sooner than suggested by average bond maturities, is to consider that QE has created a big maturity mismatch in the central bank's balance sheet, financing long assets with short liabilities. So as policy rates go up, this immediately leads to losses for the central bank via rising interest costs on its short reserves, lowering seigniorage revenue for the government budget (or even requiring explicit recapitalisation from the treasury). In summary, the fiscal consequences of current monetary tightening will materialise sooner due to past QE programs. While the central bank balance

[33]This is a necessary feature to pin down the price level and have a unique determinate equilibrium in the presence of an ultimately passive fiscal rule that stabilises public debt via raising primary budget surpluses (in our scenarios the fiscal rule is switched on only after 20 years though.)

sheet is not explicitly modelled in QUEST, we can think of public debt in the model as a "synthetic liability" for the consolidated government. If so, the above mechanism due to QE warrants considering an alternative scenario with shorter effective debt maturity in the model.

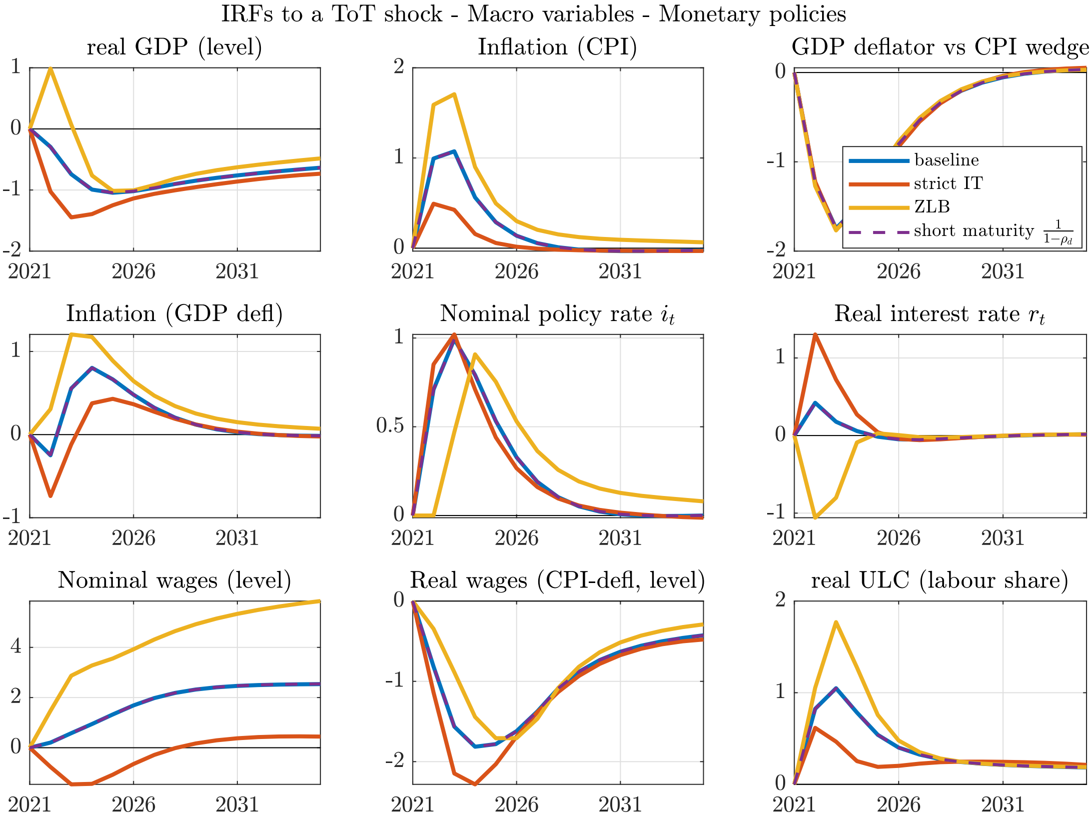

Figure 5: Impulse responses to an adverse terms-of-trade shock under alternative monetary and debt management policies. Inflation and interest rates are in percentage point deviations from steady state, while other variables are in percentage deviations. *Source: Commission services.*

The results of these sensitivity analyses can be seen on Figure 5. Relative to the baseline scenario, more accommodative monetary policy ("ZLB scenario") supports aggregate demand via lower *real* interest rates, leading to relatively higher GDP growth and inflation. Stronger domestic price pressures via second round effects also mean that nominal wages grow more strongly. In contrast, a stricter Taylor rule would lead to changes in the opposite direction.

The fiscal consequences of these different macroeconomic dynamics are shown in Figure 6. In the ZLB scenario, results change even qualitatively as the above effects are so large that they lead to an outright *falling* debt-to-GDP ratio in response to an adverse terms-of-trade shock. This is mainly driven by a hugely beneficial interest-growth differential (snowball effect) that is dominated by the inflation revaluation term (see Figure 13). A more "dovish" monetary stance also lowers the effective nominal interest rate on public debt, thereby directly helping to keep interest payments in check. Together with higher inflation, this leads to a markedly negative effective *real* interest rate on bonds: the implicit fiscal consolidation behind the debt reduction is essentially paid for by levying an inflation tax on long term bondholders. That said, the response of the primary budget balance also flips sign and contributes to lower public debt. The main reasons behind this are that higher real GDP lowers the GDP share of expenditure items that are fixed in real terms (like transfers, see Figure 14), and also that higher nominal wage growth increases the average labour tax rate in a progressive tax system due to the fiscal drag effect.

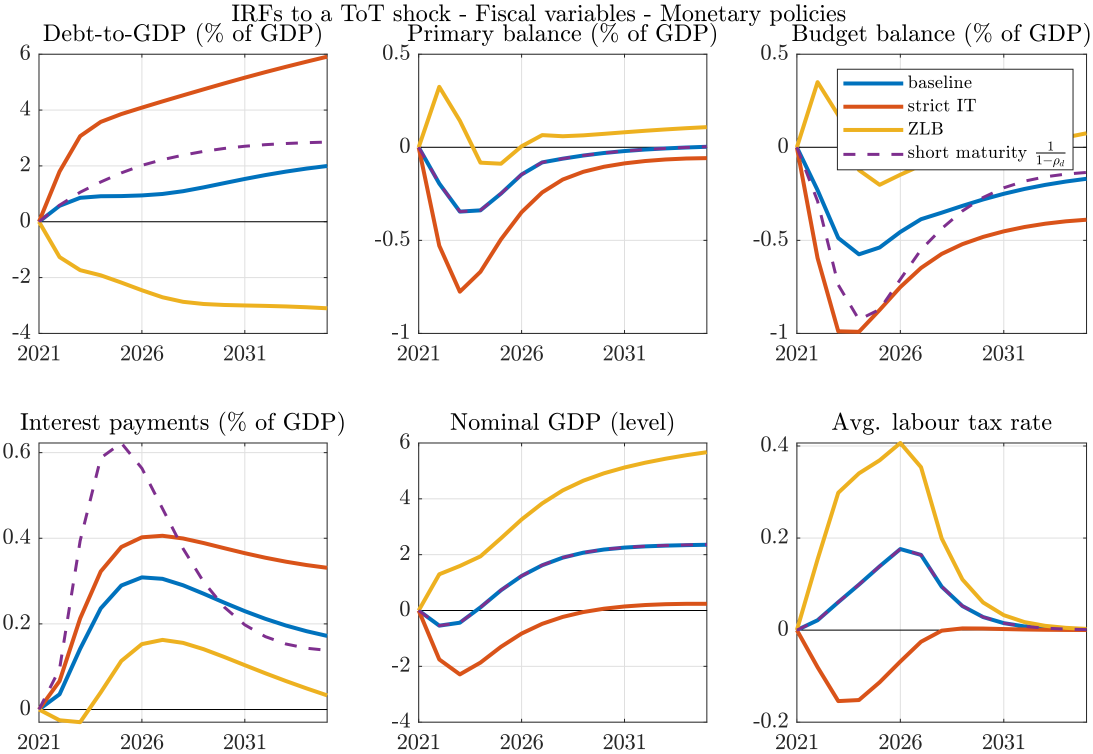

Figure 6: Impulse responses to an adverse terms-of-trade shock under alternative monetary and debt management policies. Nominal GDP is in percentage deviation from steady state, while others are in percentage point deviations as a share of actual GDP. *Source: Commission services.*

In summary, under the less responsive monetary policy of the ZLB scenario, the beneficial effect of higher inflation for easing public debt burdens can outweigh the other detrimental effects of slower real growth, higher interest rates and widening primary deficits, which are themselves mitigated relative to the baseline scenario. However, with more agressive monetary policy reaction (strict Taylor rule) the differences would go in the opposite direction.

Shorter debt maturity speeds up debt-to-GDP dynamics (Figure 6), as higher short term interest rates feed much quicker and to a larger extent into effective government financing costs. However, when the debt stabilisation rule is switched off ($\phi^b = 0$ as in the baseline), this does not affect the primary budget balance, only interest payments. Therefore, without additional fiscal impulse, maturity length does not matter much for the wider macroeconomic effect of the shock either (as can be seen in Figure 5) – but it is quite consequential for debt dynamics.

To put this another way, by shortening the maturity of the *consolidated* government debt vis-a-vis the private sector, QE has also limited the government's ability to erode the real value of its outstanding liabilities by surprise inflation (i.e. its capacity to "inflate away"). Those shortened liabilities are being repriced more quickly at the higher nominal interest rates brought about by rising inflation, thereby offsetting the beneficial effect of higher inflation in the snowball term (i.e. limiting to how low the *ex post* real effective government interest rate can fall).[34]

In summary, the top left panel of Figure 6 illustrates that even in response to the very same underlying inflationary shock process, the trajectory of public debt-to-GDP can span a very wide range depending on monetary policy and debt maturity. These policy settings are therefore highly consequential.

4.2 Fiscal policies – primary expenditures

In the baseline scenario, one of the factors which drove the increase in the debt-to-GDP ratio after an adverse terms-of-trade shock was a deteriorating primary balance, mainly due to transfers being fixed in CPI-deflated real terms. This sensitivity analysis addresses this part of the transmission channel by changing the indexation of some primary expenditure items, as detailed in Table 2.

		a) baseline	b) real fix	c) nominal fix	d) wage-indexed
Government spending					
nominal	$P_t^C(G_t + I_t^G)$	$P_t^Y Y_t$	P_t^C	1	$P_t^Y Y_t$
real (GDP-defl)	$\frac{P_t^C}{P_t^Y}(G_t + I_t^G)$	Y_t	$\frac{P_t^C}{P_t^Y}$	$\frac{1}{P_t^Y}$	Y_t
real (CPI-defl)	$G_t + I_t^G$	$\frac{P_t^Y}{P_t^C} Y_t$	1	$\frac{1}{P_t^C}$	$\frac{P_t^Y}{P_t^C} Y_t$
as % of GDP	$\frac{P_t^C(G_t+I_t^G)}{P_t^Y Y_t}$	1	$\frac{P_t^C}{P_t^Y}\frac{1}{Y_t}$	$\frac{1}{P_t^Y Y_t}$	1
Transfers (e.g. pensions)					
nominal	TR_t	P_t^C	P_t^C	1	W_t
real (GDP-defl)	$\frac{TR_t}{P_t}$	$\frac{P_t^C}{P_t^Y}$	$\frac{P_t^C}{P_t^Y}$	$\frac{1}{P_t^Y}$	$w_t = \frac{W_t}{P_t^Y}$
real (CPI-defl)	$\frac{TR_t}{P_t^C}$	1	1	$\frac{1}{P_t^C}$	$w_t \frac{P_t^Y}{P_t^C} = \frac{W_t}{P_t^C}$
as % of GDP	$\frac{TR_t}{P_t^Y Y_t}$	$\frac{P_t^C}{P_t^Y}\frac{1}{Y_t}$	$\frac{P_t^C}{P_t^Y}\frac{1}{Y_t}$	$\frac{1}{P_t^Y Y_t}$	$\frac{W_t}{P_t^C}\left(\frac{P_t^C}{P_t^Y}\frac{1}{Y_t}\right)$

Table 2: Growth of primary expenditure items under various indexation scenarios. The expressions show the combination of macro variables which a particular expenditure item is indexed to. A value of 1 signals that fiscal policy keeps that expenditure item constant in the indicated terms.

In particular, we consider varying the rules governing government consumption G_t, public investment I_t^G and transfers TR_t.

a) **baseline:** government consumption and investment are fixed as a percentage of GDP, while transfers are indexed to consumer prices (i.e. expressed as a percentage of GDP, transfers fluctuate according to $\frac{P_t^C}{P_t^Y}\frac{1}{Y_t}$).

[34] Of course, the central bank (a sub-branch of the government) could decide not to raise short-term nominal interest rates in response to rising inflation, thereby ensuring that real rates are falling for whatever maturities (a passive monetary policy rule, violating the Taylor principle). That is, coordinated joint monetary-fiscal policies always have the power to inflate away their paper liabilities, even if they have very short maturities. The ZLB scenario discussed above goes some way towards such a policy mix, but only to a limited extent. A truly permanent passive monetary + active fiscal policy regime, however, is not considered in this paper.

b) **real fix:** relative to the baseline, government consumption and investment are fixed in (CPI-deflated) real terms

c) **nominal fix:** government consumption, investment and transfers are all frozen in nominal terms, which means that expressed as a percentage of GDP, they fluctuate inversely with nominal GDP $\frac{1}{P_t Y_t}$

d) **wage-indexed transfers:** transfers are indexed to nominal wages (instead of consumer prices), meaning that relative to the baseline, the growth of transfers changes by the growth of (CPI-deflated) real wages W_t/P_t^C

Given that these alternative fiscal policies do not change the macroeconomic picture significantly relative to the baseline scenario, key macroeconomic variables (like real GDP, nominal GDP or the wedge between CPI and GDP deflator) evolve similarly to what has already been discussed above. With these in mind, the growth factors displayed in the last rows of Table 2 are useful in understanding how the different indexation rules change the contribution of these expenditure items to the primary budget balance as a percentage of GDP.

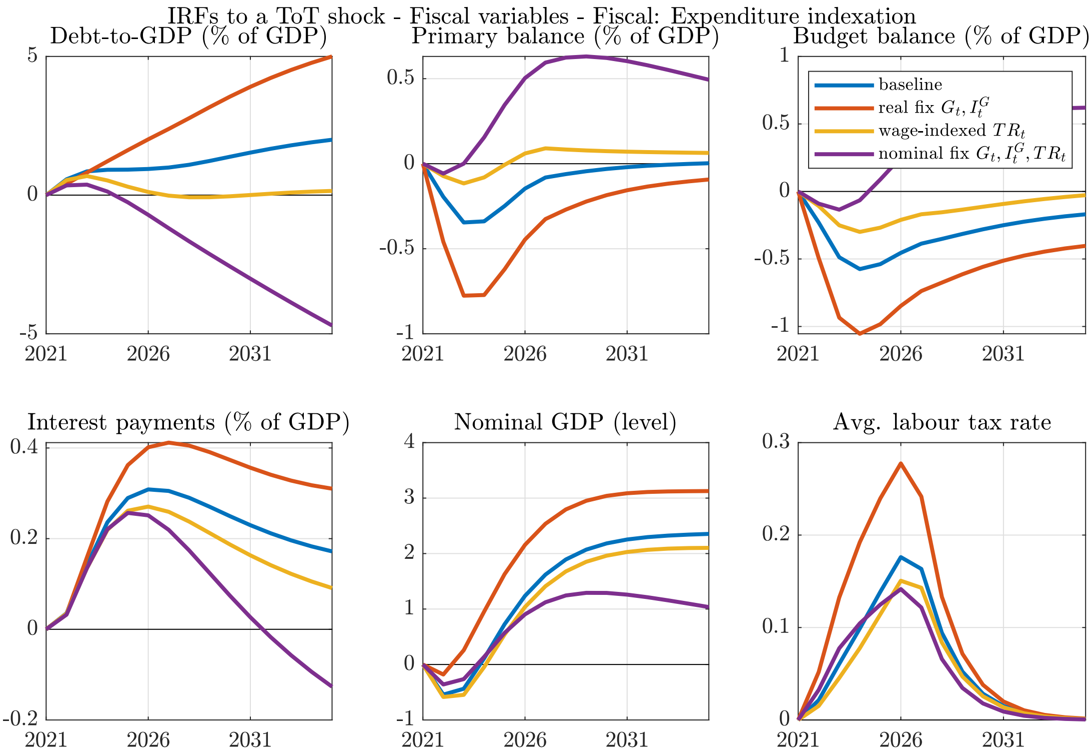

Figure 7: Impulse responses to an adverse terms-of-trade shock under alternative expenditure side fiscal policies. Nominal GDP is in percentage deviation from steady state, while others are in percentage point deviations as a share of actual GDP. *Source: Commission services.*

The results of this analysis can be seen on Figure 7. Fixing government consumption and investment in absolute (CPI-deflated) real terms, when CPI-deflated GDP (i.e. real gross domestic *income*) is falling,[35] raises their contribution to the primary deficit-to-GDP ratio (relative to

[35] Equivalently, when real GDP is falling and a wedge opens up between consumer prices and the GDP deflator.

the baseline where these items were fixed as a percentage of GDP). This is shown in Figure 15. In effect, instead of having nominal expenditures evolve in line with GDP, these items are now indexed to consumer prices which grow at a faster pace during a terms-of-trade shock, so the government has to spends more on them relative to the baseline. This in turn raises the primary deficit, and leads to a faster-rising debt-to-GDP ratio.

Conversely, freezing these primary expenditures in nominal terms, while *nominal (!) GDP* will eventually be higher (as is the case for this shock), helps lower them as a percent of GDP, therefore raising the primary balance and contributing to a lower debt-to-GDP ratio. In essence, the recipients of these expenditures (e.g. pensioners) see their real disposable income eroded by CPI inflation, and they are the ones footing the bill for the fiscal consolidation. Notice how this "benefit erosion" has potentially very different distributional implications compared to the ZLB scenario above, where instead of transfers it was the real value of long term nominal bonds that was inflated away, with bondholders bearing most of the losses.

Wage-indexed transfers also support public finances relative to CPI-indexation in the baseline scenario, since real wages are falling throughout the simulation horizon after a terms-of-trade shock (recall Figure 2). This indexation rule leads to less extreme outcomes than a complete nominal freeze, but it still manages to stabilise the debt-to-GDP ratio, and prevent primary deficits from widening too much.

Potential discretionary fiscal measures in response to the rise in cost-of-living to support the private sector are not considered in this analysis. Beyond the direct further squeeze on the primary balance, the growth and inflationary effect of such measures can vary greatly depending on their precise design and targeting. However, one approximation of possible effects could be gained by comparing the baseline scenario (that has more generous transfer policy) to the alternative scenarios providing lower real transfers to households.

The main takeaway from this exercise is that different expenditure rules are highly consequential for debt dynamics, as demonstrated by the top left panel of Figure 7. However, apart from the extreme case of a complete nominal freeze on expenditures, our baseline result that the debt-to-GDP ratio increases after an adverse terms-of-trade shock, seems fairly robust.

4.3 Fiscal policies – primary revenues

As mentioned in Section 3.2.2, the baseline scenario assumes away any debt stabilisation motive in the fiscal rule for the first 20 years of the simulation, in order to isolate the direct fiscal effect of the terms-of-trade shock from those of the fiscal policy response to the shock. This assumption no doubt contributed to the declining primary balance, which is why in an alternative scenario we consider keeping the fiscal rule parameters at their original value throughout the simulation horizon. This means that in response to a rising debt-to-GDP ratio, the government would raise additional revenue through lump sum taxes sufficiently (i.e. to more than cover steady state real interest costs), thereby increasing primary surpluses in a debt-stabilising way.[36]

	a) baseline	b) linear	c) more progressive	d) debt rule ON
ψ_1 tax steepness	0.35	0	0.45	0.35
ψ_2 tax curvature	0.30	0	0.45	0.30
ϕ^b, ϕ^{def} debt stabilisation	0	0	0	> 0

Table 3: Sensitivity analysis with respect to revenue side fiscal policies. Note that in all scenarios, the labour tax function is fully operative only in the first 5 years, after which its effect gradually declines to zero, with average tax rates converging back to their steady state (as nominal tax brackets are adjusted). The fiscal rule parameters refer only to the first 20 years of the simulation, after which they take the same positive values.

In addition, we also consider two alternative labour tax systems. Under linear labour taxes, the average effective tax rate remains constant at its steady state value even as the nominal wage distribution shifts upwards. Alternatively, this case can also be interepreted as progressive tax brackets being adjusted in real time with wage inflation. This is in contrast to the baseline scenario where the effective average tax rate temporarily increased due to the fiscal drag effect stemming from a combination of rising nominal wages and nominally fixed progressive tax brackets ("tax bracket creep"). Finally, in another alternative scenario we consider even more steeply progressive tax brackets, leading to larger fiscal drag.

The results are shown on Figure 8. When the debt-stabilising fiscal rule is operative, fiscal policy raises the primary balance directly in response to increasing debt-to-GDP, via increasing lump sum tax revenues, which slows down the rise in debt-to-GDP, and eventually brings it down to be stabilised at its original value. Notice how the change in lump sum tax revenues is behind the increasing primary balance (bottom right panel of Figure 17), and how the higher primary balance is the main contributor turning debt-to-GDP dynamics around (Figure 18).

Looking at the alternative labour tax systems, we see how the average effective labour tax rate remains constant in the linear scenario, while it increases more steeply in the very progressive scenario. The fiscal drag effect changes correspondingly, contributing to a more pronounced or more muted widening in the primary deficit, via influencing labour tax revenues. As the charts show, these effects are of a somewhat smaller magnitude than those coming from switching on the fiscal rule, but they still visibly affect the trajectory of the debt-to-GDP ratio. That said, even the fiscal drag coming from a relatively progressive tax system fails to overturn our baseline result, that the public debt ratio rises after an adverse terms-of-trade shock.

Fiscal adjustment on the revenue side can slow and stabilise debt-to-GDP, with taxpayers footing the bill of the fiscal consolidation.[37]

[36] While this alternative fiscal rule is more clearly concerned with debt stabilisation, fiscal policy is categorized as ultimately *passive* also in the baseline scenario, in the sense that evenetually (after 20 years) primary surpluses would be raised to stabilise the debt-to-GDP ratio.

[37] The negative growth effects of such fiscal consolidation are relatively muted when it happens via non-

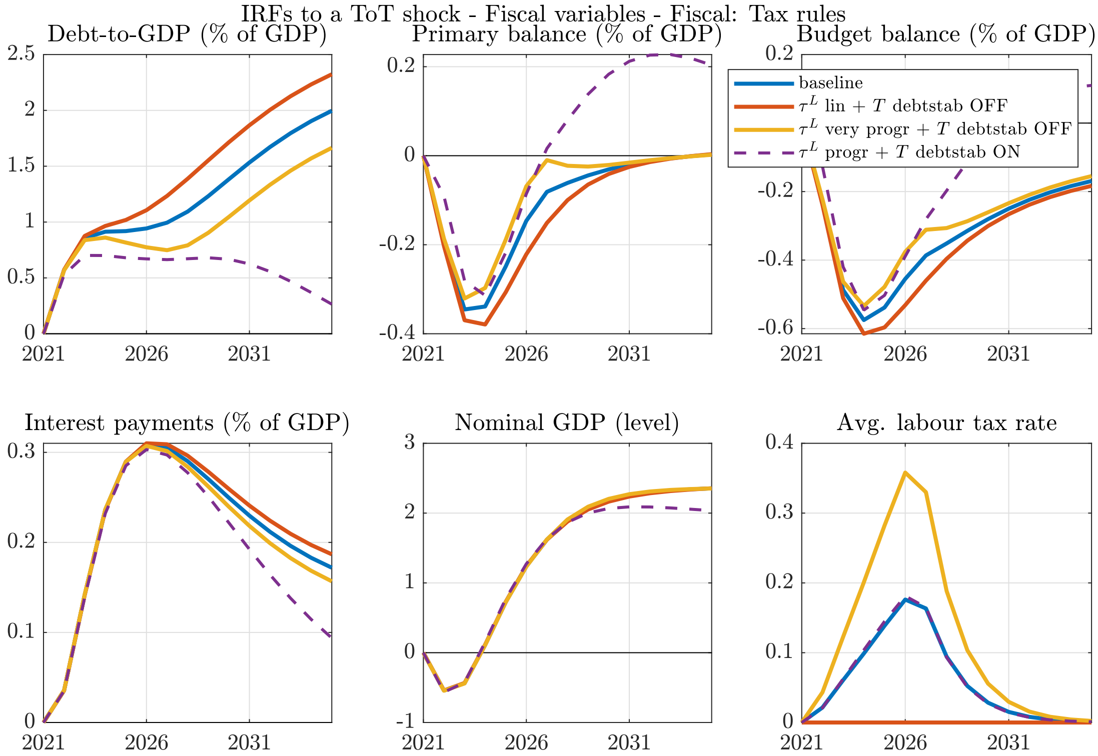

Figure 8: Impulse responses to an adverse terms-of-trade shock under alternative revenue side fiscal policies. Nominal GDP is in percentage deviation from steady state, while others are in percentage point deviations as a share of actual GDP. *Source: Commission services.*

4.4 Elasticities of substitution for imports

Recall that while discussing the transmission channels of an adverse terms-of-trade shock in Section 3, a recurring theme was the importance of the elasticity of substitution between imported and domestic goods, in the consumption basket and for intermediate inputs alike. This is important for the strength of expenditure switching effects and also for determining the income shares of domestic production factors in the face of rising relative import prices. This is also the main point of several studies considering the effect of an energy shock, like Bachmann et al. (2022), Auclert et al. (2022), Chan, Diz and Kanngiesser (2022), Lorenzoni and Werning (2023), and Guerrieri et al. (2022). It is for this reason that we consider some alternative scenarios, where there are stronger complementarities with respect to imports.

In the multi-layered production structure of QUEST there are two main parameters affecting the substitutability of imports. Most directly, σ_x is the elasticity governing substitution between imported and domestically produced tradeable goods, both in the tradeable consumption bundle and in the tradeable intermediate inputs bundle. Then, more indirectly, σ_{in} is the elasticity of substitution between intermediate inputs (including imported ones) and domestic value added

distortionary lump sum taxes, even though Ricardian equivalence does not hold in our model, where aggregate demand effects arise through affecting the disposable income of liquidity constrained agents. When the fiscal rule is specified in terms of the distortionary labour tax rate, then the damage to real growth is persistently larger due to adverse supply side effects.

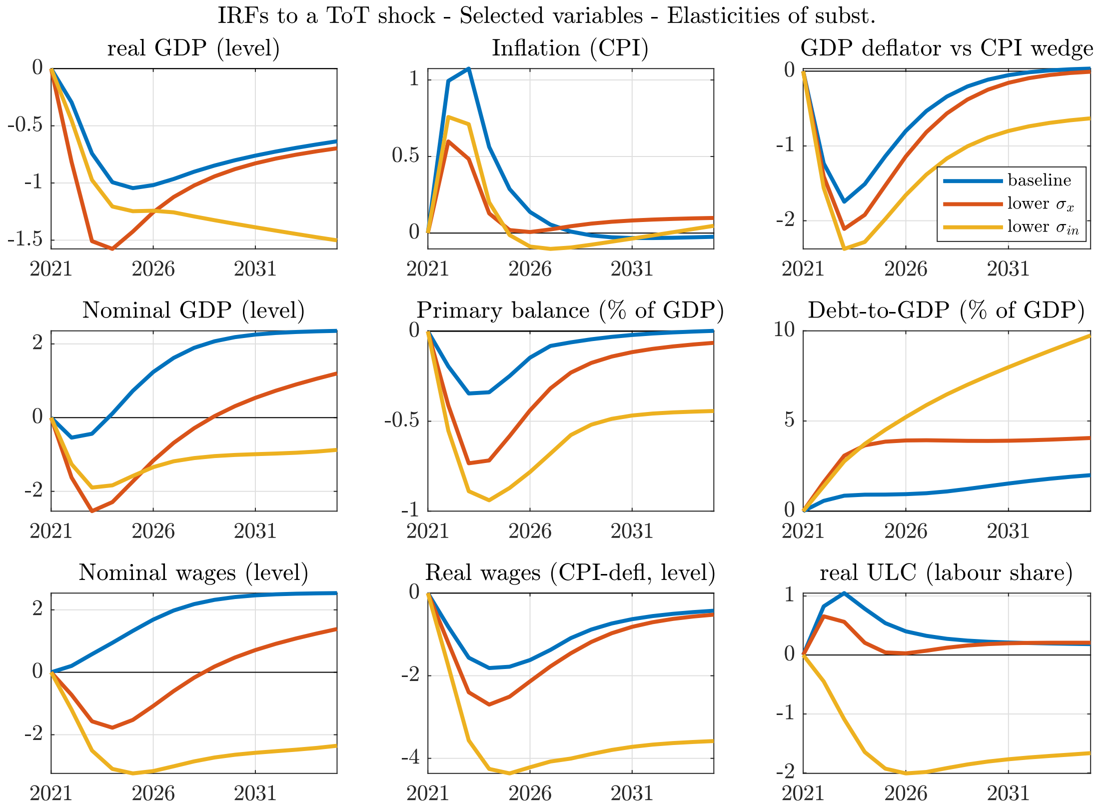

Figure 9: Impulse responses to an adverse terms-of-trade shock under alternative degrees of import substitution. Lines denote percentage deviations from steady state, except for inflation, the primary balance and debt-to-GDP, where they denote percentage point deviations. *Source: Commission services.*

in production (generated by domestic labour and capital). Each alternative scenario studies lowering one of these parameters to 0.3 from their baseline calibration.

The results are displayed in Figure 9 and in the Appendix in Figures 23 through 30. First, lower import substitution elasticities lead to a more adverse terms-of-trade loss (as seen by the wedge between CPI and GDP-deflator), since stronger complementarities with more expensive imports depress demand for domestic products and production factors as well, pulling down their relative prices (e.g wages). More difficult substitutability also weakens the expenditure switching channel, hurting domestic demand and production alike, leading to lower real GDP. As a result, real gross domestic income becomes lower, reducing aggregate demand, which in turn weakens domestic price pressures and second round effects. This also means that the GDP deflator is much lower, pulling down nominal GDP as well.

Due to labour being more complementary to imported inputs, lower nominal wages and weaker unit labour costs are an increasingly important driver behind the falling GDP-deflator, which implies that the labour share of GDP becomes lower. This phenomenon is especially pronounced in the lower σ_{in} scenario, which directly hurts substitution towards domestic production factors like labour. As Chan, Diz and Kanngiesser (2022) explains, distributing real income away from high-MPC workers (relative to consumption-smoothing firm owners) hurts aggregate demand still further, pulling domestic prices downwards more. These results are in line with Battistini et al. (2022) who find that lower labour share developments can constrain second-round effects. These findings also underline the importance of distributional outcomes in driving aggregate demand and domestic price pressures in the presence of household heterogeneity.

The fiscal consequences of this overall more bleaky macroeconomic picture are wider primary deficits and a higher debt-to-GDP path. As Figure 24 shows, the main reasons behind faster rising public debt are weaker nominal growth (via more muted inflation and real growth alike), and even more importantly, the contribution of much larger primary deficits. The latter, in turn, is driven by two main channels as depicted in Figure 23. First, the deeper decline in real gross domestic income increases the GDP share of CPI-indexed transfers. Second, the significantly lower labour income share, and the *inverse* fiscal drag effect stemming from falling nominal wages, hurt labour tax revenues as a share of GDP markedly.

In the face of an adverse terms-of-trade shock, import substitution elasticities are a highly consequential charachteristic of the economy, with important fiscal implications as well. Higher complementarities essentially amplify the effect of the shock on all fronts, except consumer price inflation.

4.5 Adverse ToT shock vs domestic demand and supply shocks

Here we consider an illustrative exercise that is meant to demonstrate the importance of what the underlying shock behind rising inflation is. A positive demand shock can have very similar inflationary effects than an adverse terms-of-trade shock, yet also have starkingly different macroeconomic and fiscal implications. In this alternative scenario, we introduce a combination of exogenous discount rate and consumption shocks raising domestic demand by households, and calibrate these shocks such that they lead to the same 1 percentage point increase in CPI inflation for the first year as it was the case for the adverse terms-of-trade shock.

The results of this comparative exercise are shown in Figure 10 and in the Appendix Figures 19 through 22. While inflation evolves fairly similarly (partly by construction), most other macroeconomic variables display almost diametrically opposite paths. Not surprisingly, this has starkly different fiscal implications, with rising primary surpluses and faling debt-to-GDP ratios, with the latter also pulled downwards by beneficial snowball effects mainly due to higher nominal growth.

This illustrates that the intuition behind the beneficial effects of higher inflation on public finances is not necessarily wrong. However, the answer crucially depends on the nature of the underlying inflationary shock. And in the case of an adverse terms-of-trade shock, the intuition is less likely to survive a more thorough general equilibrium analysis.

Figure 10 also displays additional analysis of a negative supply shock, which features an unexpected rise in *domestic* markups. Similarly to our baseline adverse terms-of-trade shock (which had rising markups for *foreign* exporters), this shock also pushes inflation upwards while de-

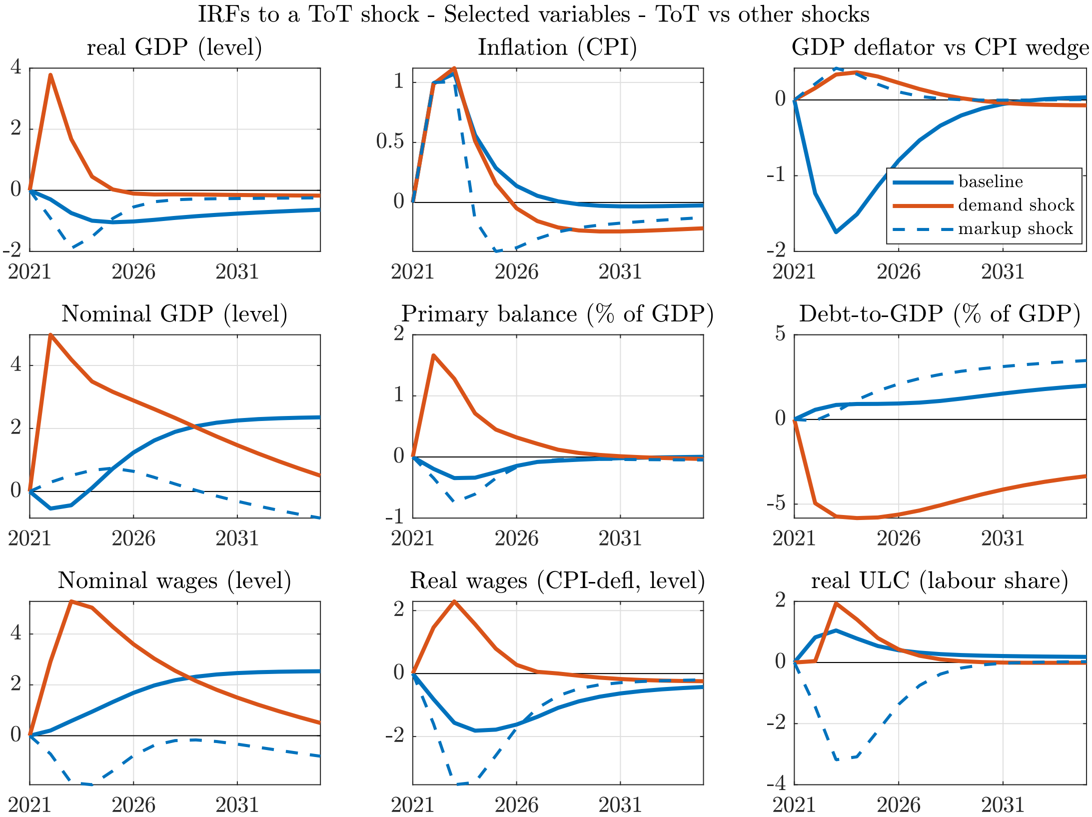

Figure 10: Impulse responses to adverse terms-of-trade (baseline), positive domestic demand and adverse domestic markup shocks such that in all cases CPI inflation rises by 1 percentage point in the first year. Lines denote percentage deviations from steady state, except for inflation, the primary balance and debt-to-GDP, where they denote percentage point deviations. *Source: Commission services.*

pressing real output, and has *qualitatively* similar overall fiscal implications, also fuelled by monetary tightening. However, there are also important differences stemming from the internal (as opposed to external) origins of rising markups. In particular, the evolution of the wedge between CPI and GDP-deflator and the strength of second round effects are quite different: with the domestic markup shock, GDP-deflator initially rises more given the domestic trigger for inflation, but it moderates more quickly thereafter as a directly suppressed labour share and larger labour market slack keep subsequent second round effects more in check. This makes for an initially less, but eventually a larger rise in public debt compared to the terms-of-trade shock.

That said, as it was mentioned in the Introduction, the actual evolution of inflation in Europe was driven not just by terms-of-trade shocks, but also by a combination of other demand and supply shocks. Given this, the relevant effects were likely a mix of the extreme and clean scenarios shown in Figure 10.

5 Concluding remarks

This paper investigated the general equilibrium fiscal implications of an adverse terms-of-trade shock. Despite the inflationary nature of such a shock, there is less scope to "inflate away" nominally fixed public debt, as negative real growth effects due to a loss of domestic purchasing power and widening budget deficits push debt-to-GDP ratios in the opposite direction, especially if monetary policy raises interest rates aggressively and debt has short maturities.

That said, the simulations have also demonstrated how consequential certain policy settings can be, e.g. fiscal policy suppressing expenditure indexation via "benefit erosion", raising tax revenues by strictly adhering to a fiscal rule, or monetary policy accommodating higher inflation and letting domestic second round effects develop to a larger extent. However, the burden of the implicit fiscal consolidation behind stabilising the debt-to-GDP ratio is always born by some domestic agents, be it transfer-recipients, taxpayers or long term bondholders. In other words, the terms-of-trade shock makes the economy *as a whole* worse off, by lowering real gross domestic incomes, so it has an inherently detrimental effect – public finances can benefit from this situation only to the extent that some other sector of the economy pays for it.

In this respect, it is worth emphasising that the simulation results in this paper are strictly descriptive. They do not aim to suggest that rising debt ratios in the face of an adverse shock are undesirable from a normative aspect, nor that more inflationary monetary policies or more austere budgetary consolidation would be called for, just so that debt-to-GDP does not increase. In fact, far from being unambigously bad, public debt can be a very useful tool in the hands of fiscal policy if it is not overused. As Blanchard and Pisani-Ferry (2022) pointed out, it can be welfare-improving if fiscal policy fulfils an insurance role and supports the groups most exposed to real income losses amid the cost-of-living crisis – while monetary policy aims to keep second round domestic price pressures in check, without trying to suppress all of the first round impact. In fact, insuring the real income of economic agents does not only protect aggregate demand (boosting real growth and inflation), but might also defuse the wage-price persistence mechanism by reducing the need to recover purchasing power losses via a struggle between labour and capital owners, thereby *mitigating* second-round domestic price pressures.

Such analyses are more suited to heterogeneous agent models where uninsured income risk plays a large role in macroeconomic dynamics, and which lend themselves more easily to study the welfare implications of distributional issues. While the QUEST simulation exercise presented in this paper does not capture these channels, this is a research agenda that we aim to join to.

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Appendix

A Further model details

A.1 Decomposition equations for charts

Note that for ease of notation, throughout this section Δ denotes level deviation *from the steady state* (instead of per period change).

A.1.1 Trade balance and terms-of-trade loss

The deviation of the value of net exports (i.e. the nominal trade balance) from its steady state can be decomposed as follows, as inspired by Oblath (2019) and Oblath (2022):

$$\begin{aligned}\Delta NX_t &\equiv (P_t^Y Y_t - P_t^C D_t) - (Y - D) = \\ &= \underbrace{(Y_t - D_t) - (Y - D)}_{\text{volume effect}} + \underbrace{(P_t^Y Y_t - P_t^C D_t) - \frac{P_t^Y Y_t - P_t^C D_t}{P_t^C}}_{\text{price level effect}} + \\ &\quad + \underbrace{\frac{P_t^Y Y_t - P_t^C D_t}{P_t^C} - (Y_t - D_t)}_{\text{ToT effect}} \\ &= \underbrace{\Delta Y_t - \Delta D_t}_{\Delta NX_t^{vol}} + \underbrace{\left(1 - \frac{1}{P_t^C}\right) NX_t}_{\Delta NX_t^{price}} + \underbrace{\left(\frac{P_t^Y}{P_t^C} - 1\right) Y_t}_{\theta_t^{ToT}} \end{aligned} \tag{A.1}$$

- Δ denotes level deviation *from the steady state*
- note that all prices are normalized to one in the steady state
- nominal domestic demand is defined as: $\mathcal{D}_t \equiv P_t^Y Y_t - NX_t = P_t^C(C_t + G_t + I_t^G) + P_t^I I_t$
- CPI-deflated real domestic demand: $D_t \equiv \frac{\mathcal{D}_t}{P_t^C} = C_t + G_t + I_t^G + \frac{P_t^I}{P_t^C} I_t$
- ΔNX_t^{vol} roughly captures the contribution of net exports to real GDP growth (barring the effect of the relative price of investments)[38]
- The bottom left panel of Figure 1 displays this equation, but with the variables expressed as a percentage of steady state GDP.

[38] Note that in QUEST the price index for investments is not exactly P_t^C as investment goods have a different composition than other consumption goods. A strict definition of the volume change term $\Delta NX_t^{vol*} = \left(Y_t - C_t - G_t - I_t^G - I_t\right) - (Y - C - G - I^G - I)$ would leave us with $\theta_t^{ToT*} = \left(\frac{P_t^Y}{P_t^C} - 1\right) Y_t - \left(\frac{P_t^I}{P_t^C} - 1\right) I_t$. However, the last term is instead pushed inside $\Delta NX_t^{vol} = \Delta Y_t - \Delta D_t = \left(Y_t - C_t - G_t - I_t^G - \frac{P_t^I}{P_t^C} I_t\right) - (Y - C - G - I^G - I)$ such that the ToT gain term lines up with that coming from the rGDI definition. The volume term therefore includes the relative price changes of investment as well, but that is not very large.

A.1.2 Real gross domestic income

$$rGDI_t \equiv \frac{P_t^Y}{P_t^C} Y_t = Y_t + \underbrace{\left(\frac{P_t^Y}{P_t^C} - 1\right) Y_t}_{\theta_t^{ToT}}$$

$$\Delta\, rGDI_t = \Delta Y_t + \theta_t^{ToT} = \tag{A.2}$$

$$= \underbrace{\Delta D_t + \Delta NX_t^{vol}}_{\Delta Y_t} + \theta_t^{ToT} = \tag{A.3}$$

$$= \Delta D_t + \underbrace{\left(\frac{NX_t}{P_t^C} - NX\right)}_{\Delta\left(\frac{NX_t}{P_t^C}\right)} \tag{A.4}$$

- where the last result follows from

$$\frac{NX_t}{P_t^C} = \frac{P_t^Y Y_t - P_t^C D_t}{P_t^C} =$$

$$= (Y_t - D_t) + \frac{P_t^Y - P_t^C}{P_t^C} Y_t$$

$$\Delta\left(\frac{NX_t}{P_t^C}\right) = \Big(\underbrace{\Delta Y_t - \Delta D_t}_{\Delta NX_t^{vol}}\Big) + \underbrace{\left(\frac{P_t^Y}{P_t^C} - 1\right) Y_t}_{\theta_t^{ToT}}$$

- Note that since prices are normalised to 1 in the steady state, $\theta^{ToT} = 0$ by construction. Therefore $\Delta\theta_t^{ToT} = \theta_t^{ToT} - \theta^{ToT} = \theta_t^{ToT}$
- The rGDI charts on Figures 1 and 11 display these equations, where the terms are all divided by steady state GDP, such that they add up to the percentage deviation of rGDI from steady state $\widehat{rGDI}_t \equiv \frac{rGDI_t - rGDI}{rGDI} = \frac{rGDI_t - Y}{Y}$

A.1.3 Nominal GDP

$$\Delta(P_t^Y Y_t) = \Delta(P_t^C D_t) + \Delta NX_t =$$

$$= \Delta D_t + (P_t^C - 1) D_t + \Delta NX_t^{vol} + \Delta NX_t^{price} + \theta_t^{ToT} =$$

$$= \underbrace{\underbrace{\Delta D_t + \Delta NX_t^{vol}}_{\Delta Y_t} + \underbrace{\theta_t^{ToT}}_{\approx \hat{P}_t^Y - \hat{P}_t^C}}_{\Delta\, rGDI_t} + \underbrace{\overbrace{\Delta NX_t^{price}}^{\approx 0} + \overbrace{(P_t^C - 1) D_t}^{\approx \hat{P}_t^C}}_{\left(1 - \frac{1}{P_t^C}\right) P_t^Y Y_t \approx \hat{P}_t^C} \tag{A.5}$$

which can be combined in other constellations:

$$\Delta(P_t^Y Y_t) = \underbrace{\Delta Y_t + \theta_t^{ToT}}_{\Delta\, rGDI_t} + \underbrace{\left(1 - \frac{1}{P_t^C}\right) P_t^Y Y_t}_{\approx \hat{P}_t^C} \tag{A.6}$$

$$= \Delta Y_t + \underbrace{\left(1 - \frac{1}{P_t^Y}\right) P_t^Y Y_t}_{\approx \hat{P}_t^Y} \tag{A.7}$$

- The charts in Figures 11 and 12 show these equations with the terms divided by steady state GDP (such that they add up to the percentage deviation of nominal GDP from steady state).

A.1.4 Real income allocation

Starting from the definition, spcifying nominal GDP as the difference of gross nominal output and intermediate input costs, with non-tradeable and domestic tradeable sectors $j \in \{NT, TD\}$:

$$P_t^Y Y_t \equiv \sum_j \left(P_t^j O_t^j - P_t^{INT,j} INT_t^j\right) =$$

$$= \sum_j \Big[\underbrace{Profits_t^j + adj_t^j}_{\equiv \Omega_t^j} + (1 + \tau^{ssc,j}) W_t L_t^j \; + \; i_t^{K,j} P_t^{I,j} K_t^j\Big] \tag{A.8}$$

$$Y_t = \sum_j \frac{\Omega_t^j}{P_t^Y} \; + \; \sum_j \frac{(1 + \tau^{ssc,j}) W_t}{P_t^Y} L_t^j \; + \; \sum_j i_t^{K,j} \frac{P_t^{I,j}}{P_t^Y} K_t^j \tag{A.9}$$

$$1 = \underbrace{\sum_j \frac{\Omega_t^j}{P_t^Y Y_t}}_{\lambda_t^{profit}} + \underbrace{\sum_j \frac{(1 + \tau^{ssc,j}) W_t L_t^j}{P_t^Y Y_t}}_{\lambda_t^{labour}} + \underbrace{\sum_j \frac{i_t^{K,j} P_t^{I,j} K_t^j}{P_t^Y Y_t}}_{\lambda_t^{capital}} \tag{A.10}$$

$$rGDI_t \equiv \frac{P_t^Y Y_t}{P_t^C} = \frac{P_t^Y}{P_t^C} \lambda_t^{profit} \, Y_t \; + \; \frac{P_t^Y}{P_t^C} \lambda_t^{labour} \, Y_t \; + \; \frac{P_t^Y}{P_t^C} \lambda_t^{capital} \, Y_t \tag{A.11}$$

- The charts in Figure 2 display the above equations with each term in level deviations from their steady state – and in the case of (A.9) and (A.11) expressed as a fraction of the steady state GDP, such that they add up the percentage deviations.
- The income shares λ_t denote shares out of *domestic value added*, i.e. GDP, and not out of gross final output. They also correspond to shares out of gross domestic income (rGDI).

A.1.5 Labour income decomposition

$$\lambda_t^{labour} = \frac{W_t}{P_t^C}\,\frac{P_t^C}{P_t^Y}\,\underbrace{\frac{\sum_j (1+\tau^{ssc,j})L_t^j}{Y_t}}_{\equiv 1/prod_t}$$

$$\mathcal{L}_t \equiv \sum_j \frac{(1+\tau^{ssc,j})W_t\,L_t^j}{P_t^C} = \frac{P_t^Y}{P_t^C}\,\lambda_t^{labour}\,Y_t$$

$$\frac{W_t}{P_t^C} = \frac{P_t^Y}{P_t^C}\,\lambda_t^{labour}\,prod_t$$

To ensure additivity, for the charts we use log deviations, which approximate percentage deviations from steady state $\Delta \log x_t \equiv \log x_t - \log x \approx \frac{x_t - x}{x}$

- labour share (real ULC):

$$\Delta \log \lambda_t^{labour} = \Delta \log \frac{W_t}{P_t^C} - \Delta \log \left(\frac{P_t^Y}{P_t^C}\right) - \Delta \log(prod_t) \tag{A.12}$$

- real labour income (CPI-deflated):

$$\Delta \log(\mathcal{L}_t) = \log \left(\frac{P_t^Y}{P_t^C}\right) + \Delta \log \lambda_t^{labour} + \Delta \log Y_t \tag{A.13}$$

- real wage (CPI-deflated):

$$\Delta \log \left(\frac{W_t}{P_t^C}\right) = \log \left(\frac{P_t^Y}{P_t^C}\right) + \Delta \log \lambda_t^{labour} + \Delta \log(prod_t) \tag{A.14}$$

A.1.6 GDP deflator and ULC

Starting from (A.8):

$$P_t^Y Y_t = \sum_j \Big[\underbrace{Profits_t^j + adj_t^j}_{\Omega_t^j} + (1+\tau^{ssc,j})W_t L_t^j \;+\; i_t^{K,j} P_t^{I,j} K_t^j\Big]$$

$$P_t^Y = \underbrace{\sum_j \frac{\Omega_t^j}{Y_t}}_{UPC_t} + \underbrace{\sum_j \frac{(1+\tau^{ssc,j})W_t L_t^j}{Y_t}}_{ULC_t} + \underbrace{\sum_j \frac{i_t^{K,j} P_t^{I,j} K_t^j}{Y_t}}_{UCC_t} \tag{A.15}$$

$$ULC_t = \lambda_t^{labour} P_t^Y = \underbrace{\frac{W_t}{P_t^C}\,P_t^C}_{W_t}\;\underbrace{\frac{\sum_j (1+\tau^{ssc,j})L_t^j}{Y_t}}_{1/prod_t}$$

$$\Delta \log(ULC_t) = \Delta \log \left(\frac{W_t}{P_t^C}\right) + \log(P_t^C) - \Delta \log(prod_t) \tag{A.16}$$

A.1.7 CPI

In QUEST, final consumption is consists of spending on non-tradables and tradables:

$$
\begin{aligned}
P_t^C C_t &= P_t^{NT} C_t^{NT} + \underbrace{P_t^{T,D} C_t^{T,D} + P_t^M C_t^M}_{P_t^T C_t^T} \\
P_t^C &= P_t^{NT}\frac{C_t^{NT}}{C_t} + \frac{C_t^T}{C_t}\underbrace{\left(P_t^{T,D}\frac{C_t^{T,D}}{C_t^T} + P_t^M \frac{C_t^M}{C_t^T}\right)}_{P_t^T} \\
\frac{\Delta P_t^C}{P^C} &\approx \frac{C^{NT}}{C}\frac{\Delta P_t^{NT}}{P^{NT}} + \frac{C^T}{C}\underbrace{\left(\frac{C^{T,D}}{C^T}\frac{\Delta P_t^{T,D}}{P^{T,D}} + \frac{C^M}{C^T}\frac{\Delta P_t^M}{P^M}\right)}_{\frac{\Delta P_t^T}{P^T}}
\end{aligned}
$$

The percentage deviation of CPI from its steady state can be well approximated by summing the percentage changes of its components weighted by their steady state shares.[39]

$$
\begin{aligned}
C_t &= \left[(1-s^T)^{\frac{1}{\sigma_{TNT}}}\left(C_t^{NT}\right)^{\frac{\sigma_{TNT}-1}{\sigma_{TNT}}} + (s^T)^{\frac{1}{\sigma_{TNT}}}\left(C_t^T\right)^{\frac{\sigma_{TNT}-1}{\sigma_{TNT}}}\right]^{\frac{\sigma_{TNT}}{\sigma_{TNT}-1}} \\
C_t^T &= \left[(1-s^M)^{\frac{1}{\sigma_x}}\left(C_t^{T,D}\right)^{\frac{\sigma_x-1}{\sigma_x}} + (s^M)^{\frac{1}{\sigma_x}}\left(C_t^M\right)^{\frac{\sigma_x-1}{\sigma_x}}\right]^{\frac{\sigma_x}{\sigma_x-1}} \\
P_t^C &= \left[(1-s^T)(P_t^{NT})^{1-\sigma_{TNT}} + (s^T)(P_t^T)^{1-\sigma_{TNT}}\right]^{\frac{1}{1-\sigma_{TNT}}} \\
P_t^T &= \left[(1-s^M)(P_t^{T,D})^{1-\sigma_x} + (s^M)(P_t^M)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}}
\end{aligned}
$$

From the CES aggregators above (see (2.3), (2.4), (2.5), (2.6)) it follows that the steady state shares are governed by $s^T = \frac{C^T}{C}$ and $s^M = \frac{C^M}{C^T}$, so we have:

$$
\begin{aligned}
p_t^C &\approx (1-s^T)p_t^{NT} + s^T\underbrace{\left((1-s^M)p_t^{T,D} + s^M p_t^M\right)}_{p_t^T} = \\
&= \underbrace{(1-s^T)p_t^{NT} + s^T(1-s^M)p_t^{T,D}}_{\text{2nd + indirect 1st round}} + \underbrace{s^T s^M p_t^M}_{\text{dirct 1st round}} \qquad \text{(A.17)} \\
\pi_t^C &\approx \underbrace{(1-s^T)\pi_t^{NT} + s^T(1-s^M)\pi_t^{T,D}}_{\text{2nd + indirect 1st round}} + \underbrace{s^T s^M \pi_t^M}_{\text{dirct 1st round}} \qquad \text{(A.18)}
\end{aligned}
$$

where $p_t \equiv \frac{P_t - P}{P} = P_t - 1$ and $\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$

[39] Composition effects are not significant up to first-order:

$$
\begin{aligned}
P_t^C &= P_t\frac{C_t^D}{C_t} + P_t^M\frac{C_t^M}{C_t} && \Rightarrow \text{price+composition effects} \\
&\neq P_t\frac{C^D}{C} + P_t^M\frac{C^M}{C} && \Rightarrow \text{price effect only} \\
\Delta P_t^C &= \underbrace{P_t\left(\frac{C_t^D}{C_t} - \frac{C^D}{C}\right) + P_t^M\left(\frac{C_t^M}{C_t} - \frac{C^M}{C}\right)}_{\text{composition effect} \approx 0} + \underbrace{\Delta P_t\frac{C^D}{C} + \Delta P_t^M\frac{C^M}{C}}_{\text{price effect}}
\end{aligned}
$$

Non-tradeable output and domestically-produced tradeable output are themselves are made up of domestic value added and intermediate inputs. Moreover, those intermediate inputs are not all imported, but there is domestic value added included in them (see CES aggregators (2.1), (2.3), (2.4)).

$$p_t^{TD} \approx (1 - s_{in}^{TD}) p_t^{VA,TD} + s_{in}^{TD} p_t^{INT,TD}$$
$$p_t^{NT} \approx (1 - s_{in}^{NT}) p_t^{VA,NT} + s_{in}^{NT} p_t^{INT,NT}$$

$$p_t^{INT,TD} \approx (1 - s_{INT}^{TD}) p_t^{NT} + s_{INT}^{TD} \Big[(1 - s^M) p_t^{TD} + s^M p_t^M \Big]$$
$$p_t^{INT,NT} \approx (1 - s_{INT}^{NT}) p_t^{NT} + s_{INT}^{NT} \Big[(1 - s^M) p_t^{TD} + s^M p_t^M \Big]$$

The above system is solved such that we arrive at:

$$p_t^{TD} \approx \phi^{VA,TD} p_t^{VA,TD} + \phi^{VA,NT} p_t^{VA,NT} + \phi^M p_t^M$$
$$p_t^{NT} \approx \xi^{VA,NT} p_t^{VA,NT} + \xi^{VA,TD} p_t^{VA,TD} + \xi^M p_t^M$$

where the parameters ξ^k and ϕ^k are functions of the steady state shares $s_{in}^{TD}, s_{in}^{NT}, s_{INT}^{TD}, s_{INT}^{NT}, s^T, s^M$ such that they solve the above system.[40] The above equations can then be plugged back into (A.17), yielding our desired decomposition:

$$p_t^C \approx \underbrace{\Big[(1-s^T)\xi^{VA,NT} + s^T(1-s^M)\phi^{VA,NT}\Big] p_t^{VA,NT} + \Big[(1-s^T)\xi^{VA,TD} + s^T(1-s^M)\phi^{VA,TD}\Big] p_t^{VA,TD}}_{2^{\text{nd}}\text{ round}}$$
$$+ \underbrace{\Big[(1-s^T)\xi^M + s^T(1-s^M)\phi^M\Big] p_t^M}_{\text{indirect } 1^{\text{st}} \text{ round}} + \underbrace{s^T s^M p_t^M}_{\text{direct } 1^{\text{st}} \text{ round}} \tag{A.19}$$

$$\pi_t^C \approx \underbrace{\Big[(1-s^T)\xi^{VA,NT} + s^T(1-s^M)\phi^{VA,NT}\Big] \pi_t^{VA,NT} + \Big[(1-s^T)\xi^{VA,TD} + s^T(1-s^M)\phi^{VA,TD}\Big] \pi_t^{VA,TD}}_{2^{\text{nd}}\text{ round}}$$
$$+ \underbrace{\Big[(1-s^T)\xi^M + s^T(1-s^M)\phi^M\Big] \pi_t^M}_{\text{indirect } 1^{\text{st}} \text{ round}} + \underbrace{s^T s^M \pi_t^M}_{\text{direct } 1^{\text{st}} \text{ round}} \tag{A.20}$$

- **direct 1st round effects:** coming from imported consumption good prices
- **indirect 1st round effects:** coming from domestic output prices due to imported intermediate inputs
- **2nd round effects:** coming from domestic price pressures due to the price index of domestic value added (GDP deflator)

[40] Sectoral value added prices are implicitly calculated from p_t^{TD} and p_t^{NT}, that are governed by Phillips-curves:

$$p_t^{VA,NT} = \frac{1}{1-\alpha^{NT}} \left[p_t^{NT} - \alpha^{NT} p_t^{INT,NT} \right]$$
$$p_t^{VA,TD} = \frac{1}{1-\alpha^{TD}} \left[p_t^{TD} - \alpha^{TD} p_t^{INT,TD} \right]$$

A.1.8 Debt-to-GDP

Starting from the nominal government budget constraint:

$$B_t = (1 + i^g_{t-1})B_{t-1} - PB_t$$

$$b_t \equiv \frac{B_t}{P^Y_t Y_t} = (1 + i^g_{t-1})\frac{B_{t-1}}{P^Y_{t-1}Y_{t-1}}\frac{P^Y_{t-1}Y_{t-1}}{P^Y_t Y_t} - \frac{PB_t}{P^Y_t Y_t}$$

$$b_t = -pb_t + \underbrace{\left(\frac{1 + i^g_{t-1}}{1 + \pi^Y_t}\right)}_{\equiv 1 + r^g_{t-1}} \frac{1}{1 + g_t}\, b_{t-1} \tag{A.21}$$

$$= -pb_t + (1 + i^g_{t-1})\underbrace{\frac{1}{(1 + g_t)(1 + \pi^Y_t)}}_{\equiv \frac{1}{1+\mathcal{G}_t}}\, b_{t-1}$$

$$\Delta b_t \equiv b_t - b_{t-1} = -pb_t + \frac{r^g_{t-1} - g_t}{1 + g_t}\, b_{t-1} =$$

$$= -pb_t + \frac{i^g_{t-1} - \mathcal{G}_t}{1 + \mathcal{G}_t}\, b_{t-1} =$$

$$= -pb_t + \frac{i^g_{t-1} - \pi^Y_t - g_t - g_t\pi^Y_t}{(1 + \pi^Y_t)(1 + g_t)}\, b_{t-1}$$

where the last equation can be separated into:

$$\Delta b_t = -pb_t + \frac{i^g_{t-1}}{(1 + \pi^Y_t)(1 + g_t)}\, b_{t-1} - \frac{g_t}{(1 + \pi^Y_t)(1 + g_t)}\, b_{t-1} - \frac{\pi^Y_t(1 + g_t)}{(1 + \pi^Y_t)(1 + g_t)}\, b_{t-1} =$$

$$= \underbrace{-pb_t + \frac{i^g_{t-1}}{(1 + \pi^Y_t)(1 + g_t)}\, b_{t-1}}_{\equiv -bb_t} \underbrace{- \frac{g_t}{(1 + \pi^Y_t)(1 + g_t)}\, b_{t-1} - \frac{\pi^Y_t}{1 + \pi^Y_t}\, b_{t-1}}_{-\frac{\mathcal{G}_t}{1+\mathcal{G}_t} b_{t-1}} \tag{A.22}$$

which is the same as (1.1) in the Introduction. Then, calculating the cumulative change:

$$\tilde{b}_t \equiv b_t - b_0 = \sum_{k=1}^{t} \Delta b_k =$$

$$= \underbrace{-\sum_{k=1}^{t} pb_k}_{-\theta^{pb}_t} + \underbrace{\sum_{k=1}^{t} \frac{i^g_{k-1}}{(1 + \pi^Y_k)(1 + g_k)}\, b_{k-1}}_{\theta^i_t} \underbrace{- \sum_{k=1}^{t} \frac{g_k}{(1 + \pi^Y_k)(1 + g_k)}\, b_{k-1}}_{-\theta^g_t} \underbrace{- \sum_{k=1}^{t} \frac{\pi^Y_k}{1 + \pi^Y_k}\, b_{k-1}}_{-\theta^\pi_t} \tag{A.23}$$

We can also arrive at a similar decomposition by log-linearising (A.21) and then solving it backward:

$$\tilde{b}_t = \frac{1 + i^g}{(1 + \pi^Y)(1 + g)}\tilde{b}_{t-1} + b\,\frac{1 + i^g}{(1 + \pi^Y)(1 + g)}\left(\frac{\tilde{i}^g_{t-1}}{1 + i^g} - \frac{\tilde{\pi}^Y_t}{1 + \pi^Y} - \frac{\tilde{g}_t}{1 + g}\right) - \widetilde{pb}_t =$$

$$= \sum_{k=0}^{t-1}\left[\frac{1 + i^g}{(1 + \pi^Y)(1 + g)}\right]^k \left[b\,\frac{1 + i^g}{(1 + \pi^Y)(1 + g)}\left(\frac{\tilde{i}^g_{t-1-k}}{1 + i^g} - \frac{\tilde{\pi}^Y_{t-k}}{1 + \pi^Y} - \frac{\tilde{g}_{t-k}}{1 + g}\right) - \widetilde{pb}_{t-k}\right] \tag{A.24}$$

which gives us essentially identical contribution terms.

- $\pi_t^Y = \frac{P_t^Y - P_{t-1}^Y}{P_{t-1}^Y}$ and $g_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$ are GDP deflator inflation and real growth rates, respectively
- The *ex post* effective real interest rate $r_{t-1}^g = \frac{1+i_{t-1}^g}{1+\pi_t} - 1$ determines the real burden of nominal public debt, which can be reduced by surprise inflation π_t as it revalues nominally fixed debt.
- the differential $(r_{t-1}^g - g)$ is a crucial determinant of debt-to-GDP dynamics, capturing the "snowball effect" $(\theta_t^i + \theta_t^g + \theta_t^\pi)$
- (A.22) demonstrates that the change in the public debt-to-GDP ratio is not equal to the headline deficit-to-GDP ratio $-bb_t$, but revaluation terms related to inflation and real growth also have to be taken into account ("denominator effect" due to nominal GDP growth $\mathcal{G}_t$, which is $\theta_t^g + \theta_t^\pi$)

On the balanced growth path, debt-to-GDP is stable, which pins down the steady state relationship with the primary deficit-to-GDP:

$$\begin{aligned} \Delta b = 0 = -pb + \frac{r^g - g}{1+g}\, b &\approx \\ \approx -pb + (r - \chi)\, b& \\ pb \approx (r-\chi)\, b& \end{aligned}$$

where r is the steady state real interest rate consistent with zero trend growth, and χ is a convenience yield that government bonds enjoy due to their liquidity and safe asset status.

- When $(r - \chi) < 0$ (as in the current QUEST calibration), then the government can run a primary deficit in the long run, and still keep the debt-to-GDP stable.

A.1.9 Primary budget balance

$$\begin{aligned} PB_t &= REV_t - EXP_t \\ \frac{PB_t}{P_t^Y Y_t} &= \frac{REV_t - EXP_t}{P_t^Y Y_t} \\ pb_t &= rev_t - exp_t = \\ &= \underbrace{\frac{1}{P_t^Y Y_t}\left(T_t^{labour} + T_t^{VAT} + T_t^{corp} + T_t\right)}_{rev_t} - \underbrace{\frac{1}{P_t^Y Y_t}\left(P_t^C G_t + P_t^C I_t^G + TR_t + BEN_t\right)}_{exp_t} \end{aligned} \tag{A.25}$$

where nominal revenue and expenditure items within REV_t and EXP_t are defined as in Section 2.

A.2 Illustrative examples

A.2.1 Income shares – illustrative

Assuming a constant share of domestic production factors from total costs, and no domestically produced intermediaries:

- Cobb-Douglas production structure (elasticity of substitution between imported inputs and domestic production factors being one) implies the share of domestic production factors vs imported inputs *out of total costs* is constant at $1-\alpha$: $O_t = \left[f(K_t, N_t)\right]^{1-\alpha} (Int_t^M)^\alpha$
- price is set with a markup over rising *average* costs, where pass-through is not perfect due to nominal rigidities, implying a fall in the markup: $\uparrow P_t = (1+\mu_t \downarrow) AC_t \uparrow\uparrow$
- together with constant expenditure shares of domestic production factors, this also implies a falling profit share out of *domestic value added* (i.e.GDP) – see (A.26)
- with lower substitution elasticities in a CES structure, the cost share of domestic value added $(1-\alpha)$ can decrease sufficiently such that this latter result flips (see Figure 27)

Nominal gross output

$$P_t O_t = \underbrace{\Omega_t + W_t N_t + i_t^K K_t}_{P_t^Y Y_t} + \uparrow P_t^M Int_t^M =$$

$$= \Omega_t + \underbrace{W_t N_t + i_t^K K_t + \uparrow P_t^M Int_t^M}_{AC_t\, O_t} =$$

$$\underbrace{(1+\mu_t) AC_t}_{P_t}\ O_t = \underbrace{\Omega_t}_{\mu_t\, AC_t O_t} + \underbrace{W_t N_t + i_t^K K_t + \uparrow P_t^M Int_t^M}_{AC_t\, O_t}$$

Nominal unit costs (based on gross output)

$$P_t = \frac{\Omega_t}{O_t} + \frac{W_t N_t + i_t^K K_t}{O_t} + \frac{P_t^M Int_t^M}{O_t} =$$

$$= \mu_t\, AC_t + AC_t \left[\underbrace{\frac{W_t N_t + i_t^K K_t}{AC_t O_t}}_{1-\alpha} + \underbrace{\frac{P_t^M Int_t^M}{AC_t O_t}}_{\alpha} \right]$$

Income shares out of gross output

$$1 = \frac{\Omega_t}{P_t O_t} + \frac{W_t N_t + i_t^K K_t}{P_t O_t} + \frac{P_t^M Int_t^M}{P_t O_t} =$$

$$= \frac{\mu_t\, AC_t}{P_t} + \frac{AC_t}{P_t} \left[\underbrace{\frac{W_t N_t + i_t^K K_t}{AC_t O_t}}_{1-\alpha} + \underbrace{\frac{P_t^M Int_t^M}{AC_t O_t}}_{\alpha} \right] =$$

$$= \downarrow \frac{\mu_t}{1+\mu_t} + \uparrow \frac{1}{1+\mu_t \downarrow} \left[(1-\alpha)\ +\ \alpha \right]$$

Income shares out of gross domestic value added (GDP)

$$\downarrow \frac{P_t^Y Y_t}{P_t O_t} = \frac{\Omega_t}{P_t O_t} + \frac{W_t N_t + i_t^K K_t}{P_t O_t} =$$
$$= \frac{\mu_t}{1+\mu_t} + \frac{1-\alpha}{1+\mu_t}$$

$$1 = \frac{\Omega_t}{P_t^Y Y_t} + \frac{W_t N_t + i_t^K K_t}{P_t^Y Y_t} =$$
$$= \downarrow \frac{\mu_t}{\mu_t + (1-\alpha)} + \uparrow \frac{(1-\alpha)}{\mu_t \downarrow + (1-\alpha)} \tag{A.26}$$

Nominal unit costs (based on value added, i.e. real GDP)

$$P_t^Y = \frac{\Omega_t}{Y_t} + \frac{W_t N_t + i_t^K K_t}{Y_t} =$$
$$= \frac{\mu_t}{\mu_t + (1-\alpha)} P_t^Y + \frac{(1-\alpha)}{\mu_t + (1-\alpha)} P_t^Y$$

A.2.2 GDP deflator and import cost pass-through – illustrative

Stylised illustrative example, in which we abstract away from domestically produced intermediate inputs, such that nominal GDP is $P_t^Y Y_t \equiv P_t O_t - P_t^M Int_t^M$.

In an analogous way to the nominal trade balance decomposition (A.1), and using (A.7):

$$\Delta(P_t^Y Y_t) \equiv (P_t O_t - P_t^M Int_t^M) - (O - Int^M) =$$

$$= \underbrace{(O_t - Int_t^M) - (O - Int^M)}_{\text{volume effect}} + \underbrace{(P_t O_t - P_t^M Int_t^M) - \frac{P_t O_t - P_t^M Int_t^M}{P_t^M}}_{\text{avg. } P_t^M \text{ price level effect}} +$$

$$+ \underbrace{\frac{P_t O_t - P_t^M Int_t^M}{P_t^M} - (O_t - Int_t^M)}_{\text{"pass-through" effect}}$$

$$= \underbrace{\Delta O_t - \Delta Int_t^M}_{\equiv \Delta Y_t} + \underbrace{\left(1 - \frac{1}{P_t^M}\right) P_t^Y Y_t}_{\Theta_t^{price}} + \underbrace{\left(\frac{P_t}{P_t^M} - 1\right) O_t}_{\Theta_t^{passthrough}} \tag{A.27}$$

$$\Delta Y_t + \underbrace{\left(1 - \frac{1}{P_t^Y}\right) P_t^Y Y_t}_{\approx \hat{P}_t^Y} = \Delta Y_t + \underbrace{\left(1 - \frac{1}{P_t^M}\right) P_t^Y Y_t}_{\Theta_t^{price}} + \underbrace{\left(\frac{P_t}{P_t^M} - 1\right) O_t}_{\Theta_t^{passthrough}}$$

$$\underbrace{\left(1 - \frac{1}{P_t^Y}\right) P_t^Y Y_t}_{\approx \hat{P}_t^Y} = \underbrace{\left(1 - \frac{1}{P_t^M}\right) P_t^Y Y_t}_{\Theta_t^{price}} + \underbrace{\left(\frac{P_t}{P_t^M} - 1\right) O_t}_{\Theta_t^{passthrough}} \tag{A.28}$$

$$(P_t^Y - 1) = \left(1 - \frac{1}{P_t^M}\right) P_t^Y + \left(\frac{P_t}{P_t^M} - 1\right) \frac{O_t}{Y_t}$$

$$P_t^Y = \uparrow P_t^M + \downarrow \left(P_t - P_t^M\right) \frac{O_t}{Y_t} \tag{A.29}$$

- Assuming no second round effects on P_t for the time being, as long as the pass-through from higher imported input costs P_t^M to final output prices P_t is not perfect due to **nominal rigidities** (weak indirect first round effects), this mechanically puts a downward pressure on the GDP deflator (via lower profits).
- In addition, with **low elasticity of substitution** with respect to imported inputs, higher import prices push down the expenditure share of domestic production factors (labour and capital) by constraining wages and capital rent, allowing for a lower P_t, thereby also lowering the GDP deflator.
- Assuming only price changes (i.e. unchanged quantities), when nominal intermediate input costs increase to a larger extent than gross nominal output, it must necessarily mean that their difference, nominal domestic value added (nominal GDP) declines. Since real GDP is unchanged by assumption (no quantity adjustment yet), falling nominal GDP corresponds to a fall in the price index of domestic value added, i.e. the GDP deflator.

- A decreasing $\frac{P_t}{P_t^M}$ is analogous to the terms-of-trade effect on the nominal trade balance. Even without changes to net export volumes, adverse relative price movements can lower the trade balance.

A.2.3 Fiscal drag effect

For an illustrative example of the fiscal drag effect ("tax bracket creep") look at a simple distribution.

- Consider a unit measure of workers i, each representing a share ω_i, working the same hours L_i and earning a nominal wage W_i, such that the average wage level in the economy is

$$W = \sum_i \omega_i W_i$$

- Then, the nominal wage distribution shifts uniformly upwards by α_t percent: $W_i \to (1 + \alpha_t)W_i$ for $\forall\, i$, leading to the same rise in average wages:

$$W_t = \sum_i \omega_i \,(1 + \alpha_t)W_i = (1 + \alpha_t)W$$

- In a progressive tax system, if tax brackets remain fixed in nominal terms, the average tax rate for an *individual* is an increasing function of their nominal wage $\tau_i(W_i)$.
- ⇒ From this it follows that the *average effective* tax rate $\tau^L(W_t)$ must also be an increasing function of the average wage:

$$\tau^L(W) \equiv \frac{T^L/L}{W} = \frac{\sum_i \omega_i\, \tau_i(W_i)W_i}{\sum_i \omega_i\, W_i} = \sum_i \frac{\omega_i W_i}{W}\, \tau_i(W_i)$$

$$\tau^L\Big([1+\alpha_t]W\Big) = \sum_i \frac{\omega_i(1+\alpha_t)W_i}{(1+\alpha_t)W}\, \tau_i\Big([1+\alpha_t]W_i\Big) = \sum_i \frac{\omega_i W_i}{W}\, \tau_i\Big([1+\alpha_t]W_i\Big)$$

- Note that $\tau^L(W_t)$ and $\tau_i(W_i)$ are not the same functions
- labour tax intake as a % of GDP depends on the aggregate tax rate $\tau^L(W_t)$ and on the labour share of income

$$\frac{T_t^L}{P_t Y_t} = \tau^L(W_t)\, \frac{W_t N_t}{P_t Y_t}$$

- we approximate $\tau^L(W_t)$ with the following stylised functional form

$$\hat{\tau}_t^L(W_t) = \iota_t \left[\bar{\tau} - \psi_1 + \psi_1 \left(\frac{W_t}{W}\right)^{\psi_2}\right] + (1 - \iota_t)\, \bar{\tau}$$

B Further figures

B.1 Baseline scenario decompositions

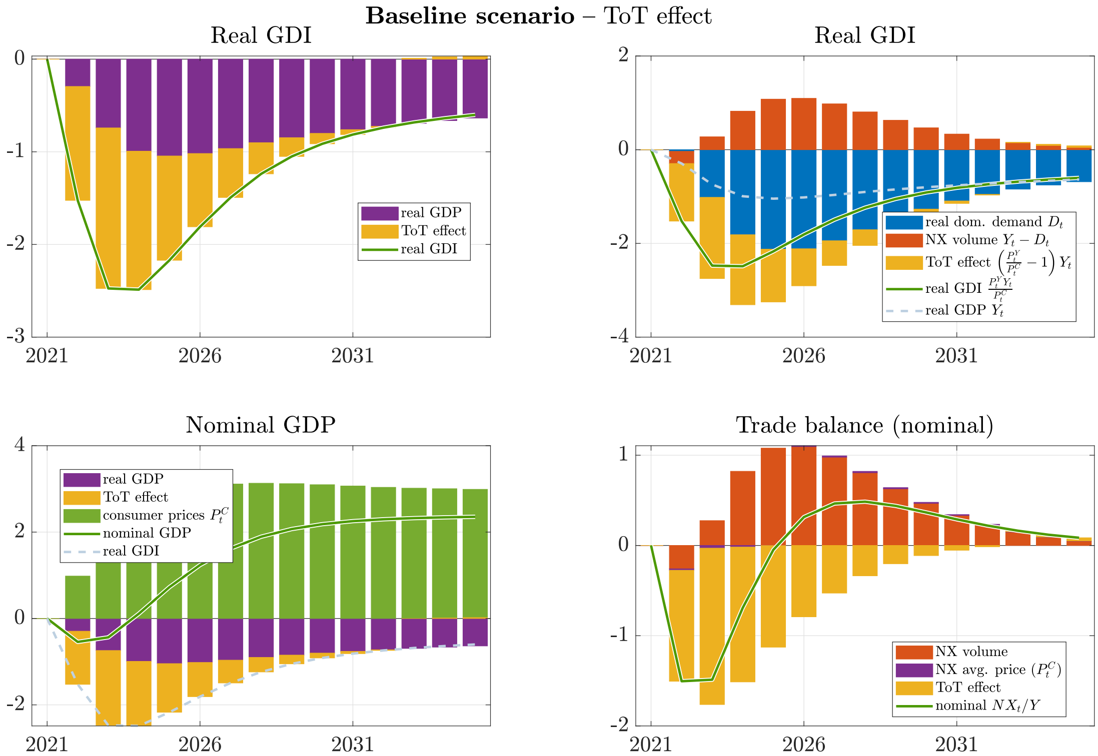

Figure 11: Green lines depict percentage deviations from steady state (except for NX_t/Y, which is in percentage point deviations). Underlying equations are (A.2), (A.3), (A.6), (A.1).

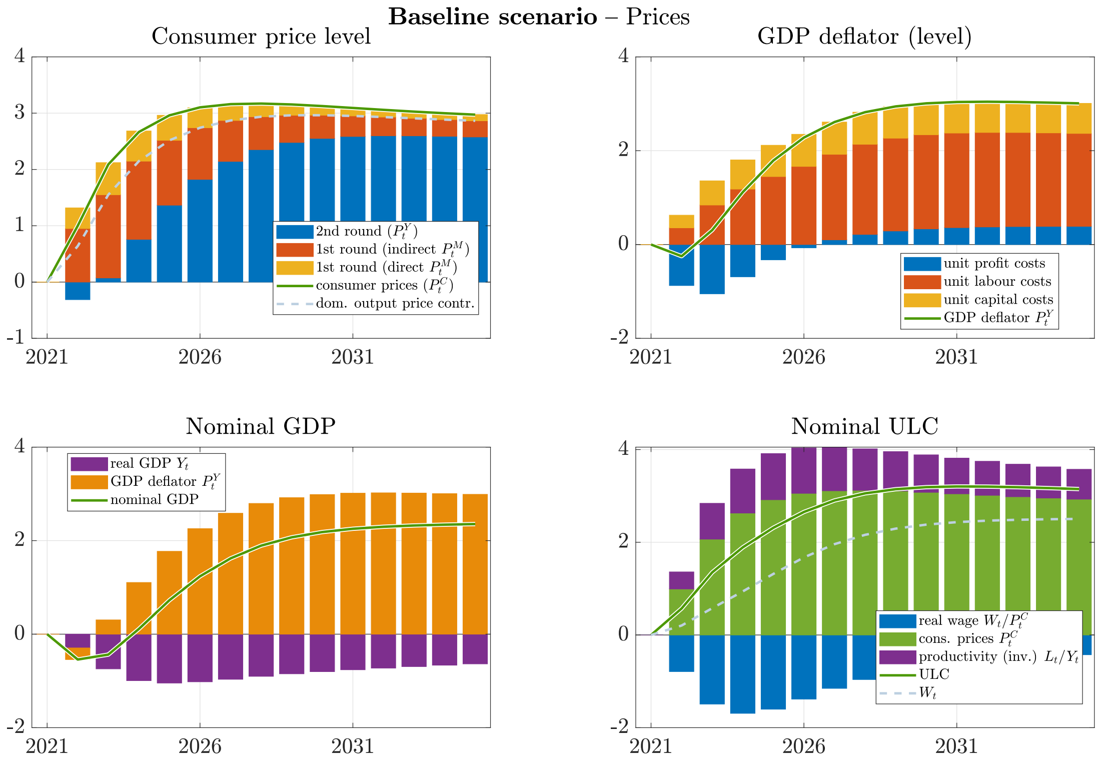

Figure 12: Underlying equations are (A.19), (A.15), (A.7), (A.16).

B.2 Monetary and debt management policies

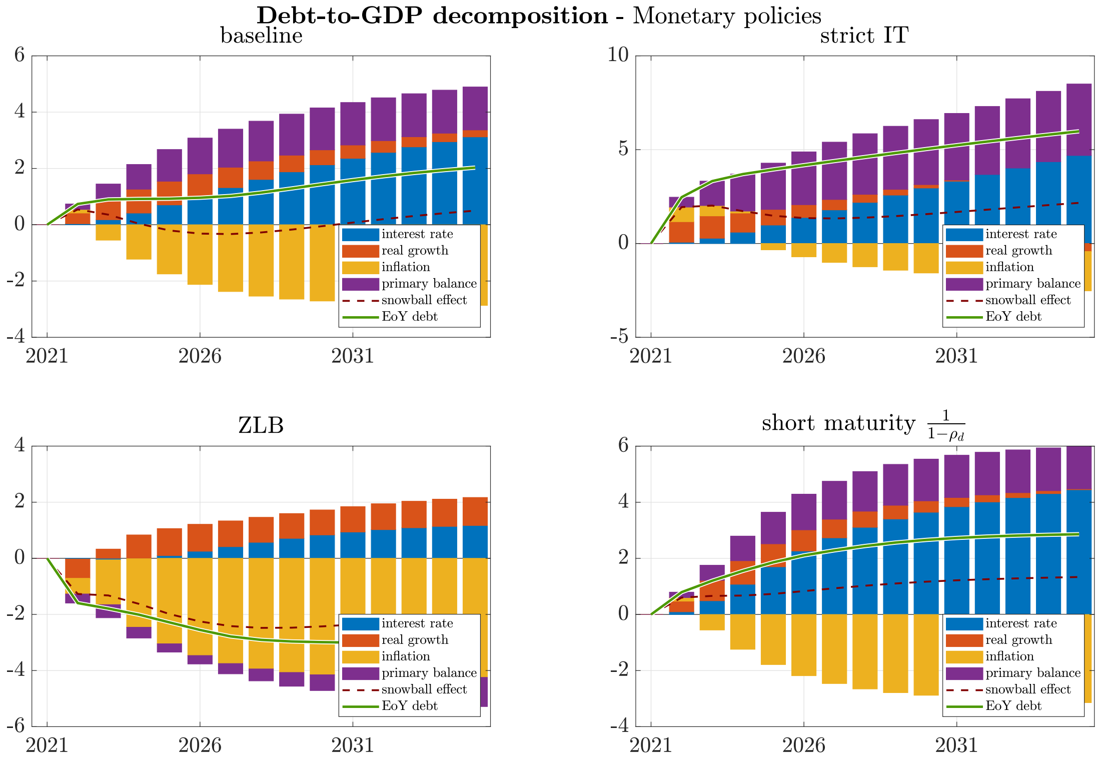

Figure 13

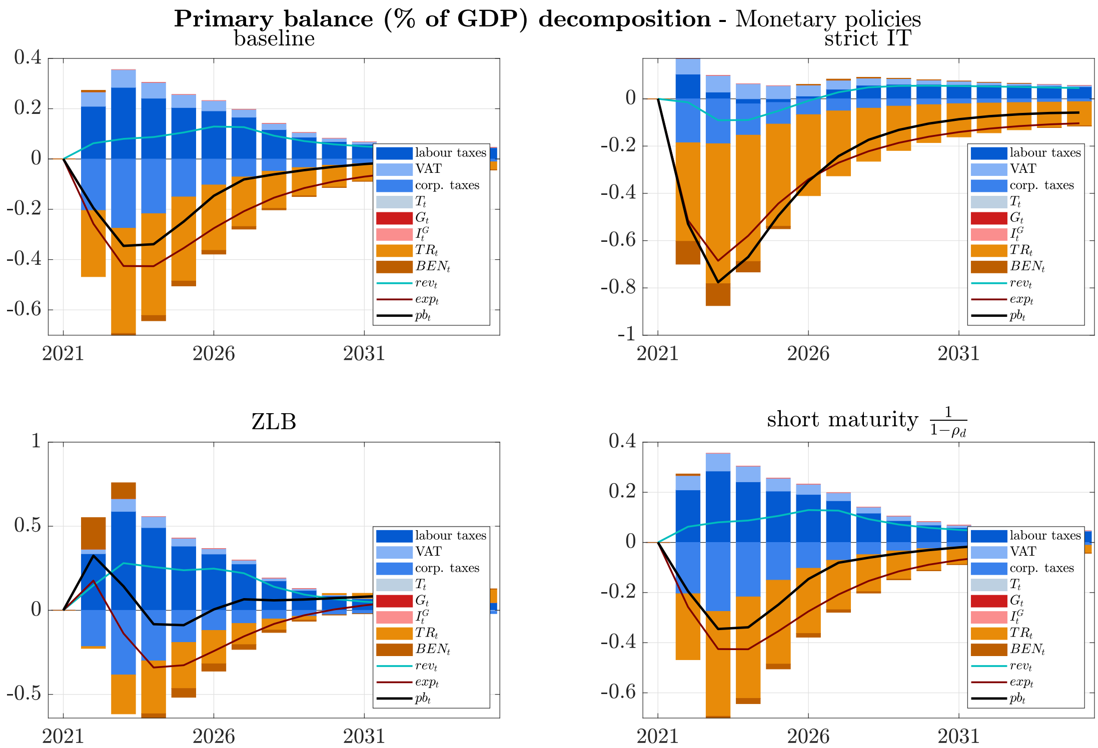

Figure 14

B.3 Expenditure rules

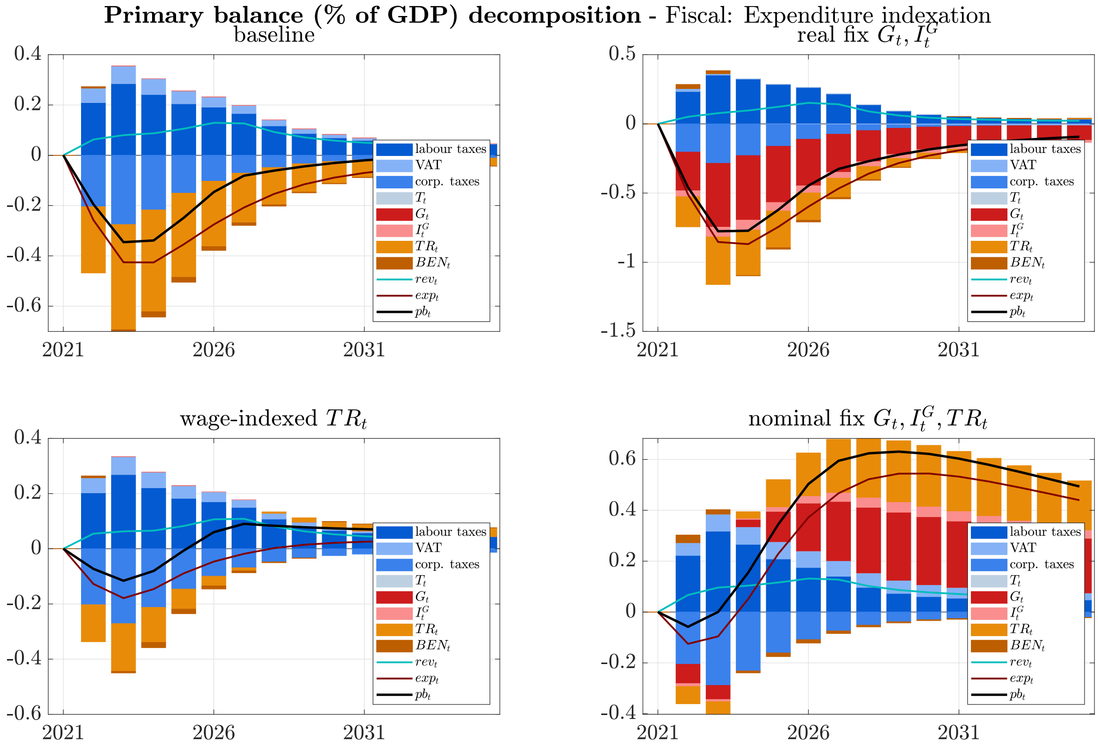

Figure 15

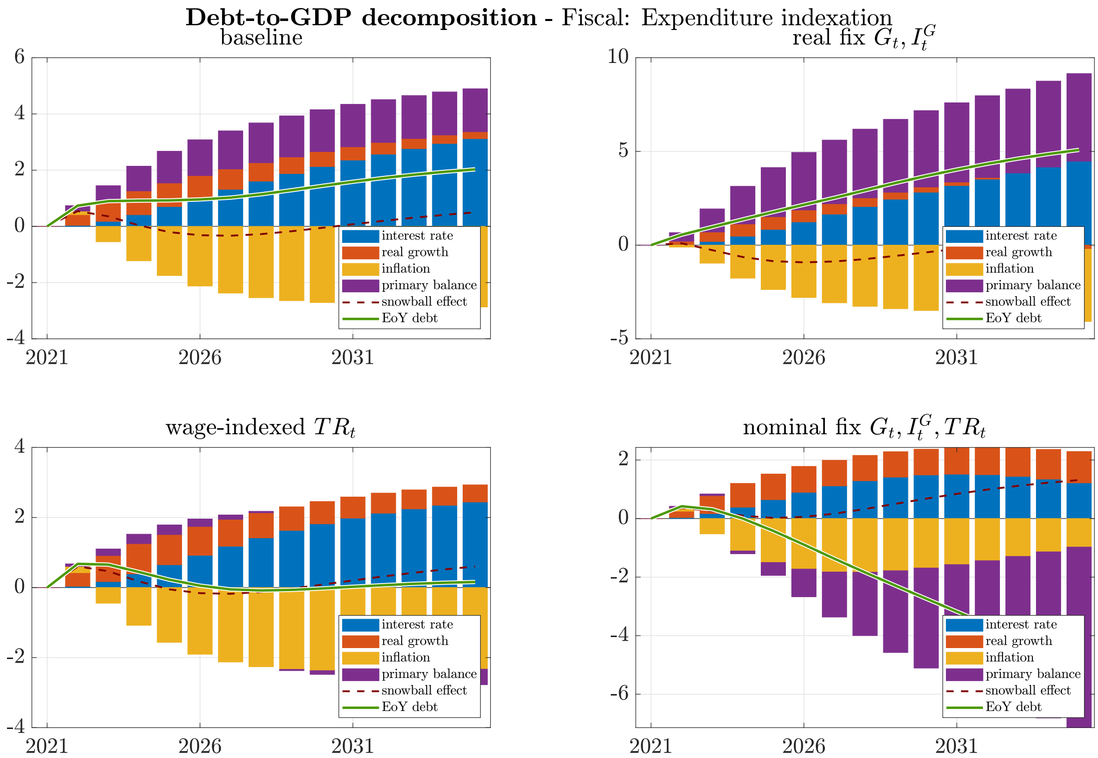

Figure 16

B.4 Tax rules

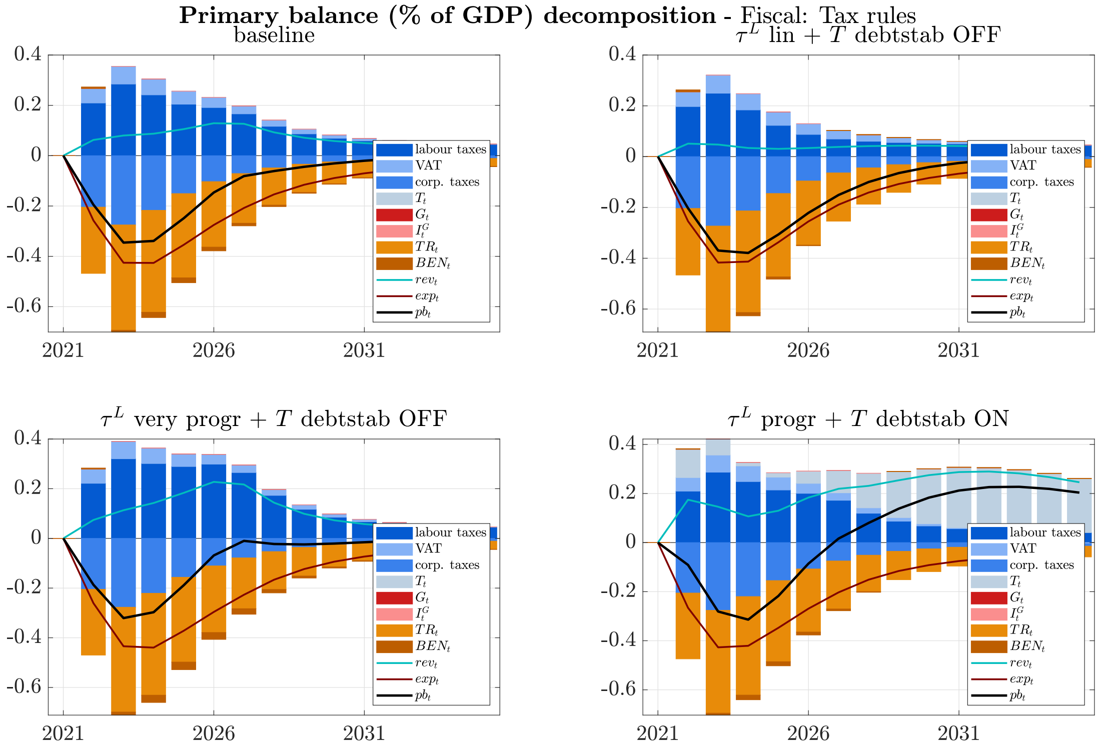

Figure 17

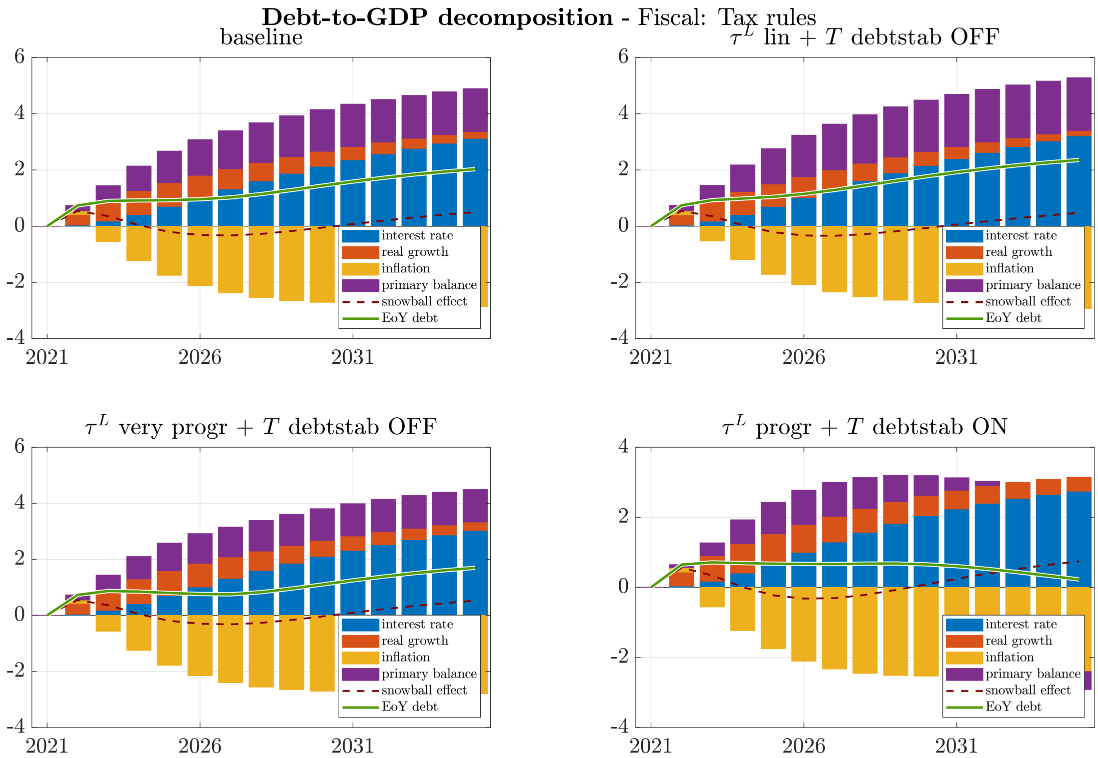

Figure 18

B.5 ToT shock (baseline) vs other shocks

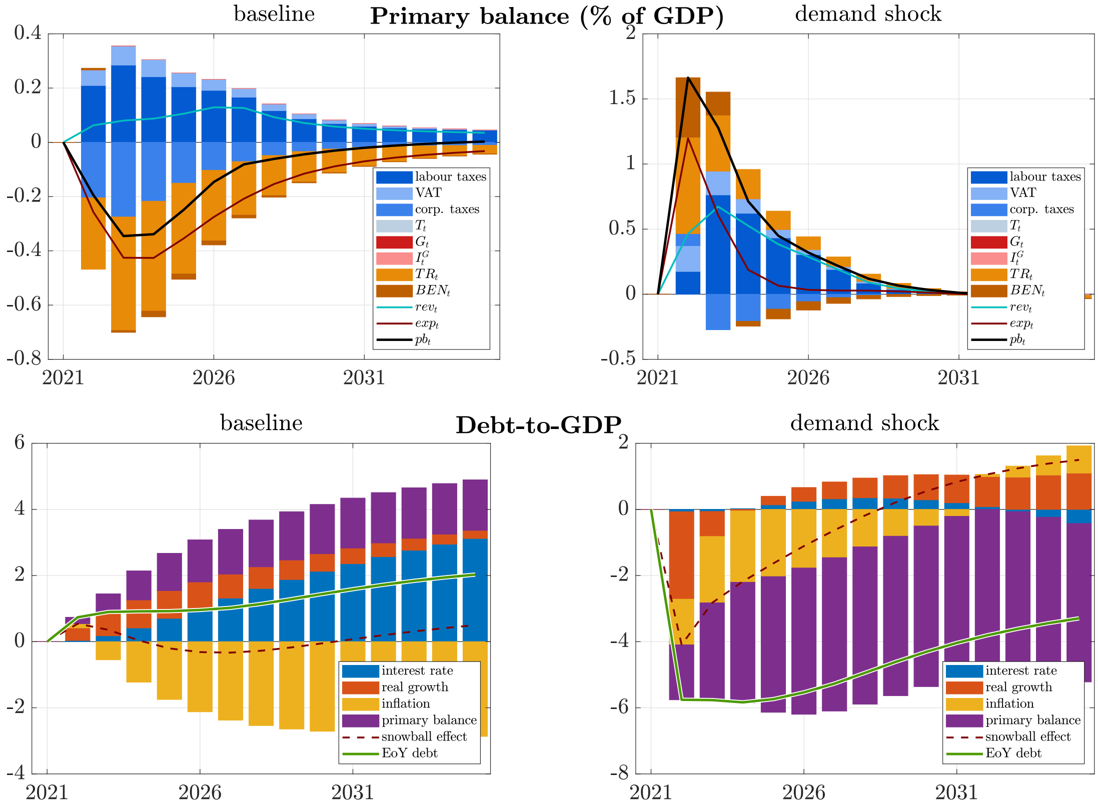

Figure 19

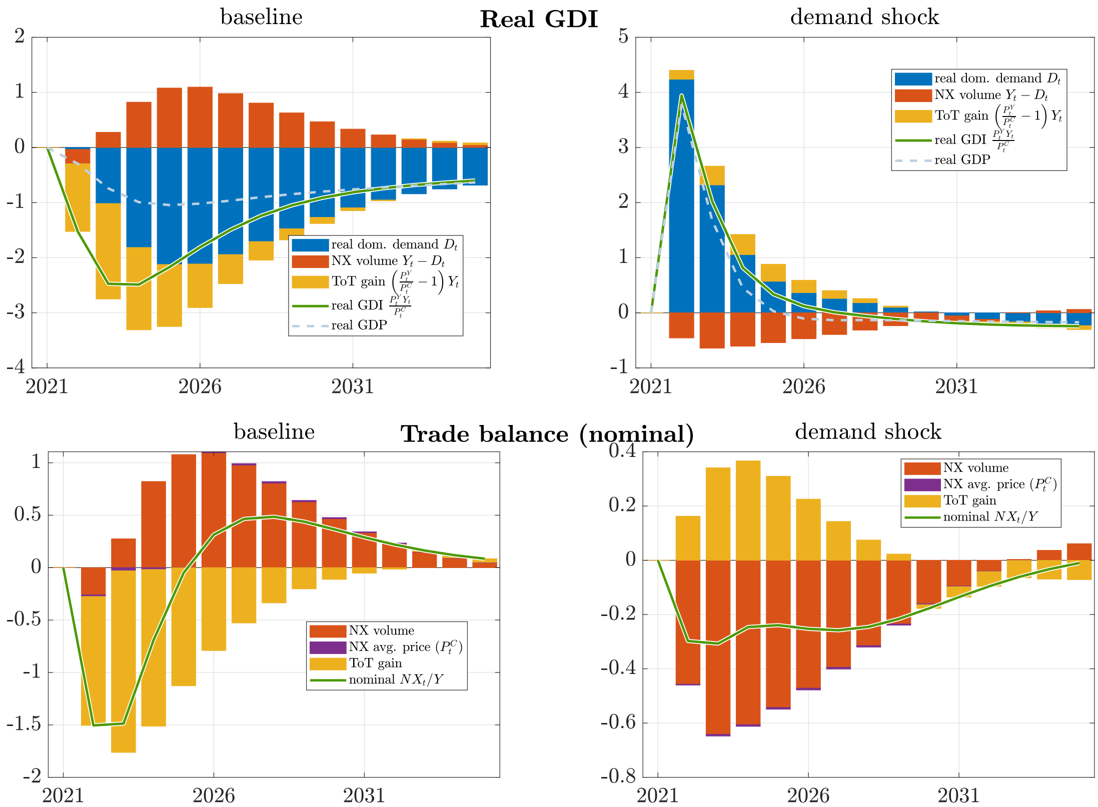

Figure 20

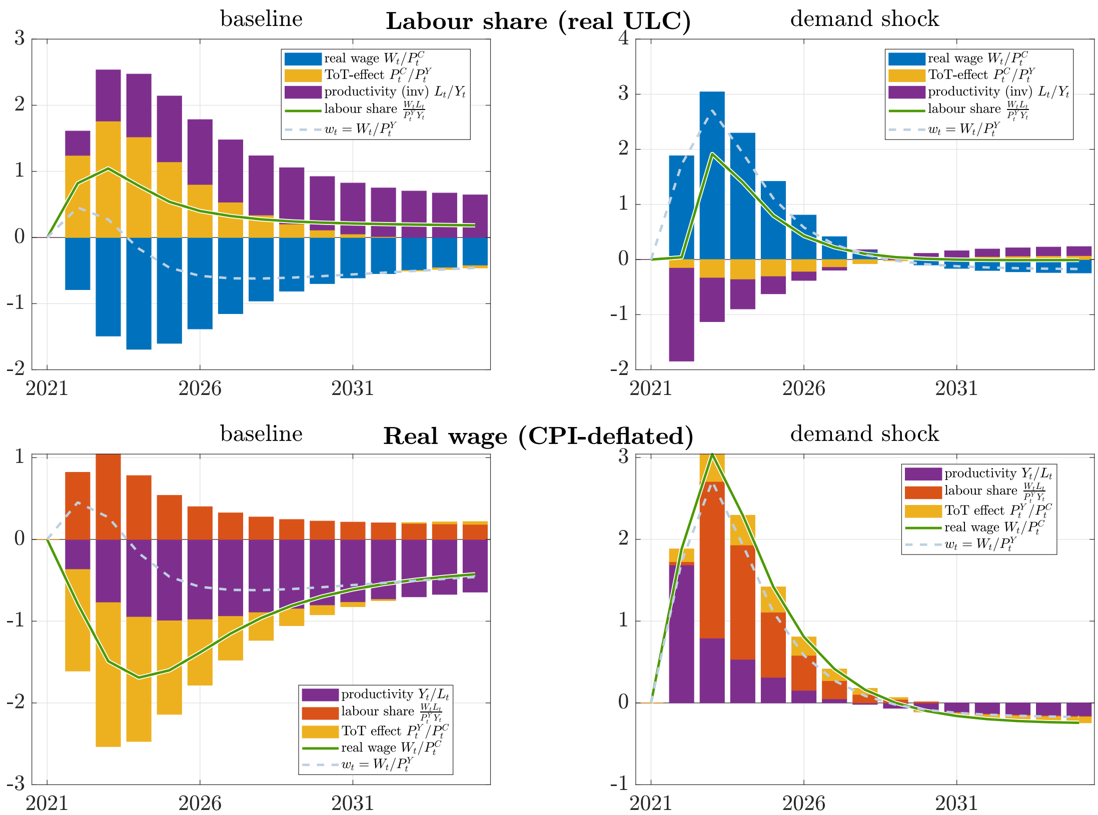

Figure 21

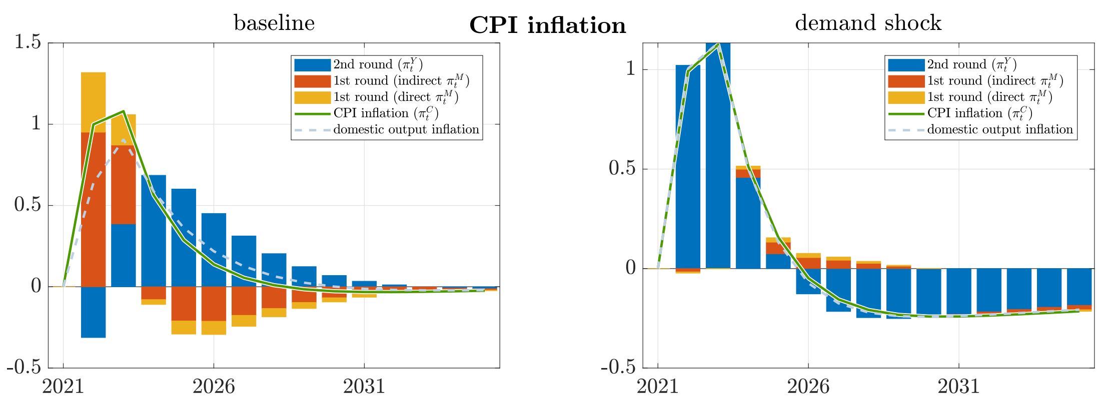

Figure 22

B.6 Elasticities of substitution

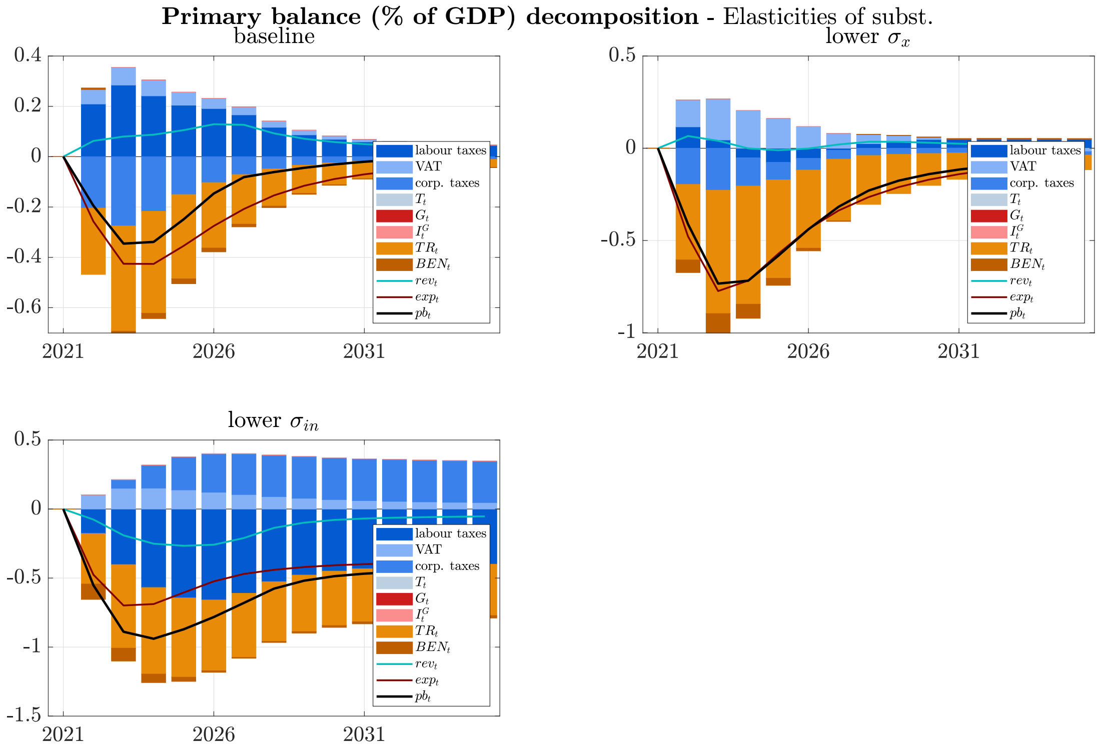

Figure 23

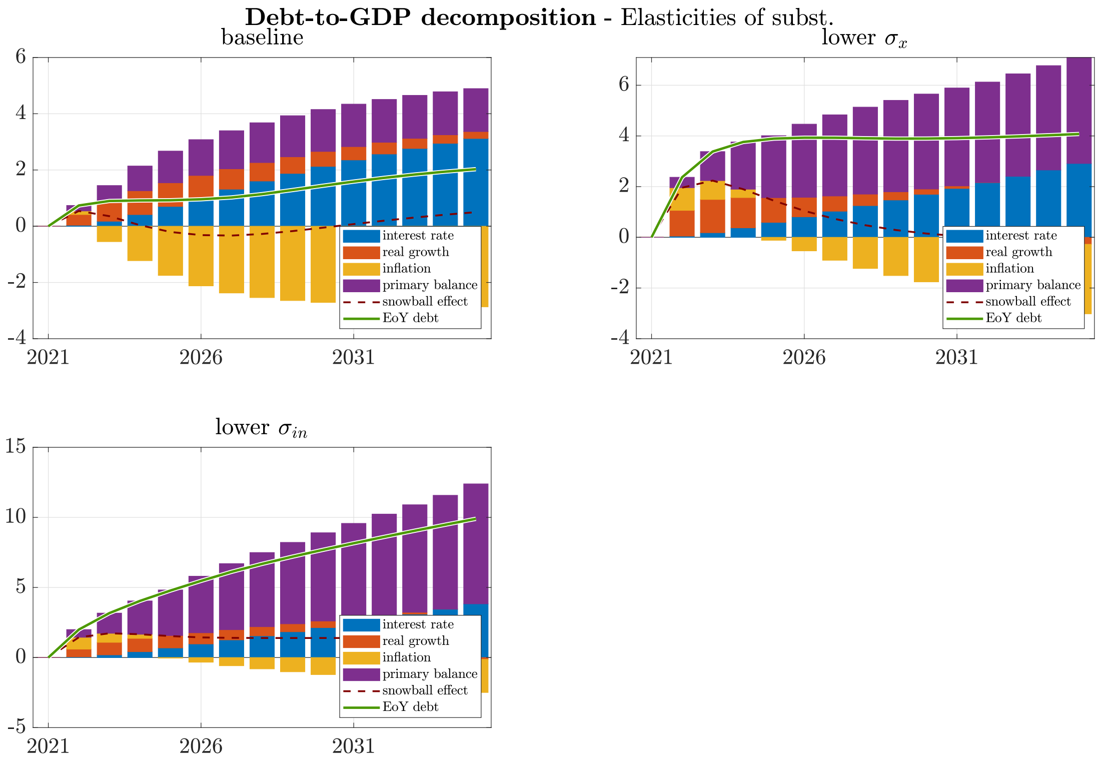

Figure 24

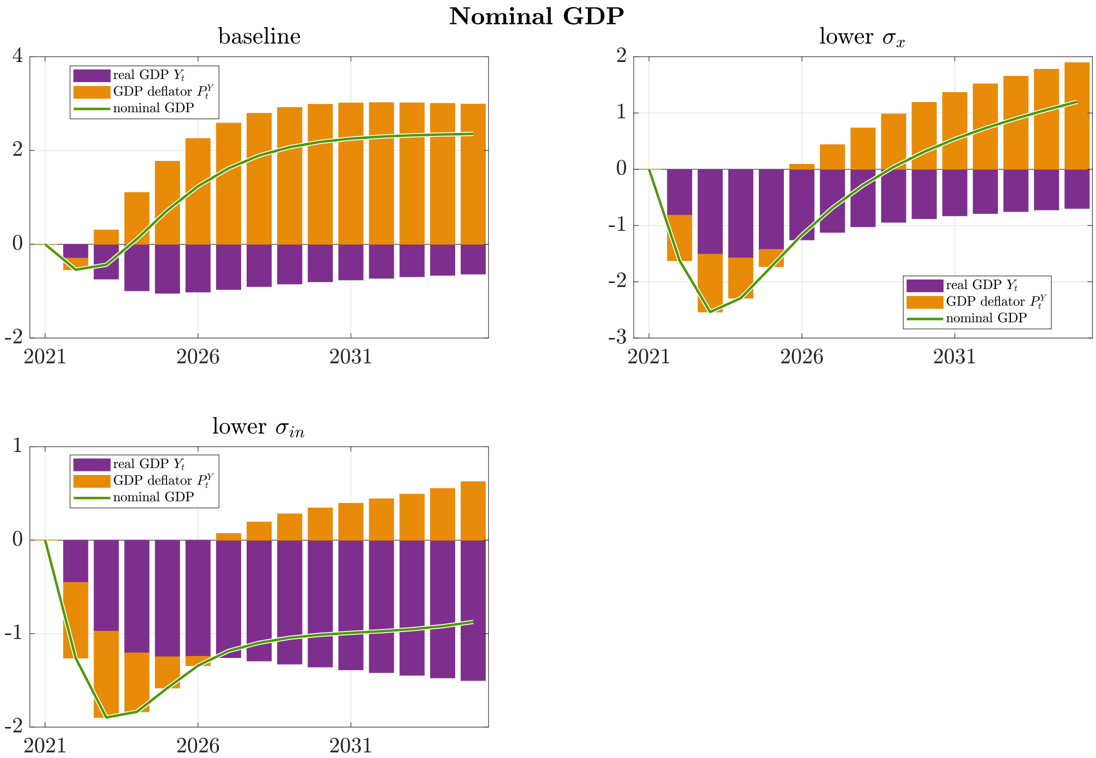

Figure 25

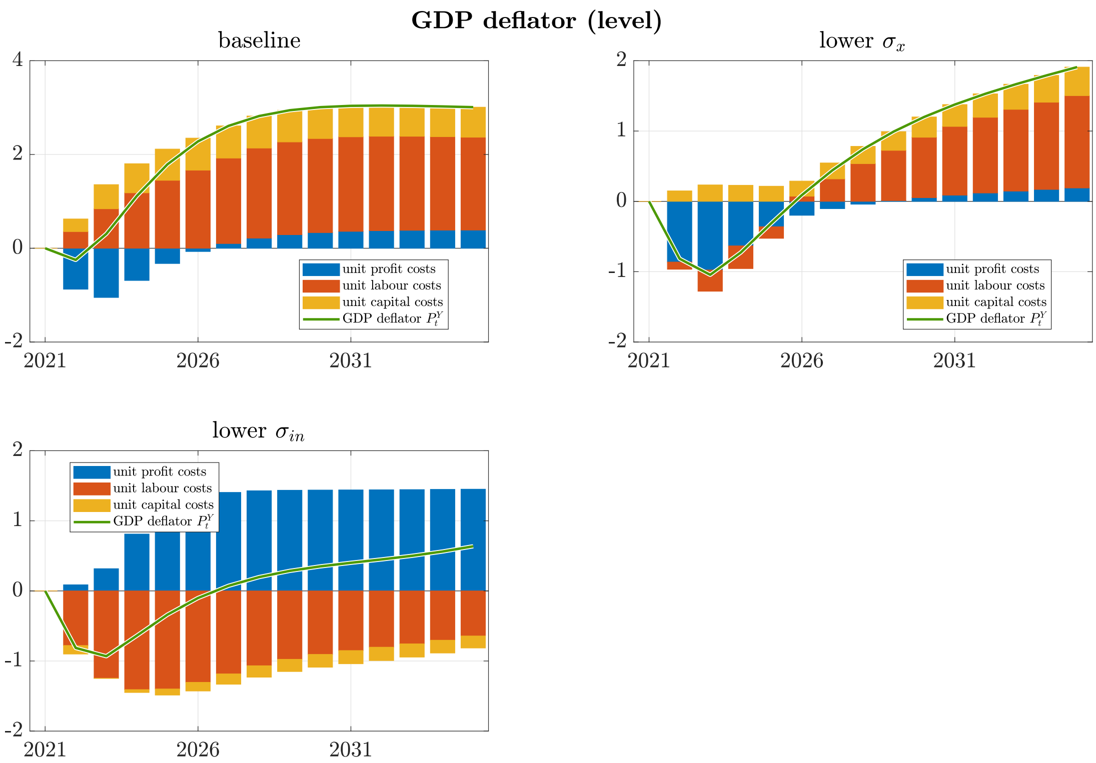

Figure 26

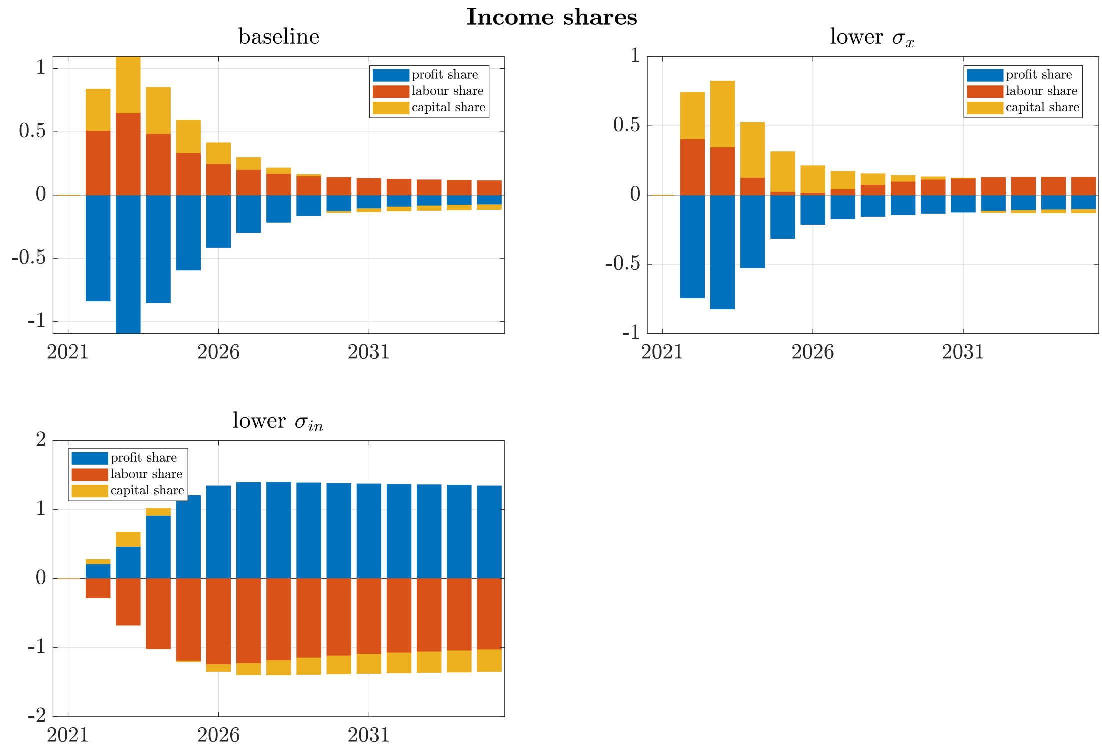

Figure 27

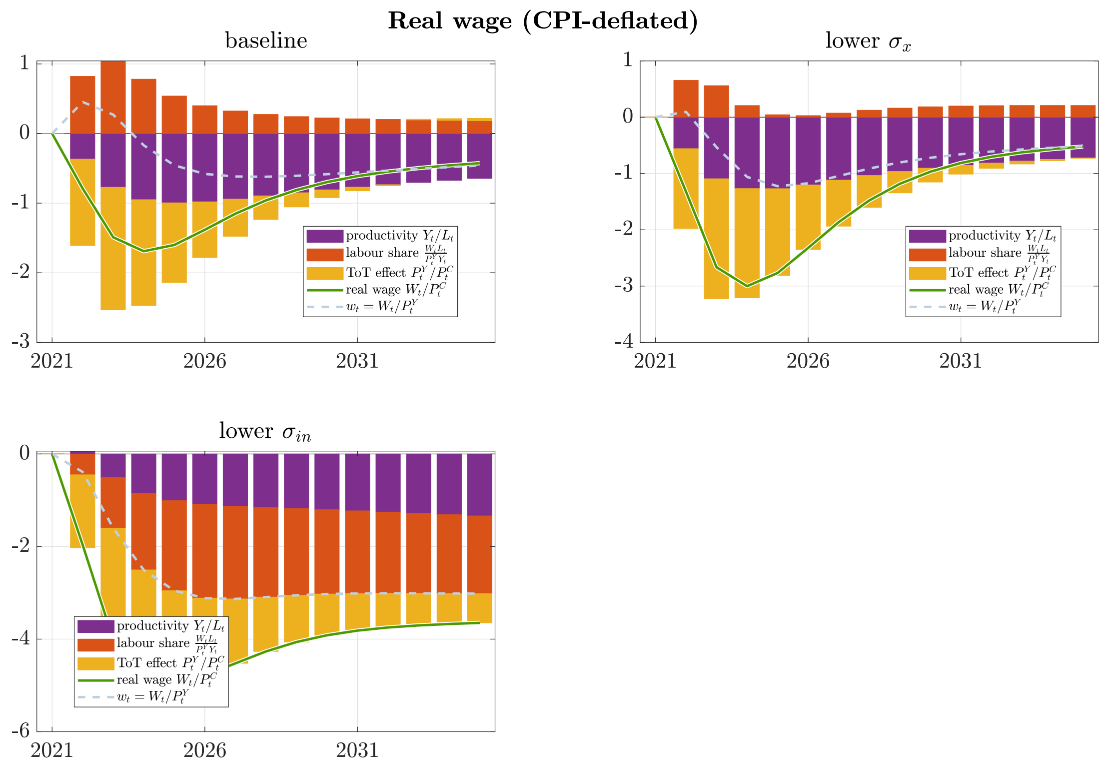

Figure 28

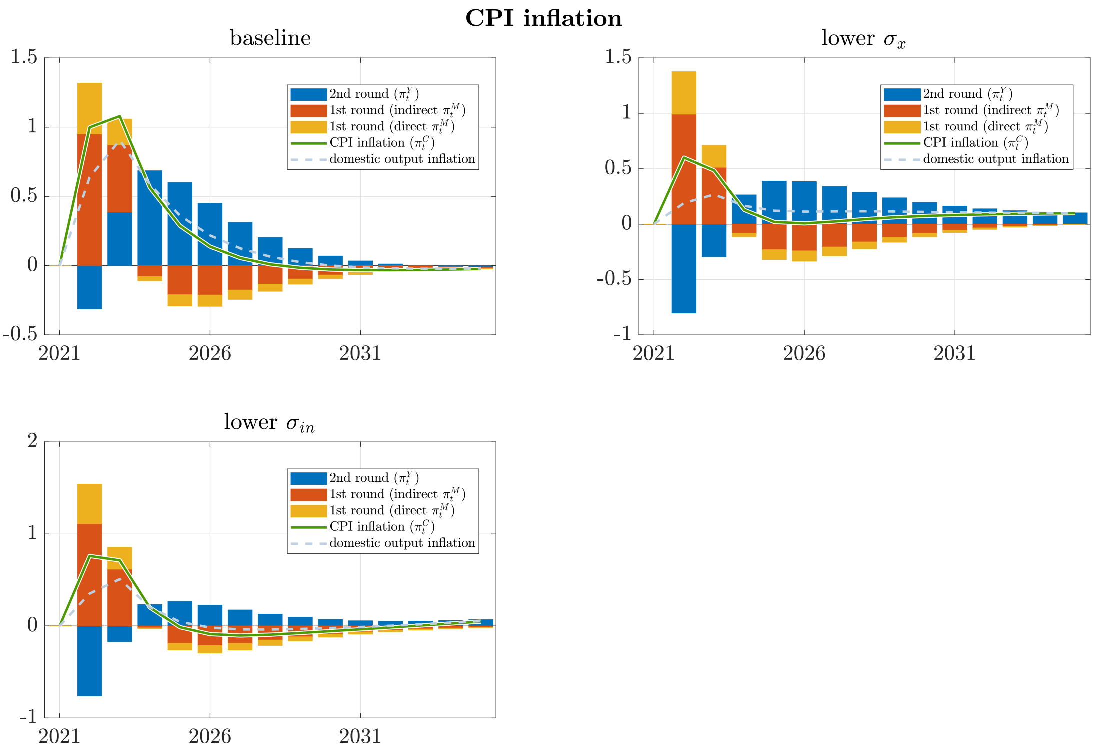

Figure 29

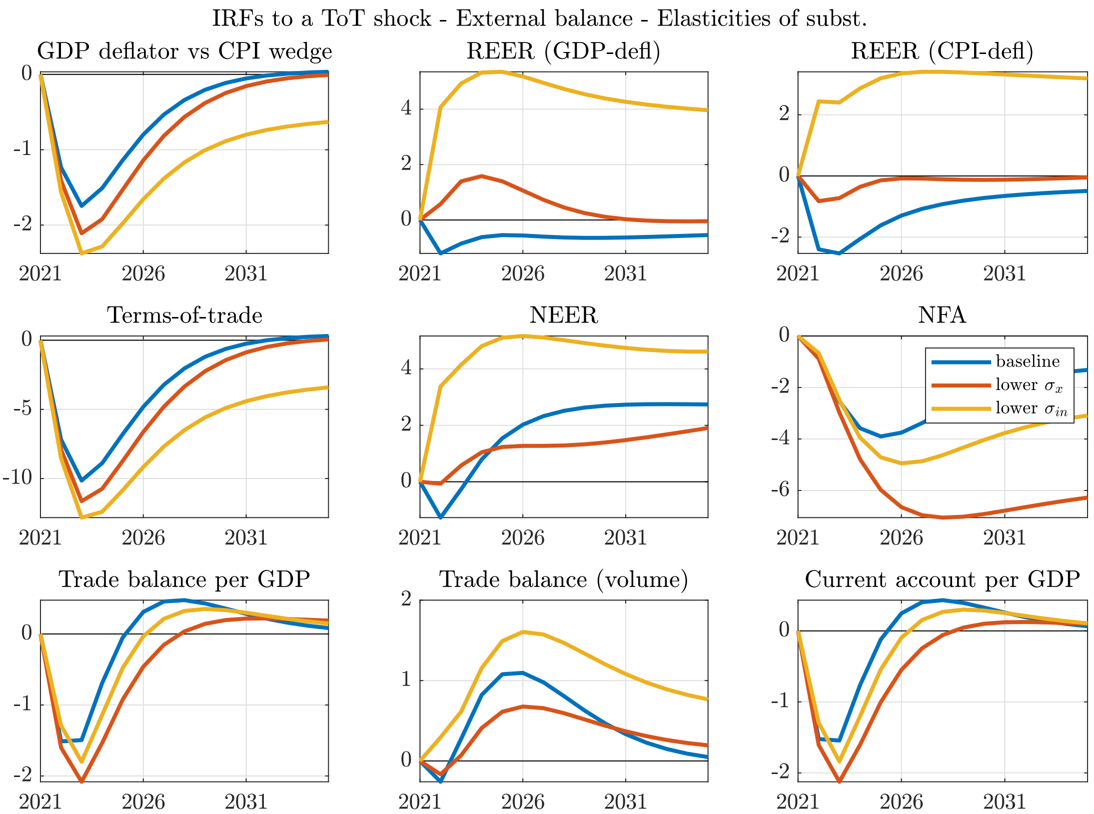

Figure 30

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