Estimating the Output Gap in Real Time: A Factor Model Approach

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- Factor model extracts a common component in the data.
  - Disregards an idiosyncratic component.
  - Revisions will have less impact (if they are measurement errors).

- Can handle a jagged edge in the data and incorporate new information on non-synchronized variables as they become available.
- End-of-sample problem is reduced through the favorable nowcasting properties of the model.
  - Further reduced by augmenting with further forecasts.
- We reduce the total errors of the real time gap to 25 percent of the standard approach.
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Factor model

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  - Main idea: The common, forecastable component of a large data set is captured by a few factors, $F$.

    \[
    X_t = \Lambda F_t + \xi_t, \quad \xi_t \sim i.i.d \, N(0, \Psi) \tag{1}
    \]

    \[
    F_t = A F_{t-1} + B u_t, \quad u_t \sim i.i.d \, N(0, I), \tag{2}
    \]

  where $t = 1, ..., T$. $\xi_t = (\xi_{1t}, \ldots, \xi_{nt})'$, is a vector of non-forecastable idiosyncratic components, $\Lambda$ is a $(n \times r)$ matrix of factor loadings and $r$ denotes the number of factors.
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- Estimate Eqs. (1) and (2) using a two-step procedure.
  1. Parameters are estimated by OLS using principal components on balanced part of data
  2. Factors are re-estimated by applying the Kalman filter and smoother to the entire data set
Bridge Equation

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- Factors represent quarterly quantities, \( \hat{F}_{q}^{\tau} \), where \( \tau = 3, 6, ..., T - 3, T \)
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2. Project quarterly GDP growth on the factors
   \[ \hat{\Delta}y_{\tau}^{q_0} = \alpha + \beta'\hat{F}_{\tau}^{q_0} \] (3)

   Transform the estimated GDP growth series to log levels, i.e.,
   \[ \hat{y}_{\tau}^{q_0} = y_0^{q_0} + \sum_{j=1}^{\tau/3} \hat{\Delta}y_{3\times j}^{q_0} \] (4)
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3. Obtain an estimate of the output gap by detrending the estimated log level series for GDP, \( \hat{y}_{\tau}^{q0} \) using the HP filter
Data set and Empirical exercise

- **Data**
  - 55 monthly real time indicators for US for period 1970M1-2006M10
    - Real time data from Philadelphia Fed, see Croushore and Stark (2001).
    - Financial variables, price indices.
    - Mostly similar to monthly variables in Bernanke and Boivin (2003).

- **Model selection**
  - All variables are transformed to induce stationarity.
  - Bai and Ng (2002, 2007) tests select q = 2 and r = 6 factors.

- **Exercise**
  - Calculate real time output gaps with and without a factor model.
  - Compare the gaps computed recursively on real time data up to the relevant point in time with a gap using the full sample of data.
  - Real time out-of-sample evaluation from 1984q1 to 2006q4.

Performance measured by relative MSFE.

Use the vintage of 2010Q3 as “Final vintage”.

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Benchmarks and Gaps

- **Benchmark**
  - \( \text{GAP}_{\text{FIN}} \): Ex-post benchmark
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  - $\text{GAP}_{\text{RT}}$: Real time gap
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- Total revisions = \( \text{GAP}_{\text{FIN}} - \text{GAP}_{\text{RT}} \)
- Data revisions = \( \text{GAP}_{\text{QRT}} - \text{GAP}_{\text{RT}} \)
**Factor model approach**

- \( \text{GAP}_{\text{RT-FM}} \): Estimated real time gap
  - Gap estimated with a factor model with real time data using real time info.

- \( \text{GAP}_{\text{QRT-FM}} \): Estimated quasi real time gap
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- **Data revisions** = $\text{GAP}_{\text{QRT-FM}} - \text{GAP}_{\text{RT-FM}}$
Output gaps with Factor model approach

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Real time Output gaps Standard vs Factor model

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Relative Mean Squared Errors

<table>
<thead>
<tr>
<th>Measure</th>
<th>Formula</th>
<th>$\lambda = 1600$</th>
<th>$\lambda = 400$</th>
<th>$\lambda = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True real-time performance</td>
<td>$\frac{\text{mean}((\text{GAP}<em>{\text{RT-FM}} - \text{GAP}</em>{\text{FIN}})^2)}{\text{mean}((\text{GAP}<em>{\text{RT}} - \text{GAP}</em>{\text{FIN}})^2)}$</td>
<td>0.59</td>
<td>0.69</td>
<td>0.64</td>
</tr>
<tr>
<td>Data revision performance</td>
<td>$\frac{\text{mean}((\text{GAP}<em>{\text{RT-FM}} - \text{GAP}</em>{\text{RT}})^2)}{\text{mean}((\text{GAP}<em>{\text{RT}} - \text{GAP}</em>{\text{FIN}})^2)}$</td>
<td>0.10</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>Quasi real-time performance</td>
<td>$\frac{\text{mean}((\text{GAP}<em>{\text{QRT-FM}} - \text{GAP}</em>{\text{FIN}})^2)}{\text{mean}((\text{GAP}<em>{\text{QRT}} - \text{GAP}</em>{\text{FIN}})^2)}$</td>
<td>0.63</td>
<td>0.76</td>
<td>0.72</td>
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Future data contains information about trend.

Add forecast from AR(1) to the data series when computing the cycle (as in Mise, Kim and Newbold (2005)).

### Relative Mean Squared Errors

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<td>0.42</td>
<td>0.55</td>
</tr>
<tr>
<td>$\text{GAP}_{\text{RT-FM-NOW-AR}}$</td>
<td>0.27</td>
<td>0.44</td>
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Real time output gaps including forecasts
Inflation forecasts based on real-time output gap estimates

Follow Stock and Watson (1999) and Orphanides and van Norden (2005) and specify the following Phillips curve regression:

$$\pi_{\tau+h}^4 = \alpha + \sum_{i=0}^{n} \beta_i \pi_{\tau-i}^4 + \sum_{i=0}^{m} \gamma_i \text{gap}_{\tau-i} + e_{\tau+h}$$

(5)

where $\pi_{\tau}^4$ denote inflation over 4 quarters ending in quarter $\tau$.

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<tr>
<td>GAP&lt;sub&gt;RT&lt;/sub&gt;</td>
<td>1.39</td>
<td>1.02</td>
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<tr>
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<td>1.12</td>
<td>0.94</td>
</tr>
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<td>GAP&lt;sub&gt;RT-FM-FOR&lt;/sub&gt;</td>
<td>0.90</td>
<td>0.80</td>
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Summary

- We found that a factor model can substantially improve the reliability of real-time output gap estimates through two mechanisms:
  - The data revision problem is considerably reduced as a factor model extract only the common component and disregards the idiosyncratic (noisy) component.
  - The end-of-sample problem is considerably reduced by combining a nowcast from a factor model with long term forecasts from an AR(1).

Newer alternative methods:
- Non-stationary factor model approach (Barigozza and Luciani (2021))
- Beveridge-Nelson decomposition based on a BVAR (Morley and Wong (2020) and Berger, Morley and Wong (2021))
- Suite of models approach (Barbarina et al. (2020), Furlanetto et al. (2020))
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