ANNEX A1

Criteria and decision trees used to assess fiscal sustainability risks

This annex presents the approach followed to assess fiscal sustainability risks over the short, medium and long term. Graph A1.1 provides an overview of the main building blocks. The general approach is the same as in the 2022 Debt Sustainability Monitor.

The remainder of this annex is organised as follows. Sections A1.1, A1.2 and A1.3 describe the approach to assess short-, medium- and long-term fiscal sustainability risks. Section A1.4 provides an overview of the thresholds used for the risk classification throughout the report.

A1.1. THE APPROACH USED TO ASSESS SHORT-TERM RISKS

The analysis of short-term fiscal sustainability risks relies on the composite S0 indicator. This early-detection indicator of fiscal stress follows a signalling approach: it flashes red when certain variables (among a set of 25) exceed critical thresholds beyond which they tended to be associated with episodes of fiscal stress in the past. S0 includes two sub-indices that cover the fiscal side and the financial-competitiveness side. The main benefit of this approach is therefore that it does not only consider purely fiscal factors, but also the risks that may arise from non-fiscal factors, thus recognising the role of structural weaknesses in triggering fiscal stress. Further details on S0 are available in Chapter I.1 (in particular in Box I.1.1) and Annex A2.

A1.2. THE APPROACH USED TO ASSESS MEDIUM-TERM RISKS

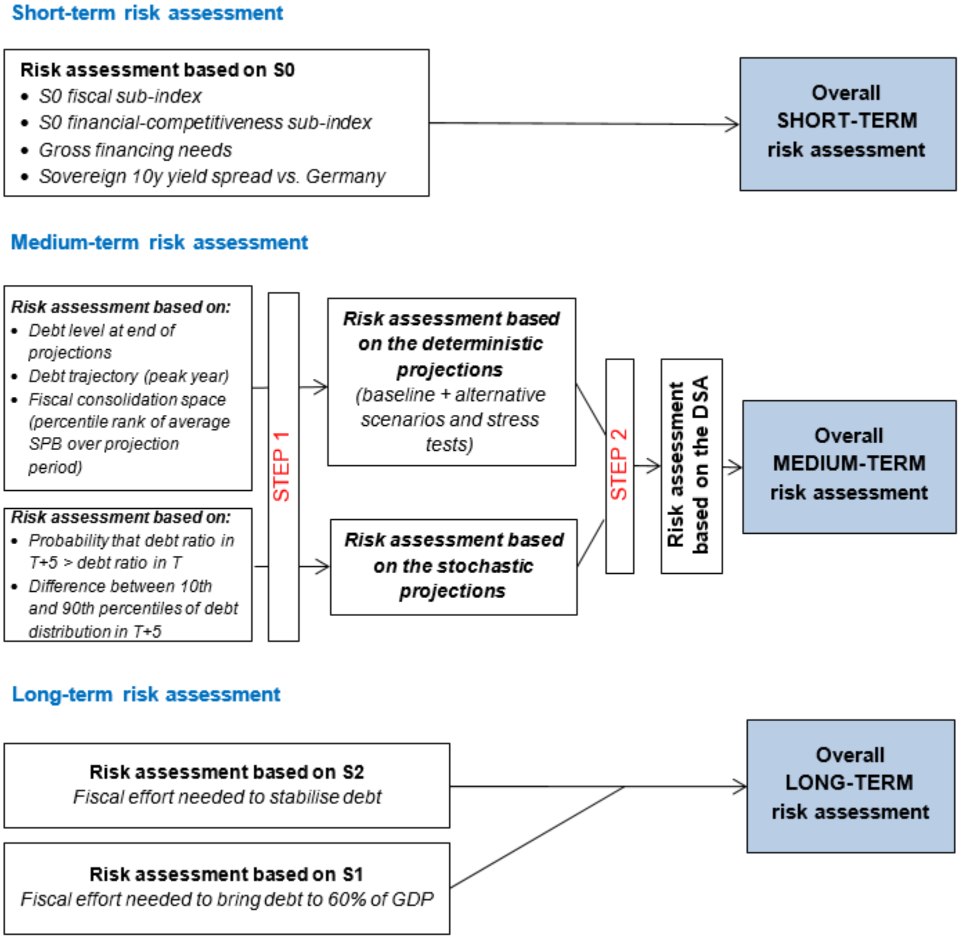
The assessment of medium-term risks is based on the debt sustainability analysis (DSA) risk classification, which is established in two steps. The first step assigns a risk category to the country under consideration for each of the deterministic projections (including the baseline) and for the stochastic projections. The second step combines the risk categories derived from the various deterministic scenarios and from the stochastic projections to conclude on the overall DSA risk classification. Further details on the DSA can be found in Chapter I.2.

In the first step, *the risk assessment based on the deterministic scenarios* depends on three criteria. These are (1) the projected debt level in 10 years' time, (2) the projected debt trajectory (as summarised by the year in which debt is projected to peak), and (3) the 'fiscal consolidation space', as measured by the percentile rank of the projected structural primary balance (SPB) in the past distribution of SPBs. The fiscal consolidation space gives an indication of whether the projected SPB is plausible in view of the country's track record, and whether the country has fiscal room for manoeuvre to take corrective measures if necessary.

The decision tree for deterministic projections describes how the three criteria interplay. First, the value of each criterion is associated with a risk category (low, medium or high, according to the thresholds reported in Table A1.1 below), then the risk categories derived from the three criteria are combined along the decision tree presented in Graph A1.2. While the risk classification starts from the risk signal associated with the projected debt level, this signal may be notched up or down by one category depending on the projected debt trajectory and the available 'fiscal consolidation space'. Fiscal consolidation space is measured by the percentile rank of the SPB within the country-specific historical distribution of the SPB. The historical distributions start at the earliest in 1980, depending on data availability. The calculations use 3-year moving averages and notably exclude major crisis years, such as the Global Financial Crisis (2008-09) and the COVID-19 pandemic (2020-21).

***The risk category based on the stochastic projections* depends on two criteria.** The first one is the probability that the debt level in 5 years' time will not exceed its current level. The second one is the amount of uncertainty, as measured by the difference between the 10th and 90th percentiles of the distribution of debt paths resulting from the stochastic projections (i.e. the difference between the worst

Graph A1.1: **The multi-dimensional approach to assess fiscal sustainability risks**



Source: European Commission.

and the best possible outcomes, leaving aside tail events). The thresholds associated with these criteria are reported in Table A1.1, and the decision tree combining the two criteria is presented in Graph A1.3.

The second step combines the signals from the deterministic and stochastic projections. Each country is first attributed a preliminary risk classification based on the baseline. This preliminary category may then be notched up, but not down. It may be adjusted from low to medium or from medium to high based on the outcome of other scenarios and stochastic projections, as described in Graph A1.4. On the other hand, if a country is considered at high risk under the baseline, the overall DSA risk category is automatically high.

Table A1.1: **DSA: thresholds for the deterministic and stochastic projections**

	Criterion	Threshold	
Deterministic projections	Debt level in 2034	**High:** above 90% of GDP **Medium:** between 60% and 90% of GDP **Low:** below 60% of GDP	
Deterministic projections	Debt trajectory (debt peak year)	**High:** peak year between T+7 (2030) and end of projections (2034), or still increasing by end of projections **Medium:** peak year between T+3 (2026) and T+6 (2029) **Low:** peak year within the T+2 forecast horizon (2023-2025)	
Deterministic projections	Fiscal consolidation space (percentile rank of average SPB in 2025-2034)	**High:** up to 25% **Medium:** between 25% and 50% **Low:** above 50%	
Stochastic projections	Probability of debt not stabilising over the next 5 years, i.e. of debt ratio in 2028 exceeding the initial debt ratio	Initial debt ratio ≥ 90%	**High:** if probability > 30% **Medium:** if 0 < probability ≤ 30% **Low:** if probability = 0
Stochastic projections		60 % ≤ initial debt ratio < 90%	**High:** if probability > 60% **Medium:** if 30% < probability ≤ 60% **Low:** if probability ≤ 30%
Stochastic projections		Initial debt ratio < 60%	**Medium:** if probability > 70% **Low:** if probability ≤ 70%
Stochastic projections	Size of macroeconomic uncertainty (diff. btw 10th and 90th percentiles of the distribution of debt paths)	**High:** the third of the countries with highest dispersion **Medium:** the third of the countries with intermediate dispersion **Low:** the third of the countries with lowest dispersion	

Source: European Commission.

Graph A1.2: **DSA, step 1: decision tree for the deterministic projections (including the baseline)**

All deterministic DSA scenarios

Case	Debt level	Debt path	Consolidation space	Overall
1	HIGH	HIGH/MEDIUM	**ANY**	HIGH
2	HIGH	LOW	HIGH/MEDIUM	HIGH
3	HIGH	LOW	LOW	MEDIUM
4	MEDIUM	HIGH	HIGH/MEDIUM	HIGH
5	MEDIUM	HIGH	LOW	MEDIUM
6	MEDIUM	MEDIUM	ANY	MEDIUM
7	MEDIUM	LOW	HIGH/MEDIUM	MEDIUM
8	MEDIUM	LOW	LOW	LOW
9	LOW	HIGH	HIGH/MEDIUM	MEDIUM
10	LOW	HIGH	LOW	LOW
11	LOW	MEDIUM/LOW	ANY	LOW

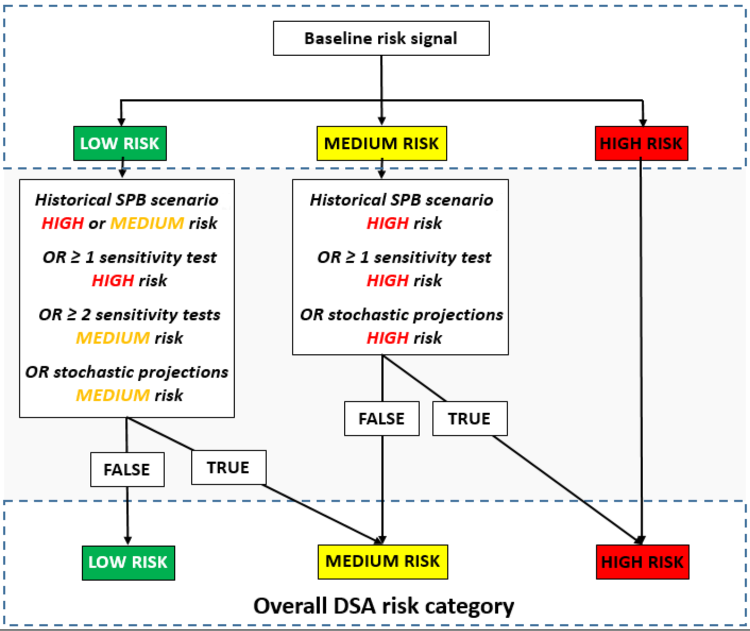
Note: the table is to be read as a decision tree, starting from the debt level then moving on to the debt path and the fiscal consolidation space. The risk category derived from the debt level in T+10 is notched up if the debt path points to high risk and the consolidation space points to medium or high risk (cases 4 and 9). Indeed, in these cases, countries have an increasing debt and limited consolidation space, meaning that there is a chance that there is no feasible adjustment path to curb the debt path. Conversely, the risk is notched down if both the debt path and the consolidation space indicator point to low risk (cases 3 and 8). In these cases, even if the projected debt level is high/medium, the debt path is decreasing, and the country has enough space to take measures in case of adverse shocks.
Source: European Commission.

Graph A1.3: **DSA, step 1: decision tree for the stochastic projections**

Probability of debt not stabilising	Size of uncertainty	Overall
HIGH	ANY	HIGH
MEDIUM	HIGH	MEDIUM
MEDIUM	MEDIUM	MEDIUM
MEDIUM	LOW	LOW
LOW	HIGH	MEDIUM
LOW	MEDIUM	LOW
LOW	LOW	LOW

Note: The table is to be read from left to right as a decision tree, starting from the probability of debt not stabilising then moving on to the size of uncertainty. It gives a strong weight to the probability of debt not stabilising over the next 5 years. Only in cases where the signal associated to this probability is medium and uncertainty is low, is the overall risk category notched down to low risk. Conversely, in cases where this probability is deemed low, but uncertainty is high, the overall risk category is notched up to medium risk.
Source: European Commission.

Graph A1.4: **DSA, step 2: decision tree for the overall DSA risk classification**



Note: it is not possible for a country to be classified at low risk under the baseline and at high risk under the stochastic projections.
Source: European Commission.

A1.3. THE APPROACH USED TO ASSESS LONG-TERM RISKS

The assessment of long-term fiscal sustainability risks is based on the S2 and S1 indicators. The S2 indicator measures the fiscal effort needed to stabilise debt in the long term, regardless of the level, based on the infinite version of the government budget constraint (see Box I.3.1). The S1 indicator measures the fiscal effort needed to bring debt to 60% of GDP by 2070. For both indicators, the risk assessment depends on the amount of fiscal consolidation needed: high risk if the required effort exceeds 6 pp. of GDP, medium risk if it lies between 2 pp. and 6 pp. of GDP, and low risk if the effort is negative or below 2 pp. of GDP (see Table A1.3). Finally, the overall long-term risk classification brings together the risk categories derived from S1 and S2. S1 may notch up the risk category derived from S2 when it signals a higher risk than S2. As a result, a country is assessed to be at high risk if (i) the S2 indicator flags high risk, irrespective of the risk category derived from S1, or (ii) S2 signals medium risk but S1 points to high risk (see Table A1.2). Similarly, a country is assessed at medium risk if S2 points to low risk but S1 flags medium or high risk. The aim of these adjustments is to capture risks linked to higher debt levels, as explained in Box I.3.1. The long-term risk classification is discussed in Chapter I.3, and technical details can be found in Annex A5.

Table A1.2: **Decision tree for the long-term risk classification**

Risk derived from S2	Risk derived from S1	Overall long-term risk category
HIGH	Any	HIGH
MEDIUM	HIGH	HIGH
	MEDIUM	MEDIUM
	LOW	
LOW	HIGH	MEDIUM
	MEDIUM	
	LOW	LOW

Source: European Commission.

A1.4. OVERVIEW OF THE THRESHOLDS USED TO ASSESS FISCAL SUSTAINABILITY RISKS

The thresholds underpinning the various heat maps presented in the report can be found in the following tables. ***The thresholds for the DSA risk classification***, both for the deterministic and stochastic projections, are reported in Table A1.1. ***For the short term,*** Table A1.3 reports the thresholds used for the S0 indicator, its sub-indices, and each of the variables that they include. The overall S0 index and its sub-indices use only one threshold, beyond which they identify vulnerabilities. For the individual variables, the upper thresholds derived from the signalling approach are complemented by lower thresholds, set at around 80% of the upper thresholds, so that variables may flash red, yellow or not flash at all. ***For the S1 and S2 indicators,*** Table A1.3 reports upper and lower thresholds to distinguish between low, medium and high risk. The percentile ranks of the SPBs required by S1 and S2 are subject to the same thresholds as average SPBs in DSA scenarios (Table A1.1).

Table A1.3: **Overview of the thresholds used for the fiscal sustainability risk classification**

	Safety	*Upper threshold*	*Lower threshold*
SHORT-TERM RISKS			
S0 overall index	<	0.46	:
S0 fiscal sub-index	<	0.36	:
S0 financial-competitiveness sub-index	<	0.49	:
Fiscal risks from the fiscal context			
Balance (% of GDP)	>	-9.6	-7.7
Primary balance (% of GDP)	>	0.2	0.3
Cyclically-adjusted balance (% of GDP)	>	-2.5	-2.0
Stabilising primary balance (% of GDP)	<	2.3	1.9
Gross debt (% of GDP)	<	68.4	54.8
Change in gross debt (% of GDP)	<	8.1	6.4
Short-term public debt (% of GDP)	<	13.2	10.6
Net debt (% of GDP)	<	59.5	47.6
Gross financing needs (% of GDP)	<	15.9	12.8
Interest-growth rate differential (%)	<	4.8	3.8
Change in governement expenditure (% of GDP)	<	1.9	1.5
Change in governement consumption (% of GDP)	<	0.6	0.5
Fiscal risks from the macro-financial context			
Yield curve (%)	>	0.6	0.7
Real GDP growth (%)	>	-0.7	-0.5
GDP per capita in PPP (% US level)	>	72.7	87.2
Net international investment position (% of GDP)	>	-19.8	-15.8
Net savings households (% of GDP)	>	2.6	3.1
Private debt (% of GDP)	<	164.7	131.8
Private credit flow (% of GDP)	<	11.7	9.4
Short-term debt non-financial corporations (% of GDP)	<	15.4	12.3
Short-term debt households (% of GDP)	<	2.9	2.3
Construction (% of value added)	<	7.5	6.0
Current account balance (% of GDP)	>	-2.5	-2.0
Change in REER (%)	<	9.7	7.7
Change in nominal ULC (%)	<	7.0	5.6
Fiscal risks from financial market developments			
Sovereign yield spreads (bp) - 10 year	<	231.0	184.8
MEDIUM-TERM RISKS			
DSA variables		see Table A1.2	
LONG-TERM RISKS			
S2 indicator	<	6	2
Percentile rank of the SPB implied by S2	>	25%	50%
S1 indicator	<	6	2
Percentile rank of the SPB implied by S1	>	25%	50%
ADDITIONAL VARIABLES			
Structure of public debt			
Share of short-term public debt (% of debt)	<	6.6	5.3
Share of public debt in foreign currency (% of debt)	<	31.6	25.0
Share of public debt held by non-residents (% of debt)	<	49.0	40.0
Contingent liabilites linked to the banking sector			
Bank loans-to-deposits ratio (%)	<	133.4	107.0
Share of non-performing loans (% of loans)	<	2.3	1.8
Change in share of non-performing loans (p.p.)	<	0.3	0.2
NPL coverage ratio (% loans)	>	66.0	33.0
Change in nominal house prix index (%)	<	13.2	11.0

Note: Variables common to the scoreboard used in the macroeconomic imbalances procedure (MIP) have different thresholds here than under the MIP, because the methodologies to calculate them are different
Source: European Commission.

ANNEX A2
The early-detection indicator of fiscal stress risk (S0)

This annex describes the methodology of the Commission's early-warning indicator S0. S0 measures fiscal stress in the following year using a signalling. It flashes red when certain variables (among a set of 25) exceed critical thresholds beyond which they tended to be associated with episodes of fiscal stress in the past. S0 includes two sub-indices that cover the fiscal side and the financial-competitiveness side.

A2.1. THE METHODOLOGY FOR THE CALCULATION OF THE THRESHOLDS

For each variable used in the composite indicator S0 the optimal threshold is chosen in a way to minimise, based on historical data, the sum of the number of fiscal stress signals sent ahead of no-fiscal-stress episodes (false positive signals – type-I error) and the number of no-fiscal-stress signals sent ahead of fiscal stress episodes (false negative signals – type-II error), with different weights attached to the two components**.** The table below reports the four possible combinations of events.

Table A2.1: **Possible cases based on type of signal sent by the variable at t-1 and state of the world at t**

	Fiscal stress episode	No-fiscal stress episode
Fiscal stress signal	True positive signal	False positive signal (Type I error)
No-fiscal stress signal	False negative signal (Type II error)	True negative signal

Source: Commission services

Formally, for each variable *i* the optimal threshold (t_i^*) is chosen to minimise the sum of type I and type II errors for variable *i* (respectively fiscal stress signals followed by no-fiscal stress episodes - False Positive signals - and no-fiscal-stress signals followed by fiscal stress episodes – False Negative signals) as from the following total misclassification error for variable *i* (TME_i): ([112])

$$t_i^* = \arg\min_{t_i \in T_i} \left(TME_i(t_i)\right) =$$

$$= \arg\min_{t_i \in T_i} \left(\frac{FN_i(t_i)}{Fs} + \frac{FP_i(t_i)}{Nfs} \right) \qquad (1)$$

$i = 1,.., n$

where T_i = set of all values taken by variable *i* over all countries and years in the panel; $FN_i(t_i)$ = total number of false negative signals sent by variable *i* (over all countries and years) based on threshold t_i; $FP_i(t_i)$ = total number of false positive signals sent by variable *i* (over all countries and years) based on threshold t_i; *Fs* = total number of fiscal stress episodes recorded in the data; *Nfs* = total number of no-fiscal-stress episodes recorded in the data; ([113]) *n* = total number of variables used.

As can be seen from the minimisation problem in (1), `false negative' signals are weighted more than `false positive' signals as:

([112]) Following this methodological approach the optimal threshold will be such as to balance between type I and type II errors. For variables for which values above the threshold would signal fiscal stress, a relatively low threshold would produce relatively more false positive signals and fewer false negative signals, meaning higher type I error and lower type II error; the opposite would be true if a relatively high threshold was chosen.

([113]) Here we simplify on the total number of fiscal stress and non-fiscal-stress episodes as in fact also these numbers vary across variables. This is due to the fact that data availability constraints do not allow us to use the whole series of episodes for all variables.

$$\frac{1}{Fs} > \frac{1}{Nfs}$$

This is due to the fact that the total number of fiscal stress episodes recorded over a (large enough) panel of countries will be typically much smaller than the total number of non-fiscal-stress episodes. This is a positive feature of the model as we might reasonably want to weigh the type II error more than the type I given the more serious consequences deriving from failing to correctly predict a fiscal stress episode relative to predicting a fiscal stress episode when there will be none.

The threshold for variable i (with $i = 1,\dots, n$) obtained from (1) is common to all countries in the panel. We define it as a common *absolute* threshold (a critical value for the level of public debt to GDP, or general government balance over GDP, for instance) but it could also be defined as a common *relative* threshold (a common percentage tail of the country-specific distributions). ([114]) In the latter case, while the optimal percentage tail obtained from (1) is the same for all countries, the associated absolute threshold will differ across countries reflecting differences in distributions (country *j*'s absolute threshold for variable *i* will reflect the country-specific history with regard to that variable). Both the aforementioned methods were applied and a decision was made to focus exclusively on the first, given that the second one tends to produce sensitive country-specific absolute thresholds for variable *i* only for those countries having a history of medium to high values for the variable concerned (or medium to low, depending on what the fiscal-stress-prone side of the distribution is), while country-specific thresholds would not be meaningful for the rest of the sample.

The TME function in equation (1) is the criterion we used to calculate the thresholds but it is not the only possible criterion used in the literature. The minimisation of the noise-to-signal ratio (*NSR*) is another possible option. ([115]) In this case the optimal threshold for variable i (t_i^*) is obtained as:

$$t_i^* = \arg\min_{t_i \in T_i} \left(NSR_i(t_i)\right) = \arg\min_{t_i \in T_i} \left(\frac{FP_i(t_i)/Nfs}{TP_i(t_i)/Fs}\right) \qquad (2)$$

$i = 1,\dots,n$

where $TP_i(t_i)$ = total number of true positive signals sent by variable i (over all countries and years) based on threshold t_i. The TME minimisation was preferred to this alternative criterion based on the size of the total errors produced.

A2.2. THE CALCULATION OF THE COMPOSITE INDICATOR S0

The early-detection indicator of fiscal stress (S0) is constructed in a similar way to what done in Baldacci et al. (2011) and Reinhart et al. (2000). ([116]) To a certain country j and year t, a 1 is assigned for every variable i that signals fiscal stress for the following year (a dummy d^i is created for each variable i such that $d_{jt}^i = 1$ if a fiscal stress signal is sent by the variable and $d_{jt}^i = 0$ otherwise, i.e. if a no-fiscal-stress signal is sent or the variable is missing). The value of the composite indicator S0 for country j and

([114]) See, for instance, Reinhart, Goldstein and Kaminsky (2000); Hemming, Kell and Schimmelpfennig (2003).
([115]) See, for instance, Reinhart, Goldstein and Kaminsky (2000); Hemming, Kell and Schimmelpfennig (2003).
([116]) See Berti et al. (2012). The difference with Baldacci et al. (2011) is that Berti et al. do not use a system of "double weighting" of each variable incorporated in the composite indicator based on the weight of the subgroup of variables it belongs to (fiscal and financial-competitiveness variables here) and the weight of the individual variable within the group. The difference with Reinhart et al. (2000) is in the way the individual variables' weights are computed (Reinhart et al. use as weights the inverse of the noise-to-signal ratios of the individual variables as they apply the NSR criterion, rather than the TME minimisation).

year t ($S0_{jt}$) is then calculated as the weighted number of variables having reached their optimal thresholds with the weights given by the "signalling power" of the individual variables:

$$S0_{jt} = \sum_{i=1}^{n} w_i d^i_{jt} = \sum_{i=1}^{n} \frac{z_i}{\sum_{k=1}^{n} h^k_{jt} \cdot z_k} d^i_{jt} \qquad (3)$$

where n = total number of variables; z_i = 1 – (type I error + type II error) = signalling power of variable i; and $h^k_{jt} \in \{0,1\}$ is an indicator variable taking value 1 if variable k is observed for country j at time t and 0 otherwise. ([117]) The variables are therefore assigned higher weight in the composite indicator, the higher their past forecasting accuracy. ([118])

([117]) This ensures that the sum of the weights is equal to 1 regardless of data availability (which is of course necessary to be able to analyse the evolution of the composite indicator).

([118]) Moreover, as evident from (3), the weight attached to each variable is decreasing in the signalling power attached to the other variables, as well as in the number of variables available for a given country and year.

ANNEX A3

Debt dynamics' projections: decomposition, interest rates and property incomes

A3.1. DECOMPOSING THE DEBT DYNAMICS

Deterministic government debt projections are based on a general identity characterising the evolution of the stock of debt. In a simplified version, the evolution of the government debt to GDP ratio can be described in the following way:

$$d_t = \alpha^n . d_{t-1} . \frac{(1+i_t)}{(1+g_t)} + \alpha^f . d_{t-1} . \frac{(1+i_t)}{(1+g_t)} . \frac{e_t}{e_{t-1}} - pb_t + f_t \quad (1)$$

where d_t represents the total government debt to GDP ratio in year t

α^n represents the share of total government debt denominated in national currency

α^f represents the share of total government debt denominated in foreign currency

i_t represents the implicit interest rate on government debt ([119])

g_t represents the *nominal* growth rate of GDP (in national currency)

e_t represents the nominal exchange rate (expressed as national currency per unit of foreign currency)

pb_t represents the primary balance over GDP

f_t represents the stock-flow adjustments over GDP.

In order to obtain the debt dynamics, d_{t-1} is subtracted from both sides of equation (1). This gives the following expression:

$$\Delta d_t = \alpha^n . d_{t-1} . \frac{(i_t - g_t)}{(1+g_t)} + \alpha^f . d_{t-1} . \frac{(i_t - g_t) + \varepsilon_t . (1+i_t)}{(1+g_t)} - pb_t + f_t \quad (2)$$

where $\varepsilon_t = \frac{e_t}{e_{t-1}} - 1$ represents the rate of depreciation of the national currency.

Decomposing further the nominal GDP growth rate, and rearranging the different terms, we obtain:

$$\Delta d_t = d_{t-1} . \frac{i_t}{(1+g_t)} - d_{t-1} . \frac{gr_t}{(1+g_t)} - d_{t-1} . \frac{\pi_t (1+gr_t)}{(1+g_t)} + \alpha^f . d_{t-1} . \varepsilon_t . \frac{(1+i_t)}{(1+g_t)} - pb_t + f_t \quad (2)'$$

where gr_t represents the *real* growth rate of GDP

π_t represents the inflation rate (in terms of GDP deflator, in national currency)

This expression allows us identifying the key drivers of the debt ratio dynamics, in particular the snow-ball effect, which can be further decomposed into four terms:

- (+) the interest rate effect: $d_{t-1} . \frac{i_t}{(1+g_t)}$

([119]) By simplicity, it is assumed that this interest rate is the same for government debt denominated in national currency and in foreign currency.

- (-) the real GDP growth effect: $-d_{t-1}.\frac{gr_t}{(1+g_{t})}$

- (-) the inflation effect: $-d_{t-1}.\frac{\pi_t(1+gr_t)}{(1+g_{t})}$

- (+) the exchange rate effect: $\alpha^f.d_{t-1}.\varepsilon_t.\frac{(1+i_t)}{(1+g_{t})}$

As can be easily seen from this expression, both the interest rate and the foreign exchange depreciation rate contribute to the increase of the debt ratio. On the other hand, higher real GDP growth and higher inflation erode the debt to GDP ratio. ([120])

Other key contributors to the debt motion are the primary balance (pb_t) (that is further decomposed in our tables between the structural primary balance before cost of ageing, the cost of ageing, the cyclical component and one-offs and other temporary measures) and stock and flow adjustments (f_t).

As can be seen from the exchange rate effect expression, both valuation effects affecting the *stock* of foreign currency denominated debt and *interest rate* payments (on this share of government debt) contribute to the debt dynamic. ([121]) Looking at historical series, Eurostat includes the exchange rate effect on the *stock* of foreign currency denominated debt in stock and flow adjustments, while the impact due to the cost of servicing debt in foreign currency is included in interest payments. In our tables, we follow this convention.

In practice, the equation used in our model is slightly more complex than equation (1), as we consider three currencies: the national currency, the EUR (foreign currency for non-euro area countries) and the USD (foreign currency for all countries). Hence, equation (1) becomes:

$$d_t = \alpha^n.d_{t-1}.\frac{(1+i_t)}{(1+g_t)} + \alpha^{eur}.d_{t-1}.\frac{(1+i_t)}{(1+g_t)}.\frac{e_t}{e_{t-1}} + \alpha^{usd}.d_{t-1}.\frac{(1+i_t)}{(1+g_t)}.\frac{\tilde{e}_{t-1}}{\tilde{e}_t}.\frac{e_t}{e_{t-1}} - pb_t + f_t \qquad (1)'$$

where α^{eur} represents the share of total government debt denominated in euros

α^{usd} represents the share of total government debt denominated in USD

e_t represents the nominal exchange rate between the national currency and the euro (expressed as national currency per EUR)

$\tilde{e}_t$ represents the nominal exchange rate between the USD and the euro (expressed as USD per EUR).

Such a specification allows taking into account the effect of exchange rate movements on government debt not only in non-euro area countries, but also in euro area countries (among which government debt issued in USD can be significant).

([120]) This presentation, based on the government debt ratio identity equation, allows grasping the impact of real GDP growth and inflation on the debt motion coming from direct valuation effects (as government debt is expressed as a share of GDP). However, the primary balance is also influenced by economic activity and inflation. Such behavioural effects are explicitly taken into account in the fiscal reaction function scenario presented in chapter 2 of the report.

([121]) An indirect effect, due to the fact that exchange rate movements affect the value of GDP in domestic currency through changes in prices in the tradable sector, could also be shown. However, in practice, in line with other institutions practices (e.g. IMF), these effects are not isolated (data limitation would require to impose further assumptions; effect likely to be of second-order).

A3.2. PROJECTING THE IMPLICIT INTEREST RATE ON GOVERNMENT DEBT

As seen from equation (1), a key driver of the debt motion is the implicit interest rate on government debt. Projecting the implicit interest rate on government debt requires not only assumptions on *market* interest rates (for newly issued debt), but also taking into account explicitly the current and future maturity structure of government debt (between short-term and long-term government debt, and between maturing, rolled-over or not, and non-maturing government debt). This allows a differential treatment in terms of interest rates applied to successive "debt vintages", and interestingly captures different levels of exposure of sovereigns to immediate financial markets' pressures.

Formally, in our model, the implicit interest rate is expressed in the following way:

$$iir_t = \alpha_{t-1}. i_t^{ST} + (1 - \alpha_{t-1}). iir_t^{LT} \qquad (3)$$

where iii_t is the implicit interest rate in year t ([122])

i_t^{ST} is the *market* short-term interest rate in year t

iir_t^{LT} is the implicit long-term interest rate in year t

α_{t-1} is the share of short-term debt in total government debt (and $(1 - \alpha_{t-1})$ is the share of long-term debt in total government debt). ([123])

Our model considers two types of government debt in terms of maturity: short-term debt (debt issued with an *original* maturity of less than one year) and long-term debt (debt issued with an *original* maturity of more than one year). Furthermore, government debt can be decomposed between new debt (debt issued to cover new financing requirements), ([124]) maturing debt (i.e. existing debt that is maturing within the year ([125]) and that needs to be repaid), rolled-over (i.e. whose repayment is covered by newly issued debt) or not, and outstanding debt (i.e. existing debt that has not reached maturity). Combining these different aspects, α_{t-1} (and $(1 - \alpha_{t-1})$) used in (3) can be described as follows:

$$\alpha_{t-1} = \frac{D_{t-1}^{STN} + D_{t-1}^{STR}}{D_{t-1}} \qquad (4)$$

$$1 - \alpha_{t-1} = \frac{D_{t-1}^{o} + D_{t-1}^{LTN} + D_{t-1}^{LTR}}{D_{t-1}} \qquad (5)$$

where D_{t-1}^{STN} is the new short-term government debt in year $t-1$

D_{t-1}^{STR} is the maturing and rolled-over short-term government debt (i.e. the existing short-term debt that has reached maturity, and whose repayment is covered by newly issued short-term debt)

D_{t-1}^{LTN} is the new long-term government debt

D_{t-1}^{LTR} is the maturing and rolled-over long-term government debt (i.e. the existing long-term debt that has reached maturity, and whose repayment is covered by newly issued long-term debt)

D_{t-1}^{o} is the outstanding (non-maturing) long-term government debt.

([122]) This corresponds to i_t in the previous section.
([123]) Hence, as indicated by the *t* index, these shares may vary through time depending on the debt dynamic.
([124]) This amount also corresponds to the yearly budgetary deficit.
([125]) Another way to describe it is that this existing debt has a *residual* maturity of less than one year.

Moreover, the implicit long-term interest rate used in (3) can be further decomposed:

$$iir_t^{LT} = \beta_{t-1}.\, i_t^{LT} + (1-\beta_{t-1}).\, iir_{t-1}^{LT} \qquad (6)$$

where β_{t-1} is the share of newly issued long-term debt (corresponding to both new debt and maturing and rolled-over debt) in total long-term government debt in year $t-1$ (and $(1-\beta_{t-1})$ is the share of outstanding long-term debt in total long-term government debt)

i_t^{LT} is the *market* long-term interest rate in year t.

The share of newly issued long-term debt (respectively outstanding debt) in total long-term government debt, used in expression (6), is described as follows:

$$\beta_{t-1} = \frac{D_{t-1}^{LTN}+D_{t-1}^{LTR}}{D_{t-1}^{O}+D_{t-1}^{LTN}+D_{t-1}^{LTR}} \qquad (7)$$

$$(1-\beta_{t-1}) = \frac{D_{t-1}^{O}}{D_{t-1}^{O}+D_{t-1}^{LTN}+D_{t-1}^{LTR}} \qquad (8)$$

Hence, replacing iir_t^{LT} in (3) by its expression in (6) gives:

$$iir_t = a_{t-1}.\, i_t^{ST} + b_{t-1}.\, i_t^{LT} + (1-a_{t-1}-b_{t-1}).\, iir_{t-1}^{LT} \qquad (3)'$$

From equation (3)', we can see that the implicit interest rate on government debt at year t is a weighted average of market short-term and long-term interest rates and of the implicit interest rate on outstanding (i.e. non-maturing) long-term debt in year $t-1$. Hence, depending on the weight of outstanding debt in total government debt, an increase of market interest rates will transmit more or less quickly to the implicit interest rate on government debt.

In the projections, the following assumptions are made:

- i_t^{LT} and i_t^{ST} are supposed to converge linearly by T+10 to the short term and 10 year long term forward rates.

- After T+10, i_t^{LT} is supposed to converge linearly to 4% in nominal terms ([126]) (2% in real terms) for all countries by the T+30 horizon;

- i_t^{ST} is supposed to converge linearly to i_t^{LT} time a coefficient corresponding to the historical (pre-crisis) EA yield curve (currently 0.5) for all countries by the T+30 horizon;

- new debt (D_{t-1}^{STN} and D_{t-1}^{LTN}) is assumed to be issued in the projections, as a proportion of the variation of government debt, based on the shares given by Estat (of short-term and long-term government debt), ([127]) whenever government debt is projected to increase; ([128])

- short-term debt issued in year $t-1$ is assumed to entirely mature within the year, and to be rolled-over (D_{t-1}^{STR}) as a proportion of past government debt, based on the share of short-term government debt given by Estat, whenever government debt is projected to increase; ([129])

([126]) For some non-euro countries, the convergence value is higher: PL, RO: 4.5%; HU: 5%, reflecting higher inflation targets by the national central banks.
([127]) More precisely, we use the average shares over the last 3 years available.
([128]) Otherwise, in the cases where government debt is projected to decrease, for instance, in case of a budgetary surplus, no new debt needs to be issued.

- a fraction of long-term debt issued in the past is assumed to mature every year, and to be rolled-over (D_{t-1}^{LTR}), whenever government debt is projected to increase. ([130]) This fraction is estimated based on Estat data on the share of long-term government debt and on ECB data on the share of existing long-term debt maturing within the year. ([131])

Finally, the values of the different variables *over the forecast horizon* (especially i_t^{LT}, i_t^{ST} and iir_{t-1}^{LT}) are set consistently with the available forecast values of the implicit interest rate (iir_t) and information on the maturity structure of debt.

A3.3. TECHNICAL OVERVIEW OF THE T+10 METHODOLOGY

The following model is solved from T+3 up to T+10 (note that as of T+6, for the EU-15 without Germany, the model for the capital and investment module deviates from the general framework below and is governed by the rules described further down in the text):

$$YPOT_{it} = LS_{it}^{\alpha} K_{it}^{(1-\alpha)} TFPS_{it}$$

$$TFP_{it} = \frac{Y_{it}}{H_{it}^{\alpha} K_{it}^{(1-\alpha)}}$$

$$K_{it} = I_{it} + (1-\delta) K_{it-1}$$

$$I_{it} = \frac{I_{it}}{YPOT_{it}} YPOT_{it}$$

$$Y_{it} = YPOT_{it}(1 + YGAP_{it}) * 100$$

1. TFP trend: Kalman-filter extension. T+10 TFP is capped (i.e. a ceiling is imposed) on the basis of US TFP growth.

2. Capital:

a) Investment to potential GDP ratio: ARIMA process to produce extended series (extension to avoid end-point bias for HP filter)

b) Depreciation rate: fixed T+2 rate which is calculated on the basis of the capital law of motion

c) Investment rule: (K_{it} and I_{it} as defined in the equation system above) up to T+5; after T+5: a mix between a capital rule (K_{it} defined as $K_{it-1}\frac{YPOT_{it}}{YPOT_{it-1}}$) and I_{it} defined by capital law of motion) and the investment rule for EU-15 (except DE); investment rule for all other member states. The weight of the capital-rule based investment is gradually decreasing.

([129]) Otherwise, in the cases where government debt is projected to decrease, for instance, in case of a budgetary surplus, only part of this maturing debt needs to be rolled-over (none when government debt is assumed to strongly decrease, for example, when a large budgetary surplus allows repaying past maturing debt).
([130]) See previous footnote.
([131]) More precisely, the starting point (currently 2023) is calculated based on the 2022 ECB data on the share of long-term debt that is maturing within the year. Beyond this year, it is assumed that the share of maturing long-term debt linearly converges from the value taken in the last available year (2023) to the country-specific historical average by the end of the T+10 projection horizon. Additionally, for post-program countries, IE, CY and PT, the redemption profile of official loans has been taken into account for the calculation of the long-term debt maturing within the year.

3. Trend labour: $LS_{it} = \left(POPW_{it}PARTS_{it}(1 - NAWRU_{it})\right)HPERES_{it}$

a) Working age population: use Eurostat projections on population growth ("proj_np")

b) Participation rate: up to T+5: HP-smoothed ARIMA process to produce extended series (extension beyond T+5 to avoid end-point bias for HP filter); for projection up to T+10 we use Ageing Working Group (AWG's) Cohort Simulation Model with a technical transition rule smoothing the break in T+6.

c) Average hours worked: ARIMA process to produce extended series up to T+5 (extension to avoid end-point bias for HP filter) and HP smoothed. From t+6 to t+10 we forecast hours using a stabilisation rule: hours(t) = hours(t-1)*1.5 – hours(t-2)*.5. Results are comparable with those from the AWG.

d) NAWRU (T+2 = last year of the ECFIN forecast):

Between T+2 and T+5:

$$NAWRU_{iT+1} = NAWRU_{iT} + \frac{NAWRU_{iT} - NAWRU_{iT-1}}{2}$$

$$NAWRU_{iT+2} = NAWRU_{iT+1}$$

$$NAWRU_{iT+3} = NAWRU_{iT+2}$$

Between T+6 and T+10: convergence rule and prudent rule

T+10 anchor based on panel regression (union density, tax wedge, almp, unemployment benefits replacement rate, demographics/education and a set of macro control variables i.e. TFP, real interest rate, construction)

4. Output gap: closure of the output gap by T+5; each year as of T+3, YGAP decreases by 1/3 of the T+2 YGAP. The gap closure rule states that if the gaps are not closed before the end of the medium term (T+5), they should be mechanically closed by that time.

A3.4. PROPERTY INCOME

The evolution of property income over time has been taken into account in the assessment of the medium and long-term sustainability of public finances since the 2007/08 round of assessments.

In the context of this report, property income received by Member States is considered to be the sum of returns from three categories of general government financial and non-financial assets: i) interest from debt securities – bonds, ii) dividends from equity securities – shares and iii) rents from tangible non-produced non-financial assets such as land and subsoil assets (i.e. natural resources water, mineral and fossil fuels). ([132])

([132]) This definition is somewhat narrower than the one used in national accounts, where property income (D.4) is as well the income from financial assets and non-produced non-financial assets, but sub-categories considered for these assets are more comprehensive. In national accounts the financial instruments giving rise to interest are, in addition to debt securities, monetary gold / SDRs, deposits, loans and other accounts. The use of produced non-financial assets such as buildings is a fee (P.11 / P.131).

Property income is projected up to 2070, affecting both the medium and long term fiscal sustainability assessment in the form of S1 and S2 indicators. ([133]) Property income projections are separate from and additional to present property income accounted for in the actual balances reported every year by Member States under the SCP scenario, as well as to property income reflected in the two-year forecast horizon.

In calculating the sustainability gaps, property income received by governments is explicitly modelled in a way that is different from government revenues in general. Government revenues in general are a function of the tax bases and the rates chosen by the government. Property income differs from this generalised assumption in that it is determined by market conditions rather than policy settings.

However, since the future stocks of assets and the expected rate of return on these assets that generate income for Member States' governments in the future are not always known, to render projections manageable, a number of simplifying assumptions are made.

In order to model the evolution of property income, the key assumption is that, except in the case of the building-up of pension funds, ([134]) there is no net sale or purchase of assets in the future. As such, projections for the three categories of property income rely on the general assumption that the stock of financial and non-financial assets generating this income remains constant over time ([135]) at the level of latest available data, i.e. at the values posted in T-1. This assumption implies that there is no future sale or redemption of government assets, that when short-term assets (such as bonds) mature, they are implicitly assumed to be replaced with other bonds of the same nominal value, and that property income flows received by a government from the current stock of assets are used to reimburse debt through its contribution to the general government balance, rather than to purchase other assets.

Consequently, future property income is assumed to be generated only from the upcoming returns on the assets stock and property income projections are modelled by just using further assumptions on the future evolution of the rate of return on assets.

In this sense, returns for equity and non-financial assets (rents) are generally considered to occur in line with GDP projections, whereas returns on bonds are underpinned by the additional assumptions described below.

All data for property income projections comes from Eurostat (general government property income subcategories bonds D41, equity D42 and rents D45).

Bond returns projection

These projections are based on an agreement reached in 2009 by the Economic Policy Committee's Working Group on Ageing Populations and Sustainability (AWG) and later supported in the context of the preparation of subsequent Ageing Reports, as well as on some additional assumptions.

Returns on bonds (D.41) have been considered to be as follows:

In the medium run (between T and T+30): country-specific yields on 10y government bonds apply as starting point in present year T to gradually converge to a 4% yield applied in T+30.

([133]) In the calculation of sustainability indicators (S1 and S2), the projected path of property income is conventionally included in the sub-indicator "initial budgetary position" (IBP).
([134]) In Finland and Luxembourg, the public pension system currently registers surpluses, recorded as part of the general government headline balance. These surpluses are then used for the building-up of pension funds – and not to reduce debt -, materialising through the acquisition of financial assets (See Part II.2). In this section, we focus on the projections of government financial assets position, *abstracting from* the change in the pension fund position.
([135]) Exception are natural resources for Denmark and the Netherlands, see below.

In the medium to long run (as of T+30): a constant 4% yield applies; this horizon and value are in line with the horizon used for government debt projections.

Equity returns projection

These projections are based on a method agreed by the AWG since 2007.

Using income from equity - D.42 which reports distributed returns - country-specific shares of paid dividends in GDP are calculated for the last year of available data, T-1; for each country it is considered this share remains constant over the projection horizon, thereby implicitly assuming continuing valuation effects in line with nominal GDP growth.

Rents projection

These projections are based on a method agreed by the AWG since 2007.

The share of rents (D45) to GDP is calculated for the last year of available data for each country, T-1. ([136]) This share is assumed to remain constant over the projection horizon for all countries except Denmark and the Netherlands. For these two countries rich in fossil fuels the stock of subsoil assets is assumed to deplete by 2050, so that the share of rents to GDP in these countries would decline linearly to reach the EU average ([137]) by 2050.

Returns on real estate (rentals on buildings etc.) are not included in property income in the National Accounts since they are produced and often consumed by the general government.

In sum, considering these hypotheses, the projected path of property income ultimately depends on the stock of bonds held at the start of the projection period (the higher the bonds stock, the steeper the decline in property income over time) given that the return on these bonds is assumed to converge to a 4% yield in the medium-long term.

Since both elements can affect property income projections markedly, mitigating assumptions on the starting point and length of bond returns convergence aim to avoid unrealistic boosts to property income projections (and thereby too large of a required SPB adjustment), in particular in countries with significant property income shares.

([136]) This is a simplification. Rents projections should combine the size of reserves, the timing of exploitation and the euro value of the commodity (assumption).
([137]) This average excludes excluding Denmark and the Netherlands.

ANNEX A4
Data and methodology for the stochastic debt projections

Stochastic debt projections capture a wide range of uncertainties surrounding debt dynamics. ([138]) Stochastic projections are essential for understanding the different ways in which shocks to government budgetary positions, economic growth, interest rates and exchange rates can affect debt dynamics. The shocks are calibrated taking into account country-specific conditions, namely the observed volatility in the past and the correlation between the different variables. Thus, unlike deterministic projections, which provide a single outcome based on a given scenario, stochastic projections provide a spectrum of possible debt trajectories.

This annex describes the methodology and data used for the Commission's stochastic debt projections and is divided into five sections. Section A4.1 lists the choice of variables subject to the stochastic shocks. Section A4.2 presents the quarterly data used to generate the stochastic shocks. Section A4.3 describes the methodology to derive the *annual* stochastic shocks. Section A4.4 shows how the shocks are used to obtain the debt dynamics. Finally, Section A4.5 describes the minor technical improvements made compared to the 2022 Debt Sustainability Monitor.

A4.1. CHOICE OF VARIABLES FOR STOCHASTIC SHOCKS

The key drivers of debt dynamics can be captured by the debt accumulation equation:

$$d_t = \alpha^n d_{t-1}\frac{1+i_t}{1+g_t} + \alpha^f d_{t-1}\frac{1+i_t}{1+g_t}\frac{e_t}{e_{t-1}} - b_t + c_t + f_t$$

where the components of the equation are the total government debt-to-GDP ratio in year t (d_t), the share of total debt denominated in national currency (α^n) and foreign currency (α^f), the implicit interest rate (i_t), the nominal GDP growth rate (g_t), the nominal exchange rate expressed in national currency per unit of foreign currency (e_t), the primary balance over GDP (before ageing costs) (b_t), the change in age-related costs over GDP in year t relative to the starting year (c_t) ([139]) and the stock-flow-adjustments (SFA) over GDP (f_t).

Stochastic shocks are simulated around the baseline for five variables of the debt accumulation equation, namely: the primary balance, the nominal short- and long-term interest rates, the nominal GDP growth rate and the exchange rate (for non-EA countries). ([140])

A4.2. QUARTERLY DATA

Quarterly data are used to compute the historical variance-covariance matrix. ([141]) Data for the government primary balance, nominal GDP growth rate and nominal exchange rate are taken from Eurostat (see Table A4.1 for an overview). Data for nominal short-term interest rates come from Eurostat and the Organisation for Economic Co-operation and Development (OECD). Nominal long-term interest rates come from Eurostat and the European Central Bank (ECB).

The definition of the variables generally follows the one used for the Commission's deterministic projections. As a rule, the definition of the variables aligns with the one used for the Commission's annual projections. The (quarterly) primary balance series is calculated as the sum of headline balance

([138]) The approach is based on Berti (2013), Stochastic public debt projections using the historical variance-covariance matrix approach for EU countries, *European Economy. Economic Papers*, No. 480. and on Beynet and Paviot (2012), Assessing the sensitivity of Hungarian debt sustainability to macroeconomic shocks under two fiscal policy reactions, *OECD Economics Department Working Paper*, No. 946.
([139]) The latter are net of taxes on pension. Property income are also included in this variable.
([140]) In the simulations, SFA are not subject to stochastic shocks.
([141]) The use of quarterly data allows to have sufficiently long time-series.

and interest payments and it is seasonally adjusted using the Census X-12-ARIMA approach. Small differences in the definitions of the interest rate series exist for Bulgaria and Estonia due to missing data availability. ([142])

The sample period ranges from Q1 2000 to Q3 2023 for most countries. This period may be shorter depending on data availability. Table A4.1 contains information on missing observations.

Outliers are identified and mitigated using a winsorising approach. For each variable and country within the sample period, the 5th and 95th percentiles are identified (based on the same sample). Observations outside these thresholds are considered outliers and replaced by the closest percentile value.

A4.3. METHODOLOGY FOR STOCHASTIC SHOCKS

The annual stochastic shocks are determined in four steps:

1. ***Transformation of (quarterly) variables into historical shocks****:* Each macroeconomic variable x is transformed into a series of historical quarterly shocks (δ_q^x), defined as the first difference of the quarterly time series of the five macroeconomic variables:

$$\delta_q^x = x_q - x_{q-1}$$

with x equal to pb, i^{ST}, i^{LT}, g and e (for non-EA countries).

2. ***Calculation of the variance-covariance matrix***: The variance-covariance matrix for the historical quarterly shocks of the five variables is calculated. The variance-covariance matrix captures country-specific conditions, namely the observed volatility in the past and the correlation between the different variables, and provides the basis for simulations over the 5-year projection period.

3. ***Run Monte Carlo simulations***: 10 000 random vectors of quarterly shocks are generated over the 5-year projection period, assuming a joint normal distribution with zero mean and variance-covariance matrix identical to that of historical quarterly shock. Although the assumption of a joint normal distribution may not perfectly match the empirical distributions observed in the data, it is strategically chosen for two main reasons. First, it simplifies the computational processes involved in the simulations, making the analysis more manageable. Second, it reduces the likelihood of drawing extreme outliers that could significantly distort the projections and lead to less reliable scenarios.

4. ***Aggregation into annual shocks***: The quarterly shocks (ε_q) are then aggregated into annual shocks of the five variables as described below.

4.1. It is assumed that the shocks to the GDP growth rate, the primary balance, the exchange rate and the short-term interest rate only affect the year *t* in which they occur, but are not persistent. The annual shock to these variables (*z*) in year *t* (ε_t^z) is then determined by the sum of the quarterly shocks, i.e.:

$$\varepsilon_t^z = \sum_{q=1}^{4} \varepsilon_q^z$$

([142]) Nominal short-term interest rates, as measured by the three-month money market rates, are not available in Eurostat for Bulgaria after 1 July 2018 and were replaced by OECD data. Nominal long-term interest rates, as measured by the long-term government interest rates (EMU convergence criterion), are not available for Estonia between January 2000 and May 2020 and were replaced by ECB data on bank interest rates. For more details see Table A.4.1.

4.2. It is assumed that the shock on the long-term interest rate (i^{LT}) is persistent. The reason is that the long-term debt issued/rolled over at the time of the shock remains in the debt stock at the market rate prevailing at the time of issue for all years until maturity. ([143]) A shock to the long-term interest rate in year *t* is therefore carried over to the following projection years in proportion to the share of maturing debt that is progressively rolled over. ([144]) The definition of the annual shock to the long-term interest rate depends on the average weighted maturity of debt. ([145])

4.2.1. For countries where the *average weighted maturity of debt* is equal to or greater than the number of projection years (T = 5 years), the annual shock ($\varepsilon_t^{i^{LT}}$) in the first projection year (t = 1) is calculated by the sum of the quarterly shocks, i.e.:

$$\varepsilon_t^{i^{LT}} = \frac{1}{T}\sum_{q=1}^{4} \varepsilon_q^{i^{LT}}$$

In the following four projection years (t = 2, …, 5), the annual shocks are calculated by averaging the effect of the current year and those of the previous year(s), i.e.:

$$\varepsilon_t^{i^{LT}} = \frac{\mathrm{t}}{T}\sum_{q=-(\mathrm{n}-1)*4}^{4} \varepsilon_q^{i^{LT}}$$

where in each year q = -4, -8, -12, -16 points to the first quarter of the previous one to four years, respectively, indicating that the calculation considers the impact of interest rate changes from those quarters on the current year's long-term interest rate shock.

4.2.2. For countries where the *average weighted maturity of debt is less than the number of projection years (T < 5)*, the above equations are adjusted accordingly to reflect a shorter carry-over of past shocks. For example, for countries with an average weighted maturity of three years (T = 3), the annual shock to the long-term interest rate is defined in the first and second projection year (t = 1) is defined as:

$$\varepsilon_t^{i^{LT}} = \frac{t}{3}\sum_{q=1}^{4} \varepsilon_q^{i^{LT}}$$

In the second projection year (t = 2) the shock is:

$$\varepsilon_t^{i^{LT}} = \frac{t}{3}\sum_{q=-4}^{4} \varepsilon_q^{i^{LT}}$$

In the third, fourth and fifth projection year (t = 3, …, 5), the shock is calculated as follows:

$$\varepsilon_t^{i^{LT}} = \sum_{q=-8}^{4} \varepsilon_q^{i^{LT}}$$

([143]) The implicit assumption made here is that long-term government bonds are issued at fixed interest rates only.
([144]) Country-specific data on the share of short- and long-term debt are provided by Eurostat and are updated each autumn.
([145]) Data for the average weighted maturity of debt by country come from the ECB and national sources if needed and are updated each autumn.

Finally, the shock to the implicit interest rate i $\left(\varepsilon_t^i\right)$ is calculated as the weighted average of the annual shocks to the short- and long-term interest rates, i.e.:

$$\varepsilon_t^i = \alpha^{ST}\varepsilon^{i^{ST}} + \alpha^{LT}\varepsilon^{i^{LT}}$$

where α^{ST} is the share of short-term debt in total government debt and $\alpha^{LT} = (1-\alpha^{ST})$ reflect the share of long-term debt in total government debt. These shares are taken from Eurostat. ([146])

A4.4. DERIVING THE STOCHASTIC DEBT PROJECTIONS

The stochastic debt projections assume that the shocks to the baseline are temporary. The annual shocks are applied to the baseline value of the variables as follows:

$b_t = \bar{b}_t + \varepsilon_t^b$ with $\bar{b}_t$ = baseline (from standard deterministic projections) primary balance at year *t*

$g_t = \bar{g}_t + \varepsilon_t^g$ with $\bar{g}_t$ = baseline (from standard deterministic projections) nominal GDP growth at year *t*

$i_t = \bar{\imath}_t + \varepsilon_t^i$ with $\bar{\imath}_t$ = baseline (from standard deterministic projections) implicit interest rate at year *t*

$e_t = \bar{e}_t + \varepsilon_t^e$ with $\bar{e}_t$ = baseline (from standard deterministic projections) nominal exchange rate at year *t*

In other words, if the shock in year *t* were equal to zero, the value of the variable would be the same as in the standard deterministic baseline projections.

The shocks are then entered into the debt accumulation equation to calculate debt ratios over a five-year horizon. All the steps described in Section A4.4 are repeated 10 000 times. This provides annual distributions of the debt ratio over the 5 projection years, from which we extract the percentiles to construct the fan harts.

A4.5. TECHNICAL IMPROVEMENTS COMPARED WITH THE DEBT SUSTAINABILITY MONITOR 2022

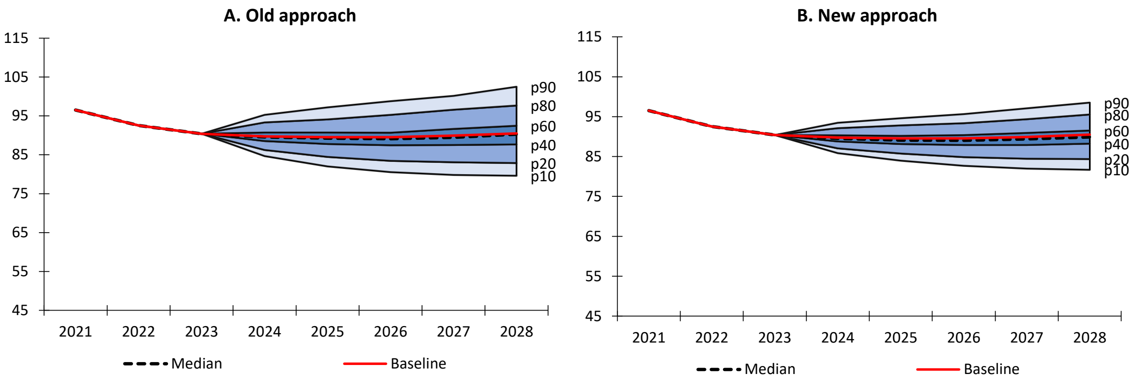
The Commission's methodology for the stochastic debt projections is well-established. The stochastic debt projections methodology was developed in 2013 ([147]) based on a variance-covariance matrix approach and has played an important role in the Commission's DSA since then, in particular in the DSA risk classification approach developed in 2016. ([148])

The following minor technical improvements were made in this current edition of the DSM:

Improvement of the quarterly dataset. The quarterly dataset used to construct the variance-covariance matrix was improved by making Eurostat the dominant source. Missing observations were filled with data from only two additional sources, namely the ECB and the OECD.

([146]) More precisely, we use the average shares over the last 3 years available.
([147]) Berti (2013).
([148]) In the Fiscal Sustainability Report 2015, an innovation was introduced compared with Berti (2013), by applying direct shocks to the primary balance (see European Commission (2016), Fiscal Sustainability Report 2015, *Institutional Paper*, No 18).

Graph A4.1: **Impact of improvements on stochastic debt projections - old vs. new approach (EA aggregate, in % of GDP)**



Source: Commission services.

Harmonisation of the sample length: The sample now starts in Q1 2000 for all countries to ensure consistency across countries, and an almost balanced panel.

Streamlining of detection and correction of outliers: Outliers are now identified and mitigated using the same winsorising approach for all countries. This helps to treat outliers consistently across countries based on a state-of-the-art methodology.

Number of random draws increased. The number of random draws was increased from 2 000 to 10 000 in line with academic standards.

Overall, the small improvements contribute to a simpler and more consistent database with limited impact on the stochastic debt projections (see Graph A4.1). A comparison with the old approach shows that the impact on the stochastic debt projections is small. For the EA aggregate, the cone width slightly decreases.

Table A4.1: **Overview of data sources to compute the historical variance-covariance matrix for the stochastic debt projections**

Variable	Frequency	Definition	Source
Exchange rate	Quarterly	Nominal exchange rate, average in national currency (= national currency for 1 euro). *Note:* Exchange rate shocks are only considered for the following six countries: CZ, DK, HU, PL, RO and SE. Since BG pegged its exchange rate to the Euro in 2005, no exchange rate volatility is expected in the future and hence no exchange rate shock is considered.	Eurostat (AVG-NAC in database ERT-BIL-EUR-Q)
Nominal GDP growth	Quarterly	Gross domestic product at current prices, million units of national currency, percentage change compared to corresponding period of previous year, seasonally and calendar adjusted data *Note:* Missing values for MT (Q1 2000 to Q4 2000)	Eurostat (national account indicator: B1GQ, unit of measure: CP_MNAC, dataset: NAMQ_10_GDP)
Short-term interest rate	Quarterly (derived from monthly averages)	Three-month money market rates, in percent per annum *Note:* • The short-term interest rate for euro area countries is identical and measured by the Euribor. For countries that joined the euro area during the sample period (EE in 2011, LV in 2014, LT in 2015 and HR in 2023), the Euribor is used between 2000 and euro area entry. • BG: Production of SOFIBOR reference rate was discontinued by the national central bank as of 1 July 2018; data filled with OECD data (see source on the right column)	Eurostat (interest rate: IRT_M3, time frequency: M, dataset: IRT_H_MR3_M OECD - Monthly Monetary and Financial Statistics (MEI)
Long-term interest rate	Quarterly (derived from monthly averages)	Government long-term interest rates (EMU convergence criterion), in percent per annum *Note*: Missing values: • CZ (Q1 2000) • CY, HU, LT, LV, MT, PL, SK (Q1 2000 to Q4 2000) • SI (Q1 2000 to Q4 2001) • BG (Q1 2000 to Q4 2002) • RO (Q1 2000 to Q1 2005) • HR (Q1 2000 to Q3 2005) • EE (Jan 2000 - May 2020) missing values are filled with ECB bank interest rate data (see source on the right column)	Eurostat (interest rate: MCBY, time frequency: M, dataset: IRT_LT_MCBY_M) ECB – MIR - MFI Interes Rate Statistics (MIR.M.EE.B.A2C.I.R.A.2250.EUR.N; MIR.M.EE.B.A2L.A.R.A.2230.EEK.N)
Primary balance - Net lending/ borrowing	Quarterly	Net lending/borrowing as percentage of GDP	Eurostat (national account indicator: B9, sector: S13, seasonal adjustment: NSA, unit c

(Continued on the next page)

Table (continued)

			measure: PC_GDP, time frequency: Q, dataset: GOV_10Q_GGNFA)
- Interest payable	Quarterly	Interest expenditure as a percentage of GDP, unadjusted data *Note:* Missing values: • AT (Q1 2000 to Q4 2000) • DE, EE, IE, LU (Q1 2000 to Q4 2001) • All countries (Q3 2023)	Eurostat (national account indicator: D41PAY, sector: S13, seasonal adjustment: NSA, unit of measure: PC_GDP, time frequency: Q, dataset: GOV_10Q_GGNFA)

Source: Commission services.

ANNEX A5
The long-term fiscal sustainability indicators (S1, S2)

A5.1. NOTATION

t: time index. Each period is one year

t_0: last year before the fiscal adjustment (2024 in this report).

$t_0 + 1$: first year of the long-term projection period (i.e. year of the fiscal adjustment).

t_1: final year of the long-term projection period (2070), which also corresponds to the target year for the debt ratio (relevant for S1).

Notice that $t_0 < t_1$.

D_t: debt-to-GDP ratio (at the end of year t).

PB_t: ratio of primary balance to GDP

$\Delta PB_t \equiv PB_t - PB_{t_0}$: change in the primary balance relative to the base year t_0. In the absence of fiscal adjustment, it equals the change in age related expenditure

$\Delta A_t \equiv A_t - A_{t_0}$: change in age-related costs relative to the base year t_0.

ΔPI_t: change in property income relative to the base year t_0.

CC_t: cyclical component of the general government balance (only relevant in the first years, by definition, it is zero over the long term as it vanishes with the closure of the output gap)

$SPB_t = PB_t - CC_t + one\text{-}offs_t$: ratio of structural primary balance to GDP, i.e. cyclically adjusted primary balance net of one-off and other temporary measures

r: differential between the nominal interest rate and the nominal GDP growth rate i.e.

$1 + r \equiv \frac{1+R}{1+G}$: where R and G are, respectively, the nominal interest rate and the nominal growth rate.

If the interest-growth rate differential is time-varying, we define:

$$\alpha_{s;v} \equiv (1 + r_{s+1})(1 + r_{s+2}) \dots (1 + r_v)$$

$$\alpha_{v;v} \equiv 1$$

as the accumulation factor that transforms 1 nominal unit in period s to its period v value.

A5.2. DEBT DYNAMICS

By definition, the debt-to-GDP ratio evolves according to:

$$D_t = (1 + r_t)D_{t-1} - \text{PB}_t. \qquad (1)$$

That is, the debt ratio at the end of year t, D_t, is a sum of three components: the debt ratio at the end of the previous year (D_{t-1}), interest accrued on existing debt during year t (rD_{t-1}), and the negative of the primary balance $(-\mathrm{PB}_t)$.

Repeatedly substituting for D_t, the debt ratio at the end of some future year $T > t$ can be expressed similarly, as:

$$D_T = D_{t-1}\alpha_{t-1;T} - \sum_{i=t}^{T}(\mathrm{PB}_i\alpha_{i;T}). \tag{2}$$

The path of the debt ratio is thus determined by the initial debt ratio, accrued interest (net of growth), and the path of primary balances from t through T.

A5.3. DERIVATION OF THE S1 INDICATOR

The S1 indicator is defined as the immediate and permanent one-off improvement in the ratio of structural primary balance to GDP that is required to bring the debt ratio to 60% of GDP by year t_1 (2070).

In addition to accounting for the need to adjust the initial intertemporal budgetary position and the debt level, it incorporates financing for any additional expenditure until the target date arising from an ageing population.

Under the assumed immediate and permanent one-off consolidation, the change in the primary balance is thus given by

$$\mathrm{PB}_i = \mathrm{SPB}_{t_0} + S_1 - \Delta A_i + \Delta PI_i + CC_i \tag{3}$$

$$\text{for } i > t_0$$

Using (2), the debt ratio target D_{t_1} can then be written as:

$$D_{t_1} = D_{t_0}\alpha_{t_0;t_1} - \sum_{i=t_0+1}^{t_1}(\mathrm{PB}_i\alpha_{i;t_1}) \tag{4}$$

Replacing (3) into (4) yields:

$$D_{t_1} = D_{t_0}\alpha_{t_0;t_1} - \sum_{i=t_0+1}^{t_1}(\mathrm{SPB}_{t_0} + S_1)\,\alpha_{i;t_2} + \sum_{i=t_0+1}^{t_1}((\Delta A_i - \Delta PI_i - CC_i)\,\alpha_{i;t_1}) \tag{5}$$

After some straightforward manipulations, ([149]) we can decompose the S1 into the following main components:

$$S_1 = \underbrace{\frac{D_{t_0}(\alpha_{t_0;t_1} - 1)}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})} - \mathrm{SPB}_{t_0} - \frac{\sum_{i=t_0+1}^{t_1}(\Delta PI_i\alpha_{i;t_1})}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})} - \frac{\sum_{i=t_0+1}^{t_1}(CC_i\alpha_{i;t_1})}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})}}_{A} + \underbrace{\frac{D_{t_0} - D_{t_1}}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})}}_{B} + \underbrace{\frac{\sum_{i=t_0+1}^{t_1}(\Delta A_i\alpha_{i;t_1})}{\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})}}_{C} \tag{6}$$

where (A) is the initial budgetary position (IBP) (i.e. the gap to the debt-stabilising primary balance); (B) the required additional adjustment due to the debt target; and (C) the additional required adjustment due to the costs of ageing.

([149]) Add and subtract D_{t_0} on the LHS of (5), divide on both sides by $\sum_{i=t_0+1}^{t_1}(\alpha_{i;t_1})$ and group the terms as in (6).

A5.4. DERIVATION OF THE S2 INDICATOR

The intertemporal budget constraint and the S2 indicator

According to a generally invoked definition, fiscal policy is sustainable in the long term if the present value of future primary balances is equal to the current level of debt, that is, if the intertemporal government budget constraint (IBC) is met. Let us define the S2 as the immediate and permanent one-off fiscal adjustment that would ensure that the IBC is met. This indicator is appropriate for assessing long-term fiscal sustainability in the face of ageing costs. ([150])

Since the S2 indicator is defined with reference to the intertemporal government budget constraint (IBC), we first discuss which conditions are required for the IBC to hold in a standard model of debt dynamics. From (2), the debt to GDP ratio at the end of any year $t > t_0$ is given by:

$$D_t = D_{t_0}\alpha_{t_0;t} - \sum_{i=t_0+1}^{t}(PB_i\alpha_{i;t}). \qquad (7)$$

Rearranging the above and discounting both sides to their time t_0 values, we obtain the debt ratio on the initial period:

$$D_{t_0} = \left(\frac{D_t}{\alpha_{t_0;t}}\right) + \sum_{i=t_0+1}^{t}\left(\frac{PB_i}{\alpha_{t_0;i}}\right). \qquad (8i)$$

Assuming an infinite time horizon ($t \to \infty$) we get:

$$D_{t_0} = \lim_{t\to\infty}\left(\frac{D_t}{\alpha_{t_0;t}}\right) + \lim_{t\to\infty}\sum_{i=t_0+1}^{t}\left(\frac{PB_i}{\alpha_{t_0;i}}\right) = \lim_{t\to\infty}\left(\frac{D_t}{\alpha_{t_0;t}}\right) + \sum_{i=t_0+1}^{\infty}\left(\frac{PB_i}{\alpha_{t_0;i}}\right) \qquad (8ii)$$

Either both of the limits on right-hand side of equation (8ii) fail to exist, or if one of them exists, so does the other.

Let us define the *no-Ponzi game condition* (also called the *transversality condition*) for debt sustainability, namely that the discounted present value of debt (in the very long term or in the infinite horizon) will tend to zero:

$$\lim_{t\to\infty}\left(\frac{D_t}{\alpha_{t_0;t}}\right) = 0 \qquad (9i)$$

Condition (9i) means that asymptotically, the debt ratio cannot grow at a rate equal or higher than the (growth-adjusted) interest rate, which is what would happen if debt and interest were systematically paid by issuing new debt (i.e. a Ponzi game).

Combining the no-Ponzi game condition (9i) with (8ii), one obtains the intertemporal budget constraint, stating that a fiscal policy is sustainable if the present discounted value of future primary balances is equal to the initial value of the debt ratio.

([150]) Note that the derivation of S2 does not assume that either the initial sequence of primary balances or the fixed annual increase (S2) are optimal according to some criterion. S2 should be considered as a benchmark and not as a policy recommendation or as a measure of the actual adjustment needed in any particular year.

$$D_{t_0} = \sum_{i=t_0+1}^{\infty} \left(\frac{PB_i}{\alpha_{t_0;i}}\right) \tag{9ii}$$

On the other hand, substituting the intertemporal budget constraint (9ii) into (8ii) implies the no-Ponzi game condition. This shows that the no-Ponzi game condition (9i) and the IBC (9ii) are, in fact, equivalent.

Assuming that the intertemporal budget constraint is satisfied through a permanent, one-off fiscal adjustment whose size is given by the S2, from $t_0 + 1$ onwards we can write:

$$PB_i = SPB_{t_0} + S_2 - \Delta A_i + \Delta PI_i + CC_i \quad \text{for} \quad i > t_0. \tag{10}$$

Then the intertemporal budget constraint (9ii) becomes

$$D_{t_0} = \sum_{i=t_0+1}^{\infty} \left(\frac{SPB_{t_0} + S_2 - \Delta A_i + \Delta PI_i + CC_i}{\alpha_{t_0;i}}\right). \tag{9iii}$$

Here the ratio of primary balance to GDP, PB_t is re-expressed in terms of the required annual additional effort, S2, and the change in age-related costs relative to the base year t_0 (as well as the change in property income and the cyclical component), combining the equation (10) with equation (9ii).

According to the theory on the convergence of series, necessary conditions for the series in equation (9ii)-(9iii) to converge are for the initial path of primary balances to be bounded and the interest rate differential in the infinite horizon to be positive ([151]). The latter is equivalent to the modified golden rule, stating that the nominal interest rate exceeds the real growth rate (i.e. $\lim_{t\to\infty} r_t > 0$). ([152])

After some rearranging, ([153]) we can decompose the S2 into the following two components:

$$S_2 = \underbrace{\frac{D_{t_0}}{\sum_{i=t_0+1}^{\infty}\left(\frac{1}{\alpha_{t_0;i}}\right)} - \mathrm{SPB}_{t_0} - \frac{\sum_{i=t_0+1}^{\infty}\left(\frac{\Delta PI_i + CC_i}{\alpha_{t_0;i}}\right)}{\sum_{i=t_0+1}^{\infty}\left(\frac{1}{\alpha_{t_0;i}}\right)}}_{A} + \underbrace{\frac{\sum_{i=t_0+1}^{\infty}\left(\frac{\Delta A_i}{\alpha_{t_0;i}}\right)}{\sum_{i=t_0+1}^{\infty}\left(\frac{1}{\alpha_{t_0;i}}\right)}}_{B} \tag{11}$$

where (A) is the initial budgetary position i.e. the gap to the debt stabilising primary balance ([154]); and (B) the additional required adjustment due to the costs of ageing.

If the interest-growth rate differential r is constant, the accumulation factor simplifies to $\alpha_{s;v} = (1 + r_{s+1})(1 + r_{s+2}) \ldots (1 + r_v) = (1 + r)^{v-s}$. Then equation (10) can be simplified further by noting that:

$$\sum_{i=t_0+1}^{\infty} \left(\frac{1}{\alpha_{t_0;i}}\right) = \sum_{i=t_0+1}^{\infty} \left(\frac{1}{(1+r)^{i-t_0}}\right) = \frac{1}{r} \tag{12}$$

Thus, for a constant discounting factor, (11) can be rewritten as:

([151]) The latter is an application of the ratio test for convergence.
([152]) See Escolano (2010) for further details on the relationships among the stability of the debt ratio, the IBC and the no-Ponzi game condition.
([153]) In addition, constant multiplicative terms are systematically taken out of summation signs.
([154]) In practical calculations, the present value of property income is also accounted for in the initial budgetary position. Property income enters the equation in an identical manner as age-related costs ΔA_t (i.e. term (B)), but with an opposite sign.

$$S_2 = \underbrace{rD_{t_0} - \text{SPB}_{t_0} - r\sum_{i=t_0+1}^{\infty}\left(\frac{\Delta PI_i + CC_i}{\alpha_{t_0;i}}\right)}_{A} + \underbrace{r\sum_{i=t_0+1}^{\infty}\left(\frac{\Delta A_i}{\alpha_{t_0;i}}\right)}_{B} \quad (13i)$$

If the interest-growth rate differential and the structural primary balance are constant after a certain date (here $t_1 = 2070$), equation (11) can be rewritten as:

$$S_2 = \frac{D_{t_0}}{\sum_{i=t_0+1}^{2069}\left(\frac{1}{\alpha_{t_0;i}}\right) + \frac{1}{r\alpha_{t_0;2069}}} - \text{SPB}_{t_0} - \frac{\sum_{i=t_0+1}^{2069}\left(\frac{\Delta PI_i + CC_i}{\alpha_{t_0;i}}\right) + \frac{\Delta PI_{2070} + CC_{2070}}{r\,\alpha_{t_0;2069}}}{\sum_{i=t_0+1}^{2069}\left(\frac{1}{\alpha_{t_0;i}}\right) + \frac{1}{r\,\alpha_{t_0;2069}}} + \frac{\sum_{i=t_0+1}^{2069}\left(\frac{\Delta A_i}{\alpha_{t_0;i}}\right) + \frac{\Delta A_{2070}}{r\,\alpha_{t_0;2069}}}{\sum_{i=t_0+1}^{2069}\left(\frac{1}{\alpha_{t_0;i}}\right) + \frac{1}{r\,\alpha_{t_0;2069}}} \quad (13ii)$$

where $r_t = r$ and $\Delta A_t = \Delta A_{2070}$ for $t \geq t_1 = 2070$.

Derivation of the steady state debt level (at the end of the projection period) corresponding to the S2

Assuming that the intertemporal budget constraint is satisfied and that the primary balance and the interest-growth rate differential are constant at their long-run levels after the end of the projection period, then the debt ratio remains constant at the value attained at the end point of the projection period (i.e. at $t_1 = 2070$).

To see this, rewrite (9ii) as:

$$D_{t_0} = \sum_{i=t_0+1}^{\infty}\left(\frac{PB_i}{\alpha_{t_0;i}}\right) = \sum_{i=t_0+1}^{t_1}\left(\frac{PB_i}{\alpha_{t_0;i}}\right) + \sum_{i=t_1+1}^{\infty}\left(\frac{PB_i}{\alpha_{t_0;i}}\right) \quad (14i)$$

Using (7) and the fact that for $t \geq t_1$ the primary balance and interest-growth rate differential stay constant at $\text{PB}_t = \text{PB}_{t_1}$ (14i) can be rearranged to obtain the debt ratio at t_1:

$$D_{t_1} = D_{t_0}\alpha_{t_0;t_1} - \sum_{i=t_0+1}^{t_1}(PB_i\alpha_{i;t_1}) = \sum_{i=t_1+1}^{\infty}\left(\frac{PB_i}{\alpha_{t_1;i}}\right) = \sum_{i=1}^{\infty}\left(\frac{PB_{t_1}}{(1+r_{t_1})^i}\right) = \frac{PB_{t_1}}{r_{t_1}} \quad (14ii)$$

Generalising the above to each $t \geq t_1$ by using (7) with the initial year changed to t_1 instead of t_0, (15) shows that for each year after t_1, the debt ratio remains unchanged at this value:

$$\begin{aligned} D_t &= D_{t_1}\alpha_{t_1;t} - \sum_{i=t_1+1}^{t}(PB_i\alpha_{i;t}) = \frac{PB_{t_1}}{r_{t_1}}(1+r_{t_1})^{t-t_1} - PB_{t_1}\sum_{i=t_1+1}^{t}(1+r_{t_1})^{t-i} = \\ &= \underbrace{\left[(1+r_{t_1})^{t-t_1} - r_{t_1}\left(\frac{1-(1+r_{t_1})^{t-t_1}}{1-(1+r_{t_1})}\right)\right]}_{=1}\frac{PB_{t_1}}{r_{t_1}} = \frac{PB_{t_1}}{r_{t_1}} \equiv \overline{\overline{D}} \quad \text{for } t \geq t_1 \end{aligned} \quad (15)$$

where $\overline{\overline{D}}$ is the constant debt ratio reached after the end of the projection period.

Using (4), the primary balance at the end of the projection period can be calculated as:

$$PB_{t_1} = SPB_{t_0} + \Delta PI_{t_1} + CC_{t_1} + S_2 - \Delta A_{t_1} \quad (16)$$

Replacing (16) into (15), the constant (steady-state) debt ratio ($\overline{\overline{D}}$) is given by:

$$\bar{\bar{D}} = \frac{PB_{t_1}}{r_{t_1}} = \frac{SPB_{t_0} + \Delta PI_{t_1} + CC_{t_1} + S_2 - \Delta A_{t_1}}{r_{t_1}} \quad \text{for} \quad t \geq t_1 \tag{17}$$

The S2 adjustment implies that the sum of debt and the discounted present value of future changes in aged-related expenditure is (approximately) constant over time.

Replacing equations (16) and (13i) into (15), and assuming a constant interest rate differential, the following equation is obtained:

$$D_t + \sum_{i=t+1}^{\infty}\left(\frac{\Delta A_i}{(1+r)^{i-t}}\right) - \sum_{i=t+1}^{\infty}\left(\frac{\Delta PI_i + CC_i}{(1+r)^{i-t}}\right) = D_{t_0} + \sum_{i=t_0+1}^{\infty}\left(\frac{\Delta A_i}{(1+r)^{i-t_0}}\right) - \sum_{i=t_0+1}^{\infty}\left(\frac{\Delta PI_i + CC_i}{(1+r)^{i-t_0}}\right) \tag{18}$$

Equation (18) can be interpreted as follows. Implementing a permanent annual improvement in the structural primary balance amounting to S2, which is both necessary and sufficient to secure intertemporal solvency, implies that the sum of explicit debt (the first term in both sides) and the variation in age-related expenditure or implicit debt (the second terms in both sides) is (approximately) constant over time. Equation (17) is exact in the steady state (e.g. after 2070), holding only as an approximation during transitory phases (i.e. for time-varying interest rate differentials). ([155])

([155]) Moreover, equations (17) and (18) imply that both the debt and the variation in age-related expenditure are constant over time in the steady state.

ANNEX A6

Estimating the potential impact of simulated bank losses on public finances based on the SYMBOL model

SYMBOL approximates the probability distributions of individual bank's losses using publicly available information from banks' financial statements. In particular, the model estimates an average implied default probability of the individual banks' asset/loan portfolios by inverting the Basel FIRB formula for capital requirements ([156]).

The main data source on banks' financial statements is Orbis Bank Focus, a commercial database of the private company Bureau van Dijk (part of Moody's analytics). For the reference year 2022, unconsolidated data for commercial, saving and cooperatives banks are included. The data as provided by Orbis Bank Focus occasionally lacks information on specific variables for some banks in the sample (e.g., capital, risk weighted assets, provisions, gross non-performing loans). In those cases, capital is imputed via a robust regression by using common equity, while risk weighted assets are approximated using the total regulatory capital ratio (at bank or country level). While gross loans are available for all banks, values for provisions and non-performing loans are available only for two thirds of the sample. Missing values for provisions have thus been estimated by country aggregates coming from the EBA dashboard ([157]), while missing values for non-performing loans have been imputed by applying a robust regression using provisions as explanatory variable. Information on the sample is presented in Table A6.1, and Table A6.2 report statistics at aggregated Member State level for non-performing loans (NPLs) and loans provisions, taken from the EBA dashboard, while recovery rates (country aggregates) are taken from the World Bank (2020). ([158]) The sample covers approximately 75% of all EU banking assets. When the sample, as illustrated in Table A6.1, either includes a small number of banks or covers a low share of total assets, results should be interpreted with caution, since a minor change to any bank's data or the addition of a new bank could have significant effects on results.

([156]) European Commission (2016) Section 5.2.2 and Annex A7 for more detail on the SYMBOL model.
([157]) EBA Risk Dashboard - data as of Q4 2022.
([158]) Due to issues in the data, the World Bank paused the 2021 Doing Business report to start a series of audits in the methodology. Thus, we use the recovery rates as of end 2020.

Table A6.1: **Descriptive statistics of samples used for SYMBOL simulations**

	Sample ratio *(Sample TA/ Population TA)*	Nbr.of banks	Total assets (TA)	Own funds (Total capital)	Risk weighted assets (RWA)	RWA/TA	Own funds (Total capital)/RWA
	%		EUR bn	EUR bn	EUR bn	%	%
BE	98.8%	24	1004.1	72.7	379.7	37.8%	19.1%
BG	92.2%	15	65.1	6.5	31.5	48.4%	20.7%
CZ	73.9%	17	253.4	21.3	96.4	38.1%	22.1%
DK	46.4%	48	530.5	49.3	201.8	38.0%	24.4%
DE	76.1%	1105	6645.8	497.6	2848.7	42.9%	17.5%
EE	101.2%	3	37.6	4.0	19.0	50.7%	21.2%
IE	31.0%	22	410.1	42.1	194.1	47.3%	21.7%
EL	96.8%	7	313.7	26.3	147.3	46.9%	17.8%
ES	88.7%	84	2535.5	208.5	1258.9	49.7%	16.6%
FR	78.2%	139	8468.4	471.1	2510.0	29.6%	18.8%
HR	106.0%	19	73.5	7.8	32.0	43.5%	24.5%
IT	73.5%	293	2752.0	228.3	1050.3	38.2%	21.7%
CY	83.9%	21	58.6	4.2	20.6	35.1%	20.7%
LV	110.4%	9	22.4	2.4	10.1	45.2%	23.8%
LT	80.4%	4	35.7	2.2	12.1	34.1%	18.4%
LU	36.4%	35	380.6	34.6	180.5	47.4%	19.2%
HU	63.3%	8	103.5	9.9	49.0	47.4%	20.2%
MT	67.5%	9	28.7	2.4	10.8	37.7%	22.3%
NL	72.9%	16	1842.4	139.4	660.3	35.8%	21.1%
AT	89.8%	405	888.9	85.2	398.7	44.9%	21.4%
PL	72.7%	122	408.2	36.0	192.1	47.1%	18.8%
PT	84.2%	90	345.4	28.9	149.3	43.2%	19.4%
RO	91.4%	14	113.8	11.5	50.8	44.7%	22.7%
SI	87.4%	9	43.2	4.7	23.5	54.5%	20.0%
SK	101.1%	8	94.2	7.8	49.4	52.5%	15.7%
FI	104.7%	114	665.6	44.1	209.2	31.4%	21.1%
SE	74.8%	79	989.9	95.9	338.6	34.2%	28.3%

(1) 2022 unconsolidated data.
Source: Commission services.

Table A6.2: **Descriptive statistics for Non-Performing Loans (NPL)**

	Gross loans	NPL Ratio Gross NPL/Gross loans	NPL/TA Gross NPL/TA	NPL/ Capital Gross NPL/Capital	Provisions	Recovery rate Baseline Scenario	NPL losses Baseline Scenario
	EUR bn	%	%	%	EUR bn	%	EUR bn
BE	560.3	0.9%	0.5%	6.6%	3.7	89.4%	0.46
BG	35.4	5.3%	2.9%	28.6%	1.4	37.7%	0.44
CZ	154.0	1.8%	1.1%	13.2%	2.5	67.5%	0.20
DK	209.5	2.3%	0.9%	9.9%	4.3	88.5%	0.04
DE	3599.8	2.2%	1.2%	15.7%	15.4	79.8%	36.04
EE	27.0	1.0%	0.8%	7.0%	0.2	36.1%	0.05
IE	122.3	3.8%	1.1%	11.0%	3.0	86.1%	0.27
EL	160.1	7.6%	3.9%	46.6%	6.1	32.0%	4.46
ES	1356.5	2.9%	1.5%	18.7%	24.9	77.5%	2.66
FR	2786.7	2.0%	0.7%	12.1%	27.5	74.8%	16.78
HR	41.0	4.5%	2.5%	23.7%	1.9	35.2%	0.09
IT	1743.7	2.9%	1.8%	22.0%	46.5	65.6%	5.60
CY	21.7	6.2%	2.3%	31.7%	0.5	73.8%	0.45
LV	12.2	3.0%	1.6%	15.1%	0.2	41.4%	0.15
LT	17.2	1.0%	0.5%	7.4%	0.1	41.4%	0.04
LU	154.5	1.8%	0.7%	7.8%	1.3	43.9%	1.00
HU	41.1	3.1%	1.2%	13.0%	1.2	44.2%	0.11
MT	13.7	4.0%	1.9%	22.7%	0.3	39.2%	0.15
NL	916.0	0.4%	0.2%	2.7%	3.5	90.1%	0.17
AT	497.6	4.7%	2.6%	27.3%	6.6	79.9%	9.27
PL	229.2	4.7%	2.6%	30.0%	9.3	60.9%	0.19
PT	192.3	2.8%	1.6%	18.7%	6.1	64.8%	0.01
RO	63.9	3.5%	2.0%	19.3%	3.0	34.4%	0.01
SI	23.6	1.7%	0.9%	8.4%	0.4	90.0%	0.00
SK	69.0	1.8%	1.4%	16.4%	1.4	46.1%	0.01
FI	267.7	1.6%	0.6%	9.7%	1.9	88.0%	0.66
SE	494.7	1.1%	0.5%	5.5%	4.9	78.1%	0.07

(1) 2022 unconsolidated data.
Source: Commission services.

1. The SYMBOL model at glance: A modelling framework for assessing public finances risks

The systemic model of banking-originated losses (SYMBOL) is a micro simulation model developed jointly by the European Commission's JRC and DG FISMA to simulate banking crises and estimate the distribution of banking sector losses at country level, accounting for all the cushioning layers of the legal safety net available to absorb shocks (capital, bail-in, resolution funds). SYMBOL can be used to assess how losses originating in banks' balance sheets potentially affect public finances due to government interventions to recapitalise banks. As input, it considers a rich dataset covering unconsolidated balance sheet data of banks in EU Member states. See for more information, the European Commission 2022 Debt Sustainability Monitor. Assessing risks for public finances with SYMBOL involves the following steps:

Overall, the SYMBOL results are estimated by calculating the Expected Shortfall of the more extreme realisations of the common factor, which might be considered as the general economic cycle. In practice, we select the simulations where the factor is above a threshold (three standard deviations) to compute the Expected Shortfall of the portfolio, namely the average value in the tail of the distribution, which represents the expected value of the portfolio losses in a crisis event. This calibration of the Expected Shortfall computation is in line with the crisis event defined in previous reports using the SYMBOL model.

1.1. Simulating banks' losses

Starting from the estimated average probability of default of the asset portfolio of each bank, SYMBOL generates realisations for each individual bank's credit losses using the Basel Foundation Internal Rating Based (FIRB) loss distribution function and assuming a correlation between simulated shocks hitting different banks in the system. ([159]) More formally, the output of the model is a matrix of losses, $L_{n,i}$:

$$L_{n,i} = LGD \cdot N\left[\sqrt{\frac{1}{1-R}}N^{-1}(IOPD_i) + \sqrt{\frac{R}{1-R}}N^{-1}(\alpha_{n,i})\right]$$

where n denotes a simulation run, i indicates the bank, LGD is the Loss Given Default, $IOPD_i$ is the average implied obligors' probability of default, R_i is the coefficient of correlation among different obligators of $Bank_i$, and N is the normal distribution function, $N^{-1}(\alpha_{n,i})$ are correlated normal random shocks with correlation ρ.

The correlation structure among the simulated shocks across different financial institutions assumes that the different banks are hit in the national system, due to their common exposure to a common factor, i.e., the business cycle. That correlation is reinforced by including a '*fire sales* mechanism', which intensity is linked to size of the common shock underpinning the degree of asset correlation and eventually the asset value. This reflects that during a major crisis, many banks will be jointly engaged in asset selling activity to keep their liquidity positions, resulting in an overall deterioration of the asset values in all banks, that in turn would generate further losses and liquidity needs. Specifically, the correlated normal random shocks $\alpha_{n,i}$ includes a bank-specific element and a common factor across financial institutions, as follows:

$$N^{-1}(\alpha_{n,i}) = l \times Z_n + \sqrt{1-l^2} \times W_{n,i}$$

1.2. Determining banks' insolvency event and obtaining the aggregated distribution of losses

Based on the matrix of correlated losses, the failure of a bank is determined by comparing the size of simulated losses L_i and the regulatory capital available to absorb the shocks. A $bank_i$ is assumed insolvent and has excess losses $ExL_{n,i}$, when simulated losses ($L_{n,i}$) exhaust the sum of expected losses (EL_i) and total actual capital K_i, as follows:

$$\text{Failure} \equiv L_{n,i} - EL_{n,i} - K_i > 0$$

$$ExL_{n,i} = max(L_{n,i} - EL_{n,i} - K_i, 0)$$

In line with the Basel rules, recapitalisation needs (i.e. funds necessary to restore the bank's minimum level of capitalisation) up to 10.5% of risk weighted assets (RWA) are also factored in the losses in excess of capital. EU27 aggregate losses and recapitalisation needs are obtained by summing the individual losses in excess of capital plus recapitalisation needs of all distressed banks at country level (both failed and undercapitalised banks) in each simulation j:

$$ExLR_{n,i} = max(L_{n,i} - EL_{n,i} - K_i + 10.5\%\ RWA_i, 0)$$

([159]) The correlation is assumed to be 0.5 for all banks in the current simulation. All EU banks are simulated together.

1.3. Accounting for asset quality and non-performing loans

The SYMBOL model reflects risks that banks face in relation to asset quality in case of a banking crisis, taking into account how current stocks of non-performing loans (NPLs) would contribute to losses in national banking systems in each country. Namely, it assumed that non-collateralised NPLs would turn into loan losses for a Member States in case of systemic banking event, while the collateralised NPL are redeemable subject to a recovery rate. This mechanism generates extra losses, which might materialise even for banks not yet failed, and are added to those coming from the SYMBOL simulations before the intervention of any safety net tools. For the simulations based on the current situation, i.e., with impacts within a year time, extra loan losses from NPLs ($NPLLosses_i$) are added to those obtained from the SYMBOL simulation before the intervention of any safety net tools. Specifically, for each bank$_i$ and each country$_j$ potential loans losses from NPLs are as follows:

$$NPLLosses_i = (1 - CollShares_i) \times NPL_i + Collshares_i \times NPL_i \times (1 - RR_i) - Provisions_i$$

where RR_j is the recovery rate, *Collshares* ([160]) represents the proportion of total loans covered by collateral in country *j*. *Provisions* and *NPL* are respectively, the amount of provisions and gross non-performing loans declared by bank *i* in its balance sheet. We consider two different modelling assumption for the recovery rates. The first method uses a constant recovery rate per Member States calibrated on data provided by the World Bank. ([161])A second more sophisticated attempt builds on a result by Jarrow and Turnbull (2000), showing that the recovery rate is related to the state of economy. As many other authors (see Schlafer and Uhrig-Homburg 2014, Madan and Unal 1998, Gaspar and Slinko 2008), we therefore assume that the recovery rate distribution follows a beta distribution with two parameters. We calibrate the parameters so that the mean of the distribution equals the country recovery rate reported by the Word Bank and the standard deviation is equal to 10%. This second approach is used as a stress testing device, in conjunction with the introduction of a common factor to mimic a fire sale mechanism. ([162])

1.4. A “fire sale” mechanism for the severe stress scenario

During a crisis, banks will sell assets to keep their liquidity positions. In this case, as many banks jointly engage in such selling activity, asset value in all banks tends to deteriorate, generating further losses and liquidity needs. This has the effect of increasing the correlation among realised losses across different financial institutions in the presence of a bigger downturn. A new improvement of the model is the attempt to reflect this mechanism by reducing the value of assets in proportion to how strong is the common shock affecting all banks in the simulation. In other words, a larger common shock implies a more intense severe fire sales’ mechanism and a larger correlation between realised gross losses.

To mimic such mechanism, we make specific assumptions on the correlation of our normal random shocks. These shocks $\alpha_{n,i}$ can be decomposed into a bank-specific element and a factor that is common across institutions and represents the status of the economy (this can be seen as the first principal component of the economic cycle). ([163]) Formally, these shocks are defined as follows:

$$N^{-1}(\alpha_{n,i}) = l \times Z_n + \sqrt{1 - l^2} \times W_{n,i}$$

([160]) Based on ECB available here: www.sdw.ecb.europa.eu/browse.do?node=9689685.
([161]) Based on country data provided by the World Banks in its Flagship Report “Doing Business 2020” available here: https://www.doingbusiness.org/en/doingbusiness
([162]) From the beta distribution, we generate the recovery rate corresponding to the common factor, after mapping the values for the common factor from a normal into those of a uniform distribution.
([163]) The methodology is based on Andersen (2003).

where $W_{n,i}$ are the idiosyncratic (bank-specific) shocks, Z_n is the common factor and l refers to the correlation with the common factor (factor loadings). Depending on the intensity of Z_n, which represents the size of the economic crisis, we set the factor loading l, which is in turn equal to the correlation ρ in the original model, between 0.5 and 0.9: the worst the status of economy, the higher the correlation between assets in bank's portfolios is. Notably, we set the following:

$$Z_n \begin{cases} \leq 1.00 \rightarrow \rho = 0.50 \\ \leq 1.25 \rightarrow \rho = 0.58 \\ \leq 1.50 \rightarrow \rho = 0.62 \\ \leq 1.75 \rightarrow \rho = 0.66 \\ \leq 2.00 \rightarrow \rho = 0.70 \\ \leq 2.25 \rightarrow \rho = 0.74 \\ \leq 2.50 \rightarrow \rho = 0.78 \\ \leq 2.75 \rightarrow \rho = 0.82 \\ \leq 3.00 \rightarrow \rho = 0.86 \\ > 3.00 \rightarrow \rho = 0.90 \end{cases}$$

Despite the fact that the standard version of the model has been using a fixed value for the correlation (namely equal to 0.5), there have been other analyses where the (fixed) correlation value has been allowed to vary and its impact on resulting losses has been assessed. For example, Benczur et al. (2017) allows for different degrees of commonality by different shock correlation structure and Di Girolamo et al. (2017) describe an attempt to capture the correlation structure existing across banks using balance sheet data.

2. Regulatory framework and scenarios assumptions

Three pieces of legislation are considered: the Capital Requirement Regulation and Directive IV (CRR, CRDIV) ([164]), which improved the definitions of regulatory capital and risk-weighted assets, increased the level of regulatory capital by introducing the capital buffers, including extra capital buffers for European Global Systematically Important Institutions (G-SIIs) and Other Systemically Important Institutions (O-SII) ([165]); the Bank Recovery and Resolution Directive (BRRD) ([166]), which introduced bail-in ([167]) and national resolution funds ([168]), and the Single Resolution Mechanism Regulation (SRMR), ([169]) which established the Single Resolution Board and the Single Resolution Fund (SRF).

The scenario comprises:

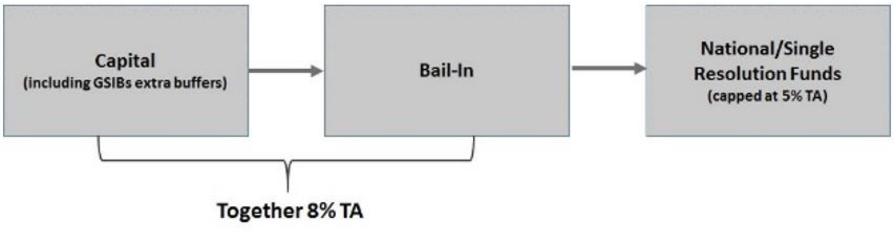
- Asset correlation is fixed to 50% (traditional SYMBOL assumption, compatible with default regulatory parameter);
- Bank total capital and initial risk-weighted assets (RWAs) taken directly from the banks' balance sheets.
- Current stocks of non-performing loans contribute to losses in the banking system of each country and their magnitude has been estimated as explained in the main text.
- Extra capital buffers for European Global Systematically Important Institutions (G-SIIs) prescribed by the Financial Stability Board (FSB) are considered.

([164]) See European Parliament and Council (2013).
([165]) Very few banks which are OSII are affected by extra buffer (not considered).
([166]) See European Parliament and Council (2014a).
([167]) A legal framework ensuring that part of the distressed banks' losses are absorbed by unsecured creditors. The bail-in tool entered into force on 01/01/2016.
([168]) Funds financed by banks to orderly resolve failing banks, avoiding contagion and other spill-overs.
([169]) See European Parliament and Council (2014b).

- Bail-in: modelled as a scenario whereby a Loss Absorbing Capacity (LAC) is built to represent, together with regulatory capital, 8% of TA.
- Resolution Funds - national (NRFs, for Member States not part of the Banking Union) and single (SRF, for Banking Union members) – completely phased-in and contributing to resolution absorbing losses up to 5% of the TA of the insolvent bank, provided that at least 8% LAC has already been called in. No backstop (other than public finances) nor ex-post contributions ([170]) are considered.
- No deposit guarantee scheme contribution or intervention is modelled.
- Extra losses generated by loans granted by the State are directly transferred to debt or deficit without passing through the safety net cascade.

Graph A6.1 illustrates the order of intervention of different tools. The first cushion assumed to absorb simulated losses is capital, the second tool is bail-in, and the last are RFs, as legally foreseen ([171]) .

Graph A6.1: **Implemented order of intervention of the safety net tools**



Source: Commission services.

3. Leftover financial needs after the safety net cascade

Throughout the cascade of safety net interventions, it can then be traced how much of each of these two types of financing needs are picked up by the different tools. If after depletion of capital, a bank is failing or left undercapitalised with respect to the minimum level established in the scenarios, the bail-in tool is applied at individual bank level up to 8% of its total liabilities and own funds (TLOF) (or total assets, TA). ([172]) If this is not enough, and a Resolution Fund (RF) is available, it is then assumed to intervene up to 5% of the total assets of each bank.[173] Given that the sample coverage in terms of the number and total assets of banks in the sample is not complete, the RF is assumed to have ex-ante funding equal to the appropriate percentage of covered deposits of the banks in the sample. Any leftover losses or recapitalisation needs not covered after all available tools have intervened are finally assumed to be covered by the government, taking into account the ratio between the total assets (TA) in the sample and the population of all banks.

([170]) Given the aim to portray worst-case fiscal consequences, ex-post contributions to the NRFs/SRF are not modelled, but these can actually go up to 3 times the ex-ante contributions, further reducing the impact on public finances.

([171]) Additional tools are available to absorb residual losses and recapitalisation needs, including additional bail-in liabilities, leftover resolution funds and the deposit guarantee scheme. See Benczur et al. (2015) for a discussion. In addition, by 2024 at the latest a common backstop to the SRF will be introduced.

([172]) The BRRD does not establish a harmonised level of liabilities eligible for bail-in, but Art. 44 sets out that the RF can kick in only after shareholders and holders of other eligible instruments have made a contribution to loss absorption and recapitalisation of at least 8% of total liabilities and own funds (TLOF). Since bank-level data on bail-inable liabilities is unavailable, the bail-in tool is modelled by imposing that individual banks hold a loss absorbing capacity of at least 8% of their TLOF. In practice banks with total capital under this threshold are assumed to meet the 8% minimum threshold via bail-inable liabilities. In the simulation, bail-in stops once the 8% of total assets limit has been reached. If a bank holds capital above 8% of TA, there would be no bail-in, but capital might be bearing losses above 8% of TLOF.

([173]) Art. 44 of the bank recovery and resolution directive sets out that the contribution of the resolution financing arrangement cannot exceed 5% of the total liabilities. In case of excess demand for SRF funds, funds are rationed in proportion to demand (i.e., proportionally to excess losses and recapitalisation needs after the minimum bail-in, capped at 5% of TA at bank level).

4. Calibrating the heat map for theoretical probability of public finances being hit by more than 3% of GDP, in the event of a severe crisis

The model allows estimating the probability distribution of the amount of public funds needed to cover losses after exhausting the protection provided by the financial safety net. To obtain the input for the heat map on government's implicit contingent liability risks, a minimum size of government's contingent liabilities is fixed, and the theoretical probability of the materialisation of the event is assessed.

Table A6.3 shows the heat map, which illustrates the relative riskiness of countries in terms of public finances being hit by at least a fixed share (3%, 5%, and 10%) of GDP, conditional on having (a) the banking sector in distress, (2) at least three countries with government's contingent liabilities. The colour coding reflects the relative magnitude of the theoretical probabilities of such event.

Table A6.3: **Model-based probabilities of public finances being hit by more than 3%, 5% or 10% of GDP, in the event of a severe crisis (i.e., involving excess losses and recapitalisation needs in at least three different EU Member States)**

	Stress (a)			Severe stress (b)		
	3% GDP	5% GDP	10% GDP	3% GDP	5% GDP	10% GDP
BE	0.02%	0.01%	0.00%	0.35%	0.13%	0.02%
BG	0.00%	0.00%	0.00%	0.04%	0.01%	0.00%
CZ	0.01%	0.00%	0.00%	0.12%	0.03%	0.00%
DK	0.08%	0.04%	0.01%	0.25%	0.13%	0.03%
DE	0.00%	0.00%	0.00%	0.09%	0.02%	0.00%
EE	0.01%	0.00%	0.00%	0.21%	0.06%	0.00%
IE	0.04%	0.01%	0.00%	0.53%	0.25%	0.06%
EL	0.07%	0.02%	0.00%	0.98%	0.43%	0.07%
ES	0.16%	0.05%	0.01%	1.32%	0.67%	0.16%
FR	0.03%	0.01%	0.00%	0.56%	0.28%	0.06%
HR	0.00%	0.00%	0.00%	0.04%	0.01%	0.00%
IT	0.02%	0.00%	0.00%	0.33%	0.10%	0.01%
CY	0.04%	0.01%	0.00%	0.57%	0.27%	0.06%
LV	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%
LT	0.00%	0.00%	0.00%	0.02%	0.00%	0.00%
LU	3.72%	1.75%	0.27%	11.88%	6.56%	2.03%
HU	0.03%	0.00%	0.00%	0.19%	0.04%	0.00%
MT	0.02%	0.01%	0.00%	0.23%	0.09%	0.02%
NL	0.05%	0.02%	0.00%	0.46%	0.24%	0.04%
AT	0.01%	0.00%	0.00%	0.15%	0.04%	0.00%
PL	0.00%	0.00%	0.00%	0.11%	0.02%	0.00%
PT	0.02%	0.01%	0.00%	0.37%	0.14%	0.02%
RO	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
SI	0.00%	0.00%	0.00%	0.08%	0.02%	0.00%
SK	0.01%	0.00%	0.00%	0.30%	0.05%	0.00%
FI	0.01%	0.01%	0.00%	0.16%	0.08%	0.02%
SE	0.01%	0.00%	0.00%	0.04%	0.01%	0.00%

(1) The scenarios for 2023 DSM and 2019 DSM estimate the potential excess losses and recapitalisation needs for 2024 and 2020 (in % of GDP 2022 and 2018) respectively. (2) In 2019 DSM, Greece was under enhanced surveillance and therefore was not included in the debt sustainability analysis. (3) The losses considered are the excess losses after the safety net (i.e., including bail-in and the resolution funds). (4) Green: low risk (model-based probability lower than 0.50%), Yellow: medium risk (model-based probability between 0.50% and 1%); Red: high risk (model-based probability higher than 1%). (5) We include the current results as well as the analysis from a pre-COVID period. The map is calibrated conditional on having (a) the banking sector in distress, and (b) at least three Member States with government's contingent liabilities. See the methodological annex for more details on the computation of the heatmap.
Source: Commission services.

Table A6.4: **Detailed scenarios description**

Components: / *Scenario:*	Asset correlation	Total regulatory capital	RWAs	Bail-in	National/ Single RF	Recapitalization	Extra losses due to NPLs	Deposit Guarantee Scheme	Banks in resolution
Stress scenario	50%	Total capital	RWA Adjusted	Yes Total capital plus bail-in 8% TA	Yes, 5% TA cap, after LAC of 8% has been called in full target No ex-post contributions	10.5% RWA Adjusted + Buffers	- Yes to all banks - NPL - Recovery rate as reported by World Bank	No	Random significant banks
Severe stress scenario	Depending on common factor	Total capital	RWA Adjusted	Yes Total capital plus bail-in 8% TA	Yes, 5% TA cap, after LAC of 8% has been called in full target No ex-post contributions	10.5% RWA Adjusted + Buffers	- Yes to all banks - NPL - Recovery rate follows a country specific	No	Random significant banks

(1) The size of the Single Resolution Fund was on Q2 2021 €52 billion (https://www.srb.europa.eu/en/content/single-resolution-fund#build-up) which is around 65% of its target size (i.e. 1% of deposits, around €80 billion)
Source: Commission services.

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